ON SOME TYPICAL KIND OF CONTINUITY IN SOFT TOPOLOGICAL SPACE

Abstract

In this paper we have made an attempt to make results on some typical kind of continuous functions of Soft J Open and Soft J Closed sets in Soft Topological spaces. This study also describes the characterization of continuity with reference to our Soft J Open sets in Soft Topological Spaces.

Keywords: Soft J Open set, Soft J Continuous Functions, Soft Strongly J Continuous Functions and Soft Perfectly J Continuous Functions.

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I. INTRODUCTION

Soft set hypothesis was proposed by Molodtsov [4] in 1999 to manage vulnerability in a parametric way. A Soft set is a defined group of sets, instinctively Soft on the grounds that the limit of the set relies upon the boundaries. One idea of a set is the idea of dubiousness.Molodtsov [6] proposed Soft set as a totally nonexclusive numerical instrument for displaying vulnerabilities. There is no restricted condition to the depiction of articles.One of the critical benefits of soft topological spaces lies in their capacity to deal with complex frameworks with deficient or problematic information. They can display questionable conditions, rough thinking, and manage fractional data in a more regular and natural way contrasted with customary topological spaces.

II. PRELIMINARIES

Definition 2.1: [6] A Soft set F_A on the universe X is defined by the set of ordered pairs F_A={(x,f_a (x)):x∈E and f_a (x)∈P(X)}, where f_a:A→P(X) such that f_a (x)=ϕ for all x∉A. Hence f_a is called an approximate function of the Soft set F_A. The value of f_a may be arbitrary, some of them may be empty, some may have non empty intersection.

Definition 2.2: [2]

1. A Soft set (F,A) over X is called as aNull Soft Set denoted by F_ϕ or ϕ˜ if for all e∈A, F(e)=ϕ.
2. A Soft set (F,E) over X is called as an Absolute Soft Set denoted by F _X or X˜ if for all e∈A, F(e)=X.

Definition 2.3: [6] Supposeτ be a collection of Soft sets over X with a fixed set E of parameters. Then, τ is called a Soft topologyon X if

1. ϕ˜,X˜ belongs to τ.
2. The artbitrary union of Soft sets in τis again inτ.
3. The finite intersection Soft sets in τis again inτ.

The term(X,τ,E) is called Soft topological Space over X. The members of τ are called Soft opensets in X and complements of them are called Soft closedsets in X.

Definition 2.4: [3] A Soft set (W,E) of a Soft topological space(X,τ,E) is known as a Soft J Closed set if s^* Cl(W,E)⊆˜Int(V,E) when (W,E)⊆˜(V,E) and (V,E) is Soft g^ -open. SJC(X,τ,E) stands for the set of all Soft J closed sets.

Definition 2.5.[3] A Soft set(Q,E) of a Soft topological space (X,τ,E) is known as a Soft J Open set if its complement is a Soft J closed set. SJO(X,τ,E) stands for the set of all Soft J open sets.

Definition 2.6.[1,5]A map f:(X,τ,E)→(Y,σ,K) is called a

1. **Soft continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft open in (X,τ,E).
2. **Soft semi-continuous** if the inverse imageof every Soft open set in (Y,σ,K) is Soft semi-open in (X,τ,E).

3. **Soft pre-continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft pre-open in (X,τ,E).
4. **Soft α-continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft α-open in (X,τ,E).
5. **Soft β-continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft β-open in (X,τ,E).
6. **Soft g continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft g open in (X,τ,E).
7. **Soft sg continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft sg-open in (X,τ,E).
8. **Soft gs continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft gs-open in (X,τ,E).
9. **Soft gp continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft gp-open in (X,τ,E).
10. **Soft gpr continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft pre-open in (X,τ,E).
11. **Soft αg-continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft αg-open in (X,τ,E).
12. **Soft gα-continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft gα-open in (X,τ,E).
13. **Soft gˆ-continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft gˆ-open in (X,τ,E).
14. **Soft JP continuous** if the inverse image of every Soft open set in (Y,σ,K) is Soft JP open in (X,τ,E).

Result 2.7.[6]

1. Each one of the Soft semi closed set remains Soft J closed.
2. Each one of the Soft closed set remains Soft J closed.
3. Each one of the Soft α-closed set remains Soft J Closed.
4. Each one of the Soft open set remains Soft J open.
5. Each one of the Soft semi-open set remains Soft J open.
6. Each one of the Soft α-open set remains Soft J open.
7. Each one of the Soft J open set remains Soft gs-open.

III. SOFT TOTALLY J CONTINUOUS FUNCTIONS

Definition 3.1: A map f:(X,τ,E)→(Y,σ,K) is known asSoft totally J continuous if the inverse-image of each one of the Soft open set in (Y,σ,K) is both Soft J closed and Soft J open (i.e Soft J clopen) in (X,τ,E).

Theorem 3.2: Each one of the Soft perfectly J continuous map is Soft totally J continuous.
Proof: Let $f:(X,\tau,E)\rightarrow(Y,\sigma,K)$ be Soft perfectly J continuous and (U,K) be a Soft open set in (Y,σ,K). Thereon (U,K) is Soft J open in (Y,σ,K). Since f is Soft Perfectly J continuous, $f^{-1}(U,K)$ is Soft clopen in (X,τ,E). By Result 2.7, f is Soft totally J continuous.

Remark 3.3: It is observed from the subsequent illustration that the reverse implication of the above theorem is incorrect.

Example 3.4: Let $X = \{x_1,x_2\}, Y = \{y_1,y_2\}, E=\{e_1,e_2\}, K = \{k_1,k_2\}$. Define $m:X\rightarrow Y$ and $n:E\rightarrow K$ as $m(x_1)=y_1, m(x_2)=y_2$ and $n(e_1)=k_1, n(e_2)=k_2$. Consider the Soft topologies $\tau=\{\tilde{\phi},\tilde{X},(M_1,E),(M_2,E)\}$ where (M_1,E) and (M_2,E) are described this way: $M_1(e_1)=\{x_1\}, M_1(e_2)=\phi, M_2(e_1)=\{x_1\}, M_2(e_2)=\{x_2\}$ and $\sigma=\{\tilde{\phi},\tilde{Y},(N_1,K),(N_2,K)\}$ where (N_1,K) and (N_2,K) are described this way: $N_1(k_1)=\phi, N_1(k_2)=\{y_1\}$ and $N_2(k_1)=\{y_1\}, N_2(k_2)=\{y_1\}$. Precisely the mapping $f:(X,\tau,E)\rightarrow(Y,\sigma,K)$ is Soft totally J continuous. The Soft set (H,K) defined as $H(k_1)=\phi, H(k_2)=\{y_1\}$ is a Soft J open set in (Y,σ,K). But $f^{-1}(H,K)=\{(e_1,\phi),(e_2,x_1)\}$ is not Soft clopen in (X,τ,E). Hence f is not Soft perfectly J continuous.

Theorem 3.5: Each one of the Soft totally J continuous map is Soft J continuous.

Proof: Let $f:(X,\tau,E)\rightarrow(Y,\sigma,K)$ is Soft totally J continuous and (A,K) be Soft open set in (Y,σ,K). Since f is Soft perfectly J continuous, $f^{-1}(A,K)$ is Soft clopen in (X,τ,E). Then $f^{-1}(A,K)$ is Soft J clopen in (X,τ,E). Thereupon f is Soft totally J continuous.

Remark 3.6: It is observed from the subsequent illustration that the reverse implication of the above theorem is incorrect.

Example 3.7: Let $X = \{x_1,x_2\}, Y = \{y_1,y_2\}, E=\{e_1,e_2\}, K = \{k_1,k_2\}$. Define $p:X\rightarrow Y$ and $q:E\rightarrow K$ as $p(x_1)=y_1, p(x_2)=y_2$ and $q(e_1)=k_1, q(e_2)=k_2$. Consider the Soft topologies $\tau=\{\tilde{\phi},\tilde{X},(F_1,E),(F_2,E)\}$ where (F_1,E) and (F_2,E) are described this way: $F_1(e_1)=\{x_2\}, F_1(e_2)=\{x_1\}, F_2(e_1)=\{x_2\}, F_2(e_2)=\{x_1,x_2\}$ and $\sigma=\{\tilde{\phi},\tilde{Y},(H,K)\}$ where (H,K) is described this way: $H(k_1)=\phi, H(k_2)=\{y_1,y_2\}$. Let $g:(X,\tau,E)\rightarrow(Y,\sigma,K)$ be a Soft mapping. Precisely g is Soft J continuous but not Soft totally J continuous, because $g^{-1}(H,K)=\{(e_1,x_1,x_2),(e_2,\phi)\}$ is not Soft J clopen in (X,τ,E).

Theorem 3.8: Each one of the Soft totally J continuous map is Soft JA continuous.

Proof:f: $(X,\tau,E)\rightarrow(Y,\sigma,K)$ is Soft J continuous. Let (A,K) be a Soft closed set in (Y,σ,K). Then $f^{-1}(A,K)$ is Soft J closed in (X,τ,E). Also, $f^{-1}(A,K)$ is Soft JA closed in (X,τ,E). Thus f is Soft JA continuous.

IV. SOFT CONTRA J CONTINUOUS FUNCTIONS

Definition 4.1: A map $f:(X,\tau,E)\rightarrow(Y,\sigma,K)$ is known asSoft Contra J continuousif the inverse-image of each one of the Soft open set in (Y,σ,K) is Soft J closed in (X,τ,E).

Example 4.2: Let $X = \{x_1,x_2\}, Y = \{y_1,y_2\}, E=\{e_1,e_2\}, K = \{k_1,k_2\}$. Define $p:X\rightarrow Y$ and $q:E\rightarrow K$ as $p(x_1)=y_1, p(x_2)=y_2$ and $q(e_1)=k_1, q(e_2)=k_2$. Consider the

Soft topologies $\tau=\{\tilde{\phi},\tilde{X},(F_1,E),(F_2,E)\}$ where (F_1,E) and $(F_{(.2)},E)$ are defined as $F_1(e_1)=\{x_2\},F_1(e_2)=\{x_1\},F_2(e_1)=\{x_2\},F_2(e_2)=\{x_1,x_2\}$ and $\sigma=\{\tilde{\phi},\tilde{Y},(H,K)\}$ where (H,K) is described this way: $H(k_1)=\{y_1,y_2\},H(k_2)=\phi$. Let $g:(X,\tau,E)\to(Y,\sigma,K)$ be a Soft mapping. Precisely g is Soft contra J continuous.

Proposition 4.3: If $f:(X,\tau,E)\to(Y,\sigma,K)$ is Soft contra semi continuous then it is Soft contra J continuous.

Proof: It is verified by Result 2.7, that each one of the Soft semi closed set is Soft J closed in (X,τ,E).

Proposition 4.4: If $f:(X,\tau,E)\to(Y,\sigma,K)$ is Soft contra continuous then it is Soft contra J continuous.

Proof: It is verified by Result 2.7, that each one of the Soft closed set is Soft J closed in (X,τ,E).

Proposition 4.5: If $f:(X,\tau,E)\to(Y,\sigma,K)$ is Soft contra α-continuous then it is Soft contra J continuous.

Proof: It is proved by Result 2.7, that each one of the Soft α-closed set is Soft J closed in (X,τ,E).

Result 4.6: It is observed from the subsequent illustration that the reverse implication of the above propositions 4.3, 4.4, 4.5 are incorrect.

Example 4.7: Consider the Soft open set (H,K) in Example 4.2. Here, $g^{-1}(H,K)=\{(e_1,x_1,x_2),(e_2,\phi)\}$ is not Softsemi-closed (Soft closed, Soft α-closed) in (Y,σ,K). Hence $g:(X,\tau,E)\to(Y,\sigma,K)$ is Soft contra J continuous but not Soft contra semi continuous (Soft contra continuous, Soft contra α-continuous).

Remark 4.8: Soft J continuity and Soft contra J continuity are independent. It is observed from the subsequent illustration.

Example 4.9:

1. Let $X=\{x_1,x_2\},Y=\{y_1,y_2\},E=\{e_1,e_2\},K=\{k_1,k_2\}$. Define $u:X\to Y$ and $v:E\to K$ as $u(x_1)=y_1,u(x_2)=y_2$ and $v(e_1)=k_1,v(e_2)=k_2$. Consider the Soft topologies $\tau=\{\tilde{\phi},\tilde{X},(F_1,E),(F_2,E)\}$ where (F_1,E) and (F_2,E) are described this way: $F_1(e_1)=x_2,F_1(e_2)=\{x_2\},F_2(e_1)=\{x_1,x_2\},F_2(e_2)=\{x_2\}$ and $\sigma=\{\tilde{\phi},\tilde{Y},(H_1,K),(H_2,K),(H_3,K)\}$ where $(H_1,K),(H_2,K)$ and (H_3,K) are described this way: $H_1(k_1)=\{y_1\},H_1(k_2)=\phi,H_2(k_1)=\phi,H_2(k_2)=\{y_2\},H_3(k_1)=\{y_1\},H_3(k_2)=\{y_2\}$. Precisely the mapping $f:(X,\tau,E)\to(Y,\sigma,K)$ is Soft J continuous. The inverse-image of the Soft open set (H_2,K), $f^{-1}(H_2,K)=\{(e_1,\phi),(e_2,x_2)\}$ is not a Soft J closed set in (X,τ,E). Hence f is not Soft contra J continuous.

2. Let $X = \{x_1,x_2\}$, $Y = \{y_1,y_2\}$, $E=\{e_1,e_2\}$, $K = \{k_1,k_2\}$. Define $p:X\to Y$ and $q:E\to K$ as $p(x_1)=y_1$, $p(x_2)=y_2$ and $q(e_1)=k_1$, $q(e_2)=k_2$. Consider the Soft topologies $\tau=\{\tilde{\phi},\tilde{X},(M_1,E),(M_2,E)\}$ where (M_1,E) and (M_2,E) are defined as $M_1(e_1)=x_2$, $M_1(e_2)=\{x_2\}$, $M_2(e_1)=\{x_1,x_2\}$, $M_2(e_2)=\{x_2\}$ and $\sigma=\{\tilde{\phi},\tilde{Y},(H,K)\}$ where (H,K) is described this way: $H(k_1)=\{y_1,y_2\}$, $H(k_2)=y_1$. Let $g:(X,\tau,E)\to(Y,\sigma,K)$ be a Soft mapping. Precisely g is Soft contra J continuous. The Soft set (R,K) defined as $R(k_1)=\phi$, $R(k_2)=y_2$ is Soft closed in (Y,σ,K) but its inverse-image $g^{-1}(R,K)=\{(e_1,\phi),(e_2,x_2)\}$ is not Soft J closed in (X,τ,E). Hence g is not Soft J continuous.

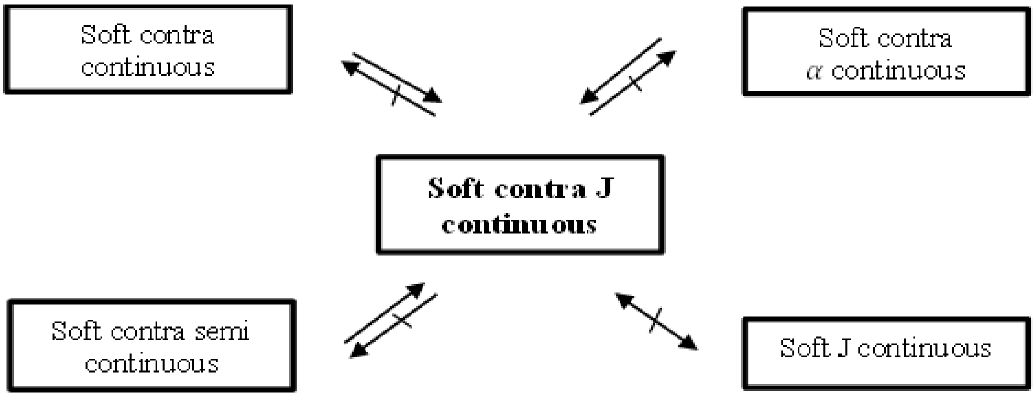

Figure 4.1. Relationship between Soft contra J continuous function and some existing Soft continuous functions

Lemma 4.10: The following properties hold for the Soft subsets $(A,E),(B,E)$ of a space (X,τ,E).
1. $(A,E)\tilde{\subset}SKer(A,E)$ and $(A,E)=SKer(A,E)$ if (A,E) is Soft open in (X,τ,E).
2. $(A,E)\tilde{\subset}(B,E)$ then $SKer(A,E)\tilde{\subset}SKer(B,E)$.

Proof: The proof is obvious.

Theorem 4.11: For a Soft mapping $f:(X,\tau,E)\to(Y,\sigma,K)$ the subsequent properties are equivalent. Assume that $SJO(X,\tau,E)$ is closed under any union and $SJC(X,\tau,E)$ is closed under any intersection.
1. f is Soft contra J continuous.
2. The inverse-image of a Soft closed set (B,K) of (Y,σ,K) is Soft J open.
3. $f(SJCl(A,E))\tilde{\subset}SKer\, f((A,E))$ for each one of the Soft subset of (A,E) of (X,τ,E).
4. $SJCl(f^{-1}(C,K))\tilde{\subset}f^{-1}(SKer(C,K))$ for each one of the subset (C,K) of (Y,σ,K).

Proof:
1. (1)⇒(2): It is evident.
2. (2)⇒(3): Let (A,E) be any Soft subset of (X,τ,E). Suppose $(y,e)\tilde{\notin}SKer(f(A,E))$. Then by

lemma 4.10, there exists (C,K) such that $f(A,E)\tilde{\cap}(C,K)=\tilde{\phi}$. Thus $(A,E)\tilde{\cap}f^{-1}(C,K)=\tilde{\phi}$ and $SJCl(A,E)\tilde{\cap}f^{-1}(C,K)=\tilde{\phi}$. Therefore, $f(SJCl(A,E))\tilde{\cap}(C,K)=\tilde{\phi}$ and $(y,e)\tilde{\notin}f(SJCl(A,E)$. Thereon $f(SJCl(A,E))\tilde{\subset}SKer\, f((A,E))$ for every Soft subset of (A,E) of (X,τ,E).

(3)⇒(4): Let (C,K) be any Soft subset of (Y,σ,K). Then by (3) and Lemma 4.10, $f(SJCl(f^{-1}(C,K)))\tilde{\subset} SKer(f(f^{-1}(C,K)))\tilde{\subset} SKer(C,K)$ and then $SJCl(f^{-1}(C,K))\tilde{\subset} f^{-1}(SKer(C,K))$.

(4)⇒(1): Let (V,K) be any Soft open subset of (Y,σ,K). Therefore, by hypothesis and by Lemma 4.10, $SJCl(f^{-1}(V,K))\tilde{\subset} f^{-1}(SKer(V,K))= f^{-1}(V,K)$. So, $SJCl(f^{-1}(V,K))=f^{-1}(V,K)$. This reveals that $f^{-1}(V,K)$ is Soft J closed in (X,τ,E). Hence f is Soft contra J continuous.

Result 4.12: The composition of two Soft contra J continuous functions need not be Soft contra J continuous and it is observed from the subsequent illustration.

Example 4.13: Let $X = \{x_1,x_2\}$, $Y = \{y_1,y_2\}$, $Z= \{z_1,z_2\}$, $E=\{e_1,e_2\}$, $K = \{k_1,k_2\}$ and $R= \{r_1,r_2\}$. Define $s:X\to Y$, $t:Y\to Z$ and $p:E\to K$, $q:K\to R$ as $s(x_1)=y_1$, $s(x_2)=y_2$, $t(y_1)=z_1$, $t(y_2)=z_2$ and $p(e_1)=k_1$, $p(e_2)=k_2$, $q(k_1)=r_1$, $q(k_2)=r_2$. Consider the Soft topologies $\tau=\{\tilde{\phi},\tilde{X},(F_1,E),(F_2,E)\}$ where (F_1,E) and (F_2,E) are described this way: $F_1(e_1)=\{x_2\}$, $F_1(e_2)=\{x_1\}$, $F_2(e_1)=\{x_2\}$, $F_2(e_2)=\{x_1,x_2\}$, $\sigma=\{\tilde{\phi},\tilde{Y},(H,K)\}$ where (H,K) is described this way: $H(k_1)=\{y_1,y_2\}$, $H(k_2)=\phi$ and $\eta=\{\tilde{\phi},\tilde{Z},(M_1,R),(M_2,R)\}$ where (M_1,R) and (M_2,R) are described this way: $M_1(r_1)=z_2$, $M_1(r_2)=\{z_2\}$, $M_2(r_1)=\{z_1,z_2\}$, $M_2(r_2)=\{z_2\}$. Let $f:(X,\tau,E)\to(Y,\sigma,K)$ and $g:(Y,\sigma,K)\to(Z,\eta,R)$ be two Soft mappings. Precisely f and g are Soft contra J continuous but their composition is not Soft contra J continuous because $(g\circ f)^{-1}(H,R)= \{(e_1,x_2),(e_2,x_2)\}$ is not Soft J closed in (X,τ,E).

Proposition 4.14: If $f:(X,\tau,E)\to(Y,\sigma,K)$ is Soft J irresolute and $g:(Y,\sigma,K)\to(Z,\eta,R)$ is Soft contra J continuous then their composition $g\circ f:(X,\tau,E)\to(Z,\eta,R)$ is Soft contra J continuous.

Proof: Let (Q,R) be a Soft closed set in (Z,η,R). Since g is Soft contra J continuous $g^{-1}(Q,R)$ is Soft J open in (Y,σ,K). Because f is Soft J irresolute, $f^{-1}(g^{-1}(Q,R))= (g\circ f)^{-1}(Q,R)$ is Soft J open in (X,τ,E). So $g\circ f$ is Soft contra J continuous.

Proposition 4.15: If $f:(X,\tau,E)\to(Y,\sigma,K)$ is Soft contra J continuous and $g:(Y,\sigma,K)\to(Z,\eta,R)$ is Soft continuous then their composition $g\circ f:(X,\tau,E)\to(Z,\eta,R)$ is Soft contra J continuous.

Proof: Let (P,R) be a Soft closed set in (Z,η,R). Since g is Soft continuous $g^{-1}(P,R)$ is Soft closed in (Y,σ,K). Since f is Soft contra J continuous $f^{-1}(g^{-1}(P,R))= (g\circ f)^{-1}(P,R)$ is Soft J open in (X,τ,E). Thus $g\circ f$ is Soft contra J continuous.

Proposition 4.16: If $f:(X,\tau,E)\to(Y,\sigma,K)$ is Soft contra semi continuous and $g:(Y,\sigma,K)\to(Z,\eta,R)$ is Soft continuous then their composition $g\circ f:(X,\tau,E)\to(Z,\eta,R)$ is Soft contra J continuous.

Proof: Let (F,R) be a Soft closed set in (Z,η,R). Since g is Soft continuous $g^{-1}(F,R)$ is Soft closed in (Y,σ,K). Since f is Soft contra semi continuous $f^{-1}(g^{-1}(F,R))= (g\circ f)^{-1}(F,R)$ is Soft semi open in (X,τ,E). By Result 2.7, $(g\circ f)^{-1}(F,R)$ is Soft J open in (X,τ,E). Thus $g\circ f$ is Soft contra J continuous.

Proposition 4.17: If f:(X,τ,E)→(Y,σ,K) is Soft contra α-continuous and g:(Y,σ,K)→(Z,η,R) is Soft continuous then their composition g∘f:(X,τ,E)→(Z,η,R) is Soft contra J continuous.

Proof: Let (S,R) be a Soft closed set in (Z,η,R). Since g is Soft continuousg^(-1) (P,R) is Soft closed in (Y,σ,K). Since f is Soft contra α-continuous, f^(-1) (g^(-1) (P,R))= 〖(g∘f)〗 ^(-1) (P,R) is Soft α-open in (X,τ,E). By Result 2.7, 〖(g∘f)〗 ^(-1) (P,R) is Soft J open in (X,τ,E). Thus g∘f is Soft contra J continuous.

Theorem 4.18: Let {(X_i,τ,E)⁄i∈I} be any family of Soft topological spaces. If f:(X,τ,E)→Q(X_i,τ,E) is Soft contra J continuous then Q_i f:(X,τ,E)→(X_i,τ,E) is Soft contra J continuous for each i∈I, where Q_i is the Soft projection of Q(X_i,τ,E) onto (X_i, τ ,E).

Proof: It has been verified by the combination of facts that Soft projection is continuous.

Theorem 4.19: If f:(X,τ,E)→(Y,σ,K) is a Soft surjective J open map and g:(Y,σ,K)→(Z,η,R) is a map such that their composition g∘f:(X,τ,E)→(Z,η,R) is Soft contra J continuous, then g is Soft contra J continuous.

Proof: Let (H,R) be a Soft closed set in (Z,η,R). Since g∘f is a Soft contra J continuous, 〖(g∘ f)〗 ^(-1) (H,R)=f^(-1) (g^(-1) (H,R)) is Soft J open in (X,τ,E). Because f is Soft surjective and Soft J open, f(〖(g∘f)〗 ^(-1) (H,R))=(g^(-1) (H,R)) is Soft J open in (Y,σ,K). Thereon g is Soft contra J continuous.

Theorem 4.20: If f:(X,τ,E)→(Y,σ,K) is Soft contra J continuous and (Y,σ,K) is Soft regular then f is Soft J continuous.

Proof: Let (x,e) be an arbitrary Soft point of X ̃ and (F,K) be a Soft open set of (Y,σ,K) containing f((x,e)). Since (Y,σ,K) is Soft regular, there exists a Soft open set (V,K) in (Y,σ,K) containing f((x,e)) such that Cl(V,K)⊂ ̃(F,K). Now, Cl(V,K) is a Soft closed set in (Y,σ,K) containing f((x,e)) and f is Soft contra J continuous. Therefore, by Theorem 4.11 there exists (U,E)∈ ̃SJPO(X,τ,E) such that f(V,K)⊂ ̃Cl(V,K). Then f(U,E)⊂ ̃(F,K). Hence f is Soft J continuous.

Proposition 4.21: If f:(X,τ,E)→(Y,σ,K) is contra Soft semi continuous and g:(Y,σ,K)→(Z,η,R) is Soft contra continuousg∘f:(X,τ,E)→(Z,η,R) is Soft J continuous.

Proof: Let (H,R) be a any Soft closed set in (Z,η,R). Since g is Soft contra continuous, g^(-1) (H,R) is Soft open in (Y,σ,K). Since f is contra Soft semi continuousf^(-1) (g^(-1) (H,R)) is Soft semi closed set in (X,τ,E). Because each one of the Soft semi closed set is Soft J closed, f^(-1) (g^(-1) (H,R)) is Soft J closed set in (X,τ,E). Thus g∘f is Soft J continuous.

Theorem 4.22: If f:(X,τ,E)→(Y,σ,K) and g:(Y,σ,K)→(Z,η,R) be any two maps then

1. g∘f:(X,τ,E)→(Z,η,R) is Soft J irresolute if both f and g are Soft J irresolute.
2. g∘f:(X,τ,E)→(Z,η,R) is Soft J continuous if f is Soft J irresolute and g is Soft J continuous.

Proof:

1. Let (V,R) be a Soft J closed set in (Z,η,R). Since g is Soft J irresolute, g^(-1) (V,R) is Soft J closed in (Y,σ,K). Because f is Soft J irresolutef^(-1) (g^(-1) (V,R))=(g∘f)^(-1) (V,R) is Soft J closed in (X,τ,E). So, g∘f is Soft J irresolute.

2. Let (V,R) be a Soft closed set in (Z,η,R). Since g is Soft J continuousg^(-1) (V,R) is Soft J closed in (Y,σ,K). Since f is Soft J irresolutef^(-1) (g^(-1) (V,R))=(g∘f)^(-1) (H,R) is Soft J closed in (X,τ,E). So, g∘f is Soft J continuous.

Theorem 4.23: If f:(X,τ,E)→(Y,σ,K) and g:(Y,σ,K)→(Z,η,R) be any two Soft maps then

1. g∘f:(X,τ,E)→(Z,η,R) is Soft continuous if f is Soft strongly J continuous and g are Soft J continuous.
2. g∘f:(X,τ,E)→(Z,η,R) is Soft strongly J continuous if both f and g are Soft strongly J continuous.
3. g∘f:(X,τ,E)→(Z,η,R) is Soft strongly J continuous if f is Soft continuous and g is Soft strongly J continuous.
4. g∘f:(X,τ,E)→(Z,η,R) is Soft continuous if f is Soft strongly J continuous and g is Soft continuous.
5. g∘f:(X,τ,E)→(Z,η,R) is Soft J irresolute if f is Soft J continuous and g is Soft strongly J continuous.

Proof:

1. Let (H,R) be a Soft closed set in (Z,η,R). Since g is Soft J continuousg^(-1) (H,R) is Soft J closed in (Y,σ,K). Since f is Soft strongly J continuousf^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft closed in (X,τ,E). So, g∘f is Soft continuous.

2. Let (H,R) be a Soft J closed set in (Z,η,R). Since g is Soft strongly J continuousg^(-1) (H,R) is Soft closed in (Y,σ,K). By Result 2.7,g^(-1) (H,R) is Soft J closed set in (Y,σ,K). Because f is also Soft strongly J continuous, f^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft closed in (X,τ,E). Thus g∘f is Soft strongly J continuous.

3. Let (H,R) be a Soft J closed set in (Z,η,R). Since g is Soft strongly J continuousg^(-1) (H,R) is Soft closed in (Y,σ,K). Since f is Soft continuousf^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft closed in (X,τ,E). Thus g∘f is Soft strongly J continuous.

4. Let (H,R) be a Soft closed set in (Z,η,R). Since g is Soft continuousg^(-1) (H,R) is Soft closed in (Y,σ,K). By Result 2.7, g^(-1) (H,R) is Soft J closed set in (Y,σ,K). As f is Soft strongly J continuous, f^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft closed in (X,τ,E). So, g∘f is Soft continuous.

5. Let (H,R) be a Soft J closed set in (Z,η,R). Because g is Soft strongly J continuousg^(-1) (H,R) is Soft closed in (Y,σ,K). Since f is Soft J continuousf^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft J closed in (X,τ,E). Thus g∘f is Soft J irresolute.

Theorem 4.24: If f:(X,τ,E)→(Y,σ,K) and g:(Y,σ,K)→(Z,η,R) be any two Soft maps then
1. g∘f:(X,τ,E)→(Z,η,R) is Soft perfectly continuous if f is Soft strongly continuous and g are Soft perfectly continuous.
2. g∘f:(X,τ,E)→(Z,η,R) is Soft perfectly J continuous if both f and g are Soft perfectly J continuous.
3. g∘f:(X,τ,E)→(Z,η,R) is Soft perfectly J continuous if f is Soft perfectly J continuous and g is Soft J irresolute.

Proof:
1. Let (H,R) be a Soft J closed set in (Z,η,R). Since g is Soft perfectly J continuousg^(-1) (H,R) is Soft clopen in (Y,σ,K). Since f is Soft strongly J continuousf^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft clopen in (X,τ,E). Thus g∘f is Soft perfectly continuous.

2. Let (H,R) be a Soft J closed set in (Z,η,R). Since g is Soft perfectly J continuousg^(-1) (H,R) is Soft clopen in (Y,σ,K). By Result 2.7, g^(-1) (H,R) is Soft J closed set in (Y,σ,K). Now, f is also Soft perfectly J continuous, then f^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft clopen in (X,τ,E). Thus g∘f is Soft perfectly J continuous.

3. Let (H,R) be a Soft J closed set in (Z,η,R). Since g is Soft J irresolute, g^(-1) (H,R) is Soft J closed set in (Y,σ,K). Since f is Soft perfectly J continuousf^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft clopen in (X,τ,E). So, g∘f is Soft perfectly J continuous.

Theorem 4.25: If f:(X,τ,E)→(Y,σ,K) and g:(Y,σ,K)→(Z,η,R) be any two Soft maps then
1. g∘f:(X,τ,E)→(Z,η,R) is Soft perfectly continuous if f is Soft perfectly J continuous and g are Soft strongly J continuous.
2. g∘f:(X,τ,E)→(Z,η,R) is Soft strongly J continuous if g is Soft perfectly J continuous and f is Soft continuous.

Proof:
1. Let (H,R) be a Soft J closed set in (Z,η,R). Since g is Soft strongly J continuousg^(-1) (H,R) is Soft closed set in (Y,σ,K). By Result 2.7, g^(-1) (H,R) is Soft JP closed set in (Y,σ,K). Since f is Soft perfectly J continuousf^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft clopen in (X,τ,E). Thus g∘f is Soft perfectly continuous.
2. Let (H,R) be a Soft J closed set in (Z,η,R). Since g is Soft perfectly J continuousg^(-1) (H,R) is Soft clopen in (Y,σ,K). Since f is Soft continuousf^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft clopen in (X,τ,E).Thus g∘f is Soft perfectly J continuous.

Theorem 4.26: Let f:(X,τ,E)→(Y,σ,K) and g:(Y,σ,K)→(Z,η,R) be any two maps then their composition g∘f:(X,τ,E)→(Z,η,R) is Soft perfectly J continuous if g is Soft strongly continuous and g is Soft perfectly J continuous.

Proof: Let (H,R) be a Soft J closed set in (Z,η,R). Since g is Soft strongly continuousg^(-1) (H,R) is Soft closed in (Y,σ,K). Since f is Soft perfectly J continuousf^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft J clopen in (X,τ,E). Thus g∘f is Soft perfectly J continuous.

Theorem 4.27: If f:(X,τ,E)→(Y,σ,K) is Soft strongly J continuous and g:(Y,σ,K)→(Z,η,R) is Soft contra J continuous then their composition g∘f:(X,τ,E)→(Z,η,R) is Soft contra continuous.

Proof: Let (H,R) be a Soft closed set in (Z,η,R). Since g is Soft contra J continuousg^(-1) (H,R) is Soft J open in (Y,σ,K). Since f is Soft strongly J continuousf^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft open in (X,τ,E). Thus g∘f is Soft contra continuous.

Theorem 4.28: If f:(X,τ,E)→(Y,σ,K) is Soft perfectly J continuous and g:(Y,σ,K)→(Z,η,R) is Soft contra J continuous then their composition g∘f:(X,τ,E)→(Z,η,R) is Soft contra J continuous.

Proof: Let (H,R) be a Soft closed set in (Z,η,R). Since g is Soft contra J continuousg^(-1) (H,R) is Soft J open in (Y,σ,K). Since f is Soft perfectly J continuousf^(-1) (g^(-1) (H,R))=(g∘f)^(-1) (H,R) is Soft clopen in (X,τ,E). Then by Result 2.7, (g∘f)^(-1) (H,R) is Soft J open in (X,τ,E). Thus g∘f is Soft contra J continuous.¬

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