APPLICATION OF FUZZY EXPONENTIAL WEIGHTED MOVING AVERAGE CONTROL CHART FOR DETECTING SMALLER SHIFTS IN MANUFACTURING INDUSTRIES

Abstract

Control charts are the tools used to identify the out-of-control process in the manufacturing industries. The traditional control charts usually deal with crisp data, but in real-life situations, data may be unclear. In this situation, a Fuzzy control chart is the best option to represent for evaluating whether the process is in control. In order to detect smaller shifts, the Fuzzy Exponentially Weighted Moving Average Control Chart (FEWMA) and Fuzzy CUSUM Control are the best tools. In this paper, application perspective discussions on the Fuzzy EWMA Control chart for detecting smaller shifts and fuzzy CUSUM control charts in the manufacturing industries with detailed illustrations.

Keywords: crisp data, fuzzy control charts, Fuzzy EWMA control charts, Fuzzy CUSUM Control charts.

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I. INTRODUCTION

The overall performance of a product can be improved by decreasing the variability. It can be achieved with the help of statistical process control (SPC) methods. Conventional control charts are the most used tools for this purpose. They are used to detect shifts in a process. However, these charts are not able to detect small shifts. In order to detect small shifts in the process mean or in the process variation, an exponentially weighted moving average scheme is used. The EWMA Control chart was introduced by Roberts in the year 1959. Thus, FEWMA control chart is accustomed to use when the statistical data under concern are unsure or hazy.

II. REVIEW OF LITERATURE

M.Mahfouf, M.F. Abbod and D. A.Linked (2000) have elaborately discussed the application of fuzzy logic in detecting and controlling the different diseases. They have given a wide idea on the type of fuzzy technology application in medicine and healthcare over time. Pandurangan and Varadharajan (2011) have discussed the construction of α –cut Fuzzy. Sayed Mojtaba Zabihinpour Jahromi et al (2012) have developed fuzzy control charts for monitoring attribute data. Average run length is the measure used to compare the methods. Mohammad Hossein Zavvar Sabegh et al.(2014) have discussed about the applications of fuzzy control charts in different fields. He also suggested the various fuzzy control charts and the situation of the adoption in the real-life situation. A.Saravanan and V.Alamelumangai (2014) have discussed about the performance of attribute charts and fuzzy control charts for variable data. P chart and CUSUM control chart are compared to analyze for identifying the better performance. Nur Ain Zafirah Ahmad Basri et al. (2016) have developed fuzzy X bar – S bar charts for solder paste thickness. The data are recorded as triangular fuzzy number. Kim-Phuong Truong et al. (2017) have designed fuzzy u-chart for the textile dyeing industry. They have demonstrated that replace the traditional control with the fuzzy u-chart gets better result. Muhammad Zakir Khan et al. (2018) have introduced Fuzzy Exponential weighted Moving average Control Chart for the attribute control to monitor the process mean. R.Sasikumar et al. (2019) proposed a short-run process control chart based on a fuzzy approach. They are used in the situation in cases where less than 20 subgroups are available. Amando dos Santos Mendes, Marcela A.G.Machado and Paloma M.S.Rocha Rizol (2019) have monitored fuzzy control charts using the mean and range of the univariate process. They have considered the illustration of a milk bag filling process. Average Run Length and Economic Quadratic Loss are compared for the traditional control and Fuzzy Control chart. Syed Muhammad Muslim Raza et al. (2019) have proposed a hybrid exponential-weighted moving average control chart based on an exponential-type estimator or mean. The control charts are measured using average run length. Akeem Ajibola et al.(2020) have extensively elaborated on the interval type-2 FEWMA control chart with an illustration by simulating the data with trapezium fuzzy numbers. Ambreen Shafqat et al. (2021) have introduced new neutrosophic double and triple exponentially weighted moving average control charts. The charts are utilized for attaining the procedure to minimize road accidents and defective products. R.M.Pratiwi et al. (2021) have designed the quality control of label stock using FEWMA control chart. It is used to detect the smaller shifts. It provides flexibility to prevent false alarms.

III. EWMA CONTROL CHARTS

EWMA control chart was alternative to a Shewhart chart as it provides a quicker response to shifts in the process mean and incorporates information from all previously collected data. The name was changed to reflect the fact that exponential smoothing serves as the basis of EWMA charts. For constructing the EWMA control chart we assume k samples of size n >=1 yielding k individual measurements $x_1, x_2, \ldots\ldots\ldots x_k$ (if n=1) or k samples means. FEWMA Control Charts are more flexible than the traditional control chart. It is used for detecting smaller shifts in the process.

IV. FUZZY TRANSFORMATION TECHNIQUES

The data used are fuzzy then it is a need to represent the fuzzy sets attached with the linguistic data in the sample by some transformations. The four most popular fuzzy transformation methods are fuzzy mean, fuzzy median, fuzzy mode and fuzzy midrange.

1. **Fuzzy Mode:** The membership function of the value of the base variable of a fuzzy set equals 1 in the fuzzy mode which is represented by

$$f_{mode} = \{x|\mu F(x)=1\} \text{ for all } x \in F.$$

2. **Fuzzy Mean:** The formula to calculate the fuzzy mean is defined by

$$F_{avg} = \frac{\int_0^1 x\mu F(x)}{\int_0^1 \mu F(x)}.$$

3. **Fuzzy Median:** It is the point which divides the curve under the membership function of a fuzzy set into two equal rejoins satisfying the following equation:

$$\int_a^{Fmed} \mu F(x)\,dx = \int_{Fmed}^{c} \mu F(x)\,dx = \frac{1}{2}\int_a^{c} \mu F(x)dx.$$

4. **Fuzzy Midrange:** The fuzzy midrange is defined by

$$F_{mr} = \frac{1}{2}(a_\alpha + c_\alpha)$$

where

$a_\alpha = \min\{F_\alpha\}$

And $c_\alpha = \max\{F_\alpha\}$.

V. ADVANTAGES OF FUZZY EWMA CONTROL CHART OVER TRADITIONAL CONTROL CHART:

EWMA control chart is used for identifying smaller shifts in the mean while traditional control charts are used for larger shifts. With EWMA control charts we are able to update our forecast as new observations arrive. It can also be applied in a situation when the distribution of the available process data is either unidentified or non-normal. The chart is more applicable, flexible, and informative in real-life quality control problems. The control charts using the EWMA statistic utilize the current and previous subgroup information about the state of the process. The construction method and the interpretation involved in a traditional Shewhart chart are different from the one in the EWMA control chart.

VI. FUZZY EXPONENTIAL WEIGHTED MOVING AVERAGE CONTROL CHART:

FEWMA control chart for unknown standard deviations are calculated for process data as follows:

$$UCL_{EWMA} = \bar{\bar{X}}_a + A_2\,\bar{R}_a\sqrt{\frac{\lambda}{2-\lambda}},\ \bar{\bar{X}}_b + A_2\,\bar{R}_b\sqrt{\frac{\lambda}{2-\lambda}},\ \bar{\bar{X}}_c + A_2\,\bar{R}_c\sqrt{\frac{\lambda}{2-\lambda}}$$

$$CL = \bar{\bar{X}}_a,\ \bar{\bar{X}}_b,\ \bar{\bar{X}}_c$$

$$LCL = \bar{\bar{X}}_a - A_2\bar{R}_a\sqrt{\frac{\lambda}{2-\lambda}}$$

$$\bar{\bar{X}}_b - A_2\,\bar{R}_b\sqrt{\frac{\lambda}{2-\lambda}}$$

$$\bar{\bar{X}}_c - A_2\,\bar{R}_c\sqrt{\frac{\lambda}{2-\lambda}}$$

The α-level fuzzy values for mean and Range are given as follows:

$$\bar{\bar{X}}_a^\alpha = \bar{\bar{X}}_a + \alpha\,(\bar{\bar{X}}_b - \bar{\bar{X}}_a)$$

$$\bar{\bar{X}}_c^\alpha = \bar{\bar{X}}_c - \alpha\,(\bar{\bar{X}}_c - \bar{\bar{X}}_b)$$

$$\bar{R}_a^\alpha = \bar{R}_a + \alpha\,(\bar{R}_b - \bar{R}_a)$$

$$\bar{R}_c^\alpha = \bar{R}_c - \alpha\,(\bar{R}_c - \bar{R}_b)$$

$$UCL_{med-EWMA}^\alpha = CL_{med-EWMA}^\alpha + \frac{1}{3}A_2\,(\bar{R}_a^\alpha + \bar{R}_b + \bar{R}_c^\alpha)\sqrt{\frac{\lambda}{2-\lambda}}$$

$$CL_{med-EWMA}^\alpha = \frac{1}{3}\,(\bar{\bar{X}}_a^\alpha + \bar{\bar{X}}_b + \bar{\bar{X}}_c^\alpha)$$

$$LCL_{med-EWA}^\alpha = CL_{med-EWA}^\alpha - \frac{1}{3}A_2(\bar{R}_c^\alpha + \bar{R}_b + \bar{R}_c^\alpha\sqrt{\frac{\lambda}{2-\lambda}}$$

$$\text{Process Control} = \begin{cases} \text{in – control, for } LCL_{med-EWA}^\alpha <^\alpha S_{med-EWA} < UCL_{med-EWMA}^\alpha \\ \text{out of control, otherwise} \end{cases}$$

S_{med}^{α}- EWM$A_{,1}$ = $\frac{1}{3}$ ($\bar{X}_{a,1}^{\alpha} + \bar{X}_{b,1}$+$\bar{X}_{c,1}^{\alpha}$)

S_{med}^{α}- EWM$A_{,2}$ = $\frac{1}{3}$ ($\bar{X}_{a,2}^{\alpha} + \bar{X}_{b,2}$+$\bar{X}_{c,2}^{\alpha}$)

S_{med}^{α} - EWMA, 25 = $\frac{1}{3}$ ($\bar{X}_{a,25}^{\alpha} + \bar{X}_{b,25}$+$\bar{X}_{c,25}^{\alpha}$)

The data are collected from the manufacturing industry for a 5kg particular brand of commodity.

Their weights are collected in a fuzzy number representation of the samples

Fuzzy Number Representation of the Samples for Sampling Units Are Given Below

Sample	X_a	X_b	X_c
1	5	5.1	4.85
2	4.95	4.9	5
3	4.9	5.1	5.05
4	5	5.05	4.9
5	4.95	5	5.05
6	4.85	5.1	5.05
7	4.95	5.05	5.1
8	5	4.95	5.1
9	4.8	5.1	4.9
10	4.9	5	5.05

The values of Ranges and Arithmetic mean are calculated as

R_{a1}=0.2 R_{b1}=0.2 R_{c1}=0.25
$\bar{X}_{a1}$ = 4.93 $\bar{X}_{b1}$ = 5.035 $\bar{X}_{c1}$ = 5.005

The Data from another Sample of 10 Units Are Given As Follows

Sample	X_a	X_b	X_c
1	4.95	4.9	5.15
2	4.85	5	5.05
3	4.9	5.05	5.15
4	5.15	5.1	5.05
5	5	5.05	5.1
6	5.05	4.95	4.9
7	4.95	4.9	4.85
8	5.1	5	4.9
9	5.05	4.95	4.85
10	5	5.05	5.1

The Ranges and Arithmetic mean of the sampling units are calculated as follows

R_{a2}=.3 R_{b2}=.2 R_{c2}=.3
$\bar{X}_{a2}$ = 5 $\bar{X}_{b2}$ = 4.995 $\bar{X}_{c2}$ = 5.01

Similarly, 20 samples are collected. Their ranges are tabulated as follows

R_{a1} = 0.2	R_{b1}= .2	R_{c1}=.25
R_{a2}= .3	R_{b2}=.2	R_{c2}=.3
R_{a3} = .25	R_{b3}=.1	R_{c3}=.2
R_{a4}= .1	R_{b4}=.25	R_{c4}=.15
R_{a5} = .15	R_{b5}=.1	R_{c5}=.05
R_{a6}= .1	R_{b6}=.15	R_{c6}=.05
R_{a7}= .2	R_{b7}=.05	R_{c7}=.1
R_{a8}= .05	R_{b8}=.1	R_{c8}=.15
R_{a9}= .1	R_{b9}=.05	R_{c9}=.1
R_{a10}= .15	R_{b10}=.1	R_{c10}=.05
R_{a11}= .05	R_{b11}=.1	R_{c11}=.15
R_{a12} = .1	R_{b12}=.05	R_{c12}=.05
R_{a13}= .05	R_{b13}=.1	R_{c13}=.15
R_{a14} = .1	R_{b14}=.05	R_{c14}=.2
R_{a15} = .05	R_{b15}=.1	R_{c15}=.05
R_{a16}= .1	R_{b16}=.05	R_{c16}=.2
R_{a17}= .15	R_{b17}=.1	R_{c17}=.05
R_{a18}= .05	R_{b18}=.15	R_{c18}=.1
R_{a19}= .1	R_{b19}=.1	R_{c19}=.05
R_{a20}= .05	R_{b20}=.15	R_{c20}=.05

$\bar{R}_a$ = .12 $\bar{R}_b$ = .1125 $\bar{R}_c$ = .1225

The Arithmetic Mean of the Twenty Samples are Tabulated Below

$\bar{X}_{a1}$=4.93	$\bar{X}_{b1}$=5.035	$\bar{X}_{c1}$=5.005
$\bar{X}_{a2}$=5	$\bar{X}_{b2}$=4.995	$\bar{X}_{c2}$=5.01
$\bar{X}_{a3}$=4.95	$\bar{X}_{b3}$=4.85	$\bar{X}_{c3}$=5.01
$\bar{X}_{a4}$=4.92	$\bar{X}_{b4}$=5.01	$\bar{X}_{c4}$=4.995
$\bar{X}_{a5}$=4.95	$\bar{X}_{b5}$=5.01	$\bar{X}_{c5}$=4.85
$\bar{X}_{a6}$=5.01	$\bar{X}_{b6}$=5.05	$\bar{X}_{c6}$=4.85
$\bar{X}_{a7}$=5	$\bar{X}_{b7}$=4.95	$\bar{X}_{c7}$=4.9
$\bar{X}_{a8}$=4.95	$\bar{X}_{b8}$=4.9	$\bar{X}_{c8}$=5.01
$\bar{X}_{a9}$=5.05	$\bar{X}_{b9}$=4.885	$\bar{X}_{c9}$=4.95
$\bar{X}_{a10}$=5.15	$\bar{X}_{b10}$=4.95	$\bar{X}_{c10}$=5
$\bar{X}_{a11}$=5.15	$\bar{X}_{b11}$=4.85	$\bar{X}_{c11}$=4.85
$\bar{X}_{a12}$=5.05	$\bar{X}_{b12}$=5.015	$\bar{X}_{c12}$=4.995
$\bar{X}_{a13}$=4.95	$\bar{X}_{b13}$=5.05	$\bar{X}_{c13}$=4.9
$\bar{X}_{a14}$=4.9	$\bar{X}_{b14}$=5	$\bar{X}_{c14}$=5.05
$\bar{X}_{a15}$=4.95	$\bar{X}_{b15}$=4.95	$\bar{X}_{c15}$=5
$\bar{X}_{a16}$=4.85	$\bar{X}_{b16}$=5.05	$\bar{X}_{c16}$=5.15
$\bar{X}_{a17}$=4.9	$\bar{X}_{b17}$=4.95	$\bar{X}_{c17}$=4.95
$\bar{X}_{a18}$=5.05	$\bar{X}_{b18}$=5.05	$\bar{X}_{c18}$=5.05
$\bar{X}_{a19}$=4.95	$\bar{X}_{b19}$=4.95	$\bar{X}_{c19}$=4.95
$\bar{X}_{a20}$=5.05	$\bar{X}_{b20}$=4.95	$\bar{X}_{c20}$=5.05

$\bar{\bar{X}}_a$ = 4.985 $\bar{\bar{X}}_b$ = 4.9725 $\bar{\bar{X}}_c$=4.97625

The values for UCL FEWMA are (4.992, 4.979, 4.9835)

The values of LCL FEWMA are (4.978, 4.9658, 4.969)

The values of CL FEWMA are (4.985, 4.9725, 4.97625)

$\bar{\bar{X}}_a^\alpha$ = 4.9768 $\bar{\bar{X}}_c^\alpha$ = 4.97385 $\bar{R}_a^\alpha$ = .120125 $\bar{R}_c^\alpha$ = .116

UCL^α= (4.984, 4.9792, 4.980)

CL^α = (4.9768, 4.9725, 4.97625)

LCL^α = (4.9698, 4.9658, 4.96695)

The values of UCL, CL and LCL for med- EWMA control chart is calculated using the formulae.

UCL^α med EWMA = 4.96551 CL^α med EWMA = 4.95855 and LCL^α med EWMA = 4.95159

If the values of S^αmed-EWMA lies between UCL and LCL then the process is said to be in control.

Control limits of FUZZY MED-EWMA, α-level fuzzy median value and the process condition

Sample	$S^\alpha_{med-EWMA}$	**4.951<S^α<5.025**
1	5.006	In Control
2	5.003	In Control
3	4.998	In Control
4	4.991	In Control
5	4.915	Out of Control
6	4.953	In Control
7	4.978	In Control
8	4.966	In Control
9	4.964	In Control
10	5	In Control
11	4.885	Out of Control
12	5.008	In Control
13	4.955	In Control
14	4.9725	In Control
15	4.9775	In Control
16	5.018	In Control
17	4.956	In Control
18	5.015	In Control
19	4.957	In Control
20	5.0166	In Control

VII. CONCLUSION

From the above table it can be found that two samples are out of control. It can be interpreted that samples 5 and 11 are out of control. Control charts are used for detecting the variation in the process, when there is a smaller shift in the process then the usage of EWMA control chart plays a vital role. When the data available are unclear or the data are inaccurate then fuzzy control charts play a major role. For detection of smaller shifts and for vague data in manufacturing industry FEWMA makes a valid contribution.

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