

VIBRATION ANALYSIS OF METALLIC THIN CYLINDERS – AN ANALYTICAL APPROACH

Abstract

Vibrations are inherent in dynamic mechanical systems. Suppression of vibrations is not an easy task. The amplitude of these vibrations can be minimized under external excitation or possibly transferred to another system. Determination of natural frequencies is paramount importance in design, so that these natural frequencies can be avoided during the operation of the element. Cylindrical shells are used for many applications, water tanks, heat exchangers, reactors, in chemical industries for distillation, smoke chimneys. These shells can be considered as thin shell depending on the ratio of diameter to the thickness. This paper evaluates the vibration characteristics of thin vibrating cylindrical shells made of metallic materials. To make a mathematical model, a thin long vertical cylinder is considered with one end fixed. The thin shell is assumed as a cantilever without fluid, a simple model was developed and frequencies are noted and this model is improved by attaching a mass at the free end, usually referred as tip mass. It is noted that there is a decrease in the fundamental frequencies. Further as the ratio of tip mass to the beam mass exceeds 1, there is a considerable decrease in the fundamental frequencies. Most commercial applications and also in industries, stain steel thin cylinder of thickness 2 mm and 3 mm were used, we considered the same for analysis.

Keywords: Thin Shells, Natural Frequency, Damping Ratio, Excitation, Vibrations, Frequency Response Function

Authors

P. Kiran Kumar

Department of Mechanical Engineering
Chaitanya Bharathi Institute of Technology
(A), Hyderabad, Telangana, India.
pkirankumar_mech@cbit.ac.in

Kiran Kumar Amireddy

Department of Mechanical Engineering
Chaitanya Bharathi Institute of Technology
(A), Hyderabad, Telangana, India.
amireddykiran@gmail.com

J.V. Subramanyam

Department of Mechanical Engineering
University College of Engineering (A)
Osmania University, Hyderabad
Telangana, India.

I. INTRODUCTION

Cylindrical structures are used in aerospace industry, aircrafts, spacecrafts and rockets; in automobiles, cars, buses and trucks as a fluid storage tanks; and others applications, computers, submarines, boats, storage tanks and the roofs of buildings [1-8]. In all the above cases, the shells are subjected to vibration or external excitation during the operation. Therefore, models should be developed that take into account the nonlinear effects, such as large structural deflections, and predictions of the structural responses to large-amplitude base excitations [9-20]. The study of the dynamic behaviour of thin shells used for storage tanks has gained significant attention in the recent years as the seismic vulnerability of these shells represents a potential source of economic loss due to structural failures [21-25]. The most common types of damage observed in nuclear reactors and fluid storage tanks are: damage to the piping connections caused by large base uplifts, damage to the roof caused by the sloshing of the free liquid surface, buckling of the tank walls caused by the high compressive stress, buckling of the tank legs caused by large axial loads coupled with lateral loads, failure of the anchorage system caused by the high overturning moment transmitted to the base, penetration of the tank wall with anchor bolts caused by the previous failure of the anchorage system and damage to the shell-base connection caused by the rotation of the base plate [26-30].



Figure 1: Shell structures damages due to earthquake

Several approaches have been proposed in this setting to model the dynamic behaviour of such shells.

II. BACKGROUND

Here in this paper, different metallic thin shells of different thickness are considered for the analysis. Two different materials, Aluminium and Stainless-steel shells of thickness 2 mm and 3 mm with and without the tip mass are considered for the analysis.

This paper is organised as explained below, it starts with the introduction followed by background, what is the importance of the vibration analysis and ways of finding the natural frequencies are explained. Then the methods, analytical approach to find the natural frequencies for the metallic shells with and without considering the cap or lid for different materials and different thickness are explained. Finally, results and discussion section are

presented and towards the end the paper is concluded with the consideration and recommendation of future scope.

III. METHODS

Mathematical Formulation Of Vibrating Thin Metallic Shells By Analytical Approach

To find the natural frequencies of the arrangement shown in figure 2 analytical expressions were developed. Mathematical equations describing the system as a cantilever shell is considered and is further refined by considering tip mass at its free end, enabling expressions for the different modes of vibrations, natural frequencies to be obtained.

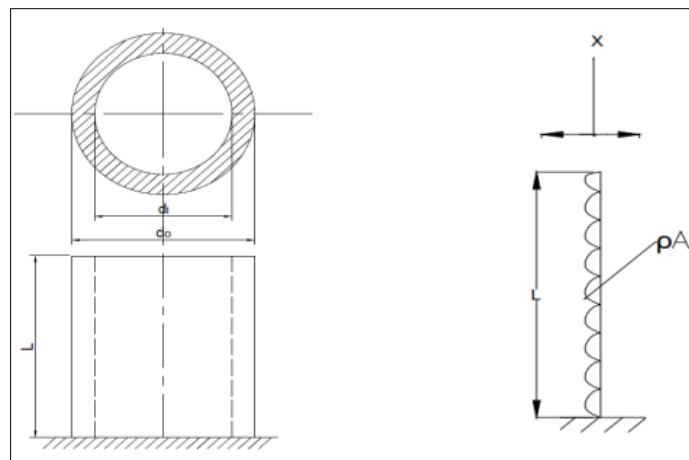


Figure 2: (a) cantilever shell (b) Mathematical model

MODEL-1: EMPTY SHELL

The system is taken as vibrating cantilever shell shown in figure 2(a) and the corresponding mathematical model is shown in figure 2(b) to find the natural frequencies. This is the simplest model.

Here,

- L = cylindrical shell length,
- d_o = cylindrical shell outer diameter,
- d_i = cylindrical shell inner diameter,
- h = shell thickness = $(d_o - d_i)/2$.

This is modelled as a cantilever executing transverse vibrations.

The equation of motion is written as

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$$\frac{EI\partial^4 y(x,t)}{\partial x^4} = -\frac{\rho A\partial^2 y(x,t)}{\partial t^2} \quad (1)$$

And this is separable as $y(x, t) = Y(x)e^{i\omega t}$

Therefore,

$$\frac{\partial^4 y(x)}{\partial x^4} - \frac{\rho A \omega^2 Y(x)}{EI} = 0 \quad (2)$$

$$\text{Let } \alpha^4 = \frac{\rho A \omega^2}{EI}$$

and the solution for

$$Y(x) = c_1 \cos \lambda x + c_2 \cos \lambda x + c_3 \cos \lambda x + c_4 \cos \lambda x \quad (3)$$

With boundary conditions

$$x = 0, y(0) = 0 \text{ and } \frac{dY(0)}{dx} = 0 \text{ at fixed end and}$$

$$x = L, \frac{d^2 Y(L)}{dx^2} = 0 \text{ and } \frac{d^3 Y(L)}{dx^3} = 0$$

At free end substituting these boundary conditions into equation (3) “Y” yields

$$\cos \alpha L \cdot \cosh \alpha L + 1 = 0 \quad (4)$$

The roots of this equation are,

$$\alpha L = 1.8751, 4.6941, 7.8547, \dots$$

Let $\alpha L = K$

$$\alpha^4 = \frac{K^4}{L^4} = \frac{\rho A \omega^2}{EI}$$

$$\omega^2 = \frac{K^4 EI}{\rho A L^4} \quad \omega = K^2 \sqrt{\frac{EI}{\rho A L^4}} \quad (5)$$

Natural frequency of the tube can be found when it is not filled in its water. As the tube is in vertical position and the base is moved in horizontal direction there is column effect. To consider this column effect, the effective length is taken. For one side is fixed and other end is free, and hence, the effective length is $2L$. The corresponding frequency equation is given below,

$$K^2 \sqrt{\frac{EI}{\rho A (2L)^4}}$$

Considering 2 mm thick tube,

For stainless steel 2 mm thick and 300 mm length,

$$E = 2.05 \times 10^{11} \text{ N/m}^2, I = 7.395 \times 10^{-7} \text{ m}^4, \rho = 7870 \text{ kg/m}^3$$

$$F = \frac{1.8751^2}{2\pi} \sqrt{\frac{EIL}{(\rho AL)(2L)^4}} = 223.695 \text{ Hz}$$

For Aluminium 2 mm thick tube

$$E = 0.72 \times 10^{11} \text{ N/m}^2, \rho = 2700 \text{ kg/m}^3$$

Similarly, the frequency is calculated and its value is 226.359 Hz

Stainless steel tube is considered for reference since these tubes are generally used compared to aluminium. The natural frequency of empty steel tube is found as 223.695 Hz. It is well known that the analytical value is in ideal environment. However, the model taken is refined considering lid as tip mass(m_r) in model-2.

MODEL-2: SHELL WITH TIP MASS AT ITS FREE END

Based on wang et al., [31], the rotary inertia is considered. As a lid is placed at the free end of the shell, it is taken as tip mas. The arrangement is shown in the following figure 3(a) shell with tip mass at its free end with the corresponding mathematical model in figure 3(b).

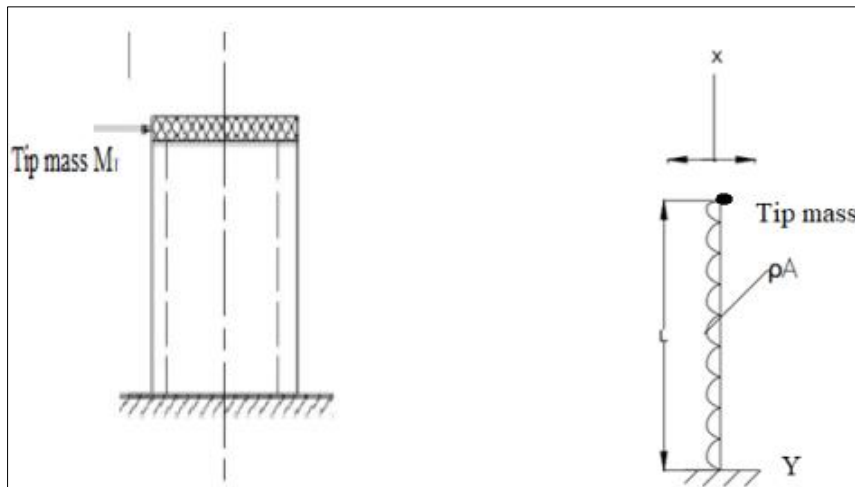


Figure 3: (a) Shell with tip mass and (b) mathematical model

The equation of motion is

$$\frac{EI \partial^4 y(x, t)}{\partial x^4} + \frac{\rho A \partial^2 y(x, t)}{\partial t^2} = 0$$

And it is separable

$$y(x, t) = \phi(x)Y(t)$$

$$\frac{d^2 Y(t)}{dt^2} + \omega^2 Y(t) = 0 \tag{6}$$

$$\frac{d^4\phi(x)}{dx^4} - \frac{\rho A}{EI} \omega^2 \phi(x) = 0 \quad (7)$$

let $\alpha^4 = \frac{\rho A}{EI} \omega^2$ and the solution

$$Y(t) = A_1 \sin(\omega t) + A_2 \cos(\omega t) \quad (8)$$

$$\phi(x) = C_1 \cos \alpha x + C_2 \sin \alpha x + C_3 \cosh \alpha x + C_4 \sinh \alpha x \quad (9)$$

The boundary conditions are

$$\text{deflection} = \phi(x) = 0$$

$$\text{Slope} = \frac{d\phi(x)}{dx} = 0$$

At free end

$$\text{Moment} = m(L) = EI \frac{d^2\phi(x)}{dx^2} \Big|_{x=L} + \omega^2 J \frac{d\phi(x)}{dx} \Big|_{x=L} = 0$$

$$\text{Shear force} = V(L) = \frac{EI d^3\phi(x)}{dx^3} \Big|_{x=L} + \omega^2 m_T \phi(x) = 0$$

Where,

$J = \text{mass moment of inertia of the Tip mass} = m_T L^2$

$m_T = \text{mass at the tip (or) tip mass}$

Substituting the boundary conditions in to the equation (9)

$$C_1 = -C_3, C_2 = -C_4$$

$$\phi(x) = C_1 (\cos \alpha x - \cosh \alpha x) + C_2 (\sin \alpha x - \sinh \alpha x)$$

With the boundary conditions at the free end the equation can be written as

$$EI \alpha^2 [C_1 (-\cos \alpha L - \sinh \alpha L) + C_2 (-\sin \alpha L - \cosh \alpha L)] + \omega^2 J_\alpha [C_1 (-\sin \alpha L - \sinh \alpha L) + C_2 (\cos \alpha L - \cosh \alpha L)] = 0$$

$$EI \alpha^3 [C_1 (-\sin \alpha L - \sinh \alpha L) + C_2 (-\cos \alpha L - \cosh \alpha L)] + \omega^2 m_T [C_1 (-\cos \alpha L - \cosh \alpha L) + C_2 (\sin \alpha L - \sinh \alpha L)] = 0$$

$$-C_1 [EI \alpha^2 (\cos \alpha L + \cosh \alpha L) + \omega^2 J_\alpha (\sin \alpha L + \sinh \alpha L)] - C_2 [EI \alpha^2 (\sin \alpha L + \sinh \alpha L) - \omega^2 m_T (\cos \alpha L - \cosh \alpha L)] - C_2 [EI \alpha^3 (\cos \alpha L + \cosh \alpha L) - \omega^2 m_T (\sin \alpha L - \sinh \alpha L)] = 0$$

$$\text{Since } \alpha^4 = \frac{\omega^2 \rho A}{EI}; \frac{\omega^2}{EI} = \frac{\alpha^4}{\rho A} = \frac{\alpha^4 L}{\rho A L} = \frac{\alpha^4 L}{m_b}$$

{where m_b is the mass of the beam}

$$C_1[(\cos \alpha L + \cosh \alpha L) + \frac{JL}{m_b} \alpha^3 (\sin \alpha L + \sinh \alpha L)] + C_2[(\sin \alpha L + \sinh \alpha L) - \frac{JL}{m_b} \alpha^3 (\cos \alpha L - \cosh \alpha L)] = 0$$

$$C_1[(\sin \alpha L - \sinh \alpha L) + \frac{m_T}{m_b} L \alpha (\cos \alpha L - \cosh \alpha L)] + C_2 [-(\cos \alpha L + \cosh \alpha L) + \frac{m_T L \alpha}{m_b} (\sin \alpha L - \sinh \alpha L)] = 0$$

Further put αL as x and where $\frac{J}{m_b L^2} = a$ and $\frac{m_T}{m_b} = b$.

Rewriting the equation as

$$C_1[(\cos x + \cosh x) + ax^3 (\sin x + \sinh x)] + C_2[(\sin x + \sinh x) - ax^3 (\cos x - \cosh x)] = 0$$

$$C_1[(\sin x - \sinh x) + bx(\cos x - \cosh x)] + C_2[-(\cos x + \cosh x) + bx(\sin x - \sinh x)] = 0$$

This can be arranged as $[A] \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = 0$

Therefore, the determinant of the A is made equal to zero to obtain the frequency equation.

$$(\sin x \cosh x + \cos x \sinh x) - abx^4(1 - \cos x \cosh x) = 0 \quad (10)$$

If there is no tip mass a & b will become zero and the first term of the equation only exists and this is classical solution for a cantilever. If $a=0$ then the equation is same as obtained by Laura et al [32]

$$m_b = \text{beam mass} = \rho AL = 1.45 \text{ Kg}$$

$$m_T = \text{mass at the end due to the cap} = \frac{\pi}{4} d_{cap}^2 \rho$$

$J =$ mass moment of inertia of the beam and mass moment of inertia of tip mass

$$= \frac{m_b L^2}{3} + m_T L^2 = 0.199$$

$$a = \frac{J}{m_b L^2} = 0.0823; b = \frac{m_T}{m_b} = 0.298$$

$$a.b = 0.0245$$

Substituting these constants into the frequency equation the values for “ x ” are found as 1.6144, 4.98.

The frequencies for 2 mm Stainless Steel shell are 189.8, 1802.82Hz and for Aluminium 2 mm shell 192.18, 1824.82 Hz. These values are much less than the values obtained by the model-1 without considering the tip mass.

It is noted that unless the tip mass is considered the natural frequencies of shells are deviating from the experimental values. With the tip mass the values are much closer.

IV. RESULTS AND DISCUSSION

Natural frequencies of the vibrating thin metallic empty shells with and without tip mass are obtained using an analytical method. The frequencies of different materials with different thickness are given in below table 1

Table 1: Natural frequencies of different thin metallic shells

Material	Thickness	Natural frequency (in Hz)
Without considering the Tip mass		
Aluminium	2 mm	278.1
Aluminium	3 mm	378.1
Stainless steel	2 mm	274.9
Stainless steel	3 mm	272.2
With considering the Tip mass		
Aluminium	2 mm	192.1
Aluminium	3 mm	194.4
Stainless steel	2 mm	189.8
Stainless steel	3 mm	192.1

With the addition of tip mass, the natural frequencies of the shells are come down significantly for both the materials Aluminium and SS steel. As general practice, the tip mass compared to the beam mass should be higher i.e., $\frac{m_T}{m_b} \geq 1$.

V. CONCLUSION

The natural frequencies of the thin metallic shells are obtained with analytically and compared with two different materials and two different thickness. It was observed that with the addition of tip mass the natural frequencies of the shells are reduced. And also, with the increase of the thickness the natural frequencies are increased. The natural frequencies of both Aluminium and SS steel are almost same because of the product EI values are almost same.

VI. CONFLICT OF INTEREST

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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