EXPLORING GEOGEBRA: AN INTERACTIVE TOOL FOR MATHEMATICAL VISUALIZATION AND ANALYSIS

Abstract

The capacity of Geogebra, a potent dynamic mathematics programmed, to combine geometry, algebra, and calculus has won it widespread acclaim. In this essay, we investigate how Geogebra can be used to improve algebraic concept teaching and learning. We seek to develop an engaging and immersive environment that promotes a deeper comprehension of algebraic concepts by utilizing Geogebra's interactive capabilities. Some of the equipment applied to perform the Monge projection implementing the GeoGebra to provide a sharper projection of more difficult surfaces of movement. GeoGebra's Aesthetics 2D and Visuals 3D are both utilized to connect the 2D building through the Monger projection while emulating a 3D circumstances within the Visuals 3D mode. In order to examine the revolution substrates, useful objects, and terminal contours of the junctions of two revolution surfaces, numerous points from various perspectives can be hired in the Visuals 3D.

Keywords: Monge, GeoGebra, Mathematical Visualization, Interactive Tool

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I. INTRODUCTION

GeoGebra is a dynamic mathematics software that combines geometry, algebra, calculus, and graphing features in an interactive and user-friendly interface. It was created by Markus Hohenwarter and a team of developers and researchers and was first released in 2001.Markus Hohenwarter, a mathematician and computer scientist from Austria, is widely recognized as the primary author and developer of GeoGebra. He initially developed the software as part of his master's thesis at the University of Salzburg. Inspired by his passion for making mathematics education more accessible and engaging, Hohenwarter collaborated with a team of researchers and programmers to refine and expand GeoGebra's capabilities over the years.Geogebra has been extensively used in both educational used in both educational and research settings. Numerous papers and publications have been dedicated to exploring its potential and effectiveness in teaching and learning mathematics. These papers typically investigate various aspects of geogebra, such as its impact on student understanding, its integration into the curriculum, and its use in different mathematical domains. “New possibilities in teaching and learning mathematics” (2006) by Markus Hohenwater, zsoltlavicza [3]. This paper presents the features and benefits of geogebra, discussing its potential to enhance teaching and learning mathematics. It highlights the software’s dynamic nature, visualizations and integration of geometry, algebra and calculus. “An innovative and multiplatform mathematics software” (2007) by Markus Hohenwarter and Judith Preiner [4]. This paper introduces geogebra as an innovative and cross platform mathematics software. It provides an overview of the software’s capabilities including its use in dynamic geometry, algebra and calculus. “The new mathematics education software” (2010) by Markus Hohenwarter, Judith Preiner and zsoltlavicza [4]. This paper discusses the role of geogebra as a mathematics education software. It explores the softwares potential for interactive and exploratory learning and its impact on teaching and learning mathematics in different educational settings. Perspectives of Research in Mathematics Education” (2013) by zsoltlavicza and Markus Hohenwarter. This paper provides an overview of the research perspectives on geogebra in mathematics education. It discusses various studies research methodologies used to investigates the impact of geogebra on student learning outcomes and teaching practices. “Dynamic Mathematics with geogebra” (2019) by Markus Hohenwarter. This paper presents an overview of geogebra as a “dynamic mathematics with geogebra” (2019) by Markus Hohenwarter. This paper presents an overview of geogebra as a dynamic mathematics tool. It discusses the software key features, such as interactive geometry, algebra and calculus and highlights its versality for both students and teachers in mathematical exploration.

II. PRELIMINARIES

Geogebra is a powerful and versatile software program that combines geometry, algebra, calculus, and other mathematical tools into a single interface. It is commonly used for teaching and learning mathematics, as well as for conducting mathematical experiments and exploring mathematical concepts. Here are some preliminaries or basic features of Geogebra:

1. **Graphing:** Geogebra allows you to graph functions, equations, and inequalities in a Cartesian coordinate system. You can plot points, draw lines, curves, and customize the appearance of graphs.Graphing: Geogebra allows you to graph functions, equations, and

inequalities in a Cartesian coordinate system. You can plot points, draw lines, curves, and customize the appearance of graphs.

2. **Geometry:** Geogebra provides a range of tools for creating and manipulating geometric objects such as points, lines, segments, circles, polygons, and conic sections. You can construct figures using precise measurements, perform transformations, and explore geometric properties.

3. **Algebra:** Geogebra has a built-in algebra system that allows you to perform symbolic computations. You can define variables, create algebraic expressions and equations, solve equations, factor polynomials, and work with functions.

4. **Calculus:** Geogebra supports calculus operations such as differentiation and integration. You can find derivatives, evaluate limits, compute definite and indefinite integrals, and visualize the behavior of functions.

5. **Spreadsheet:** Geogebra includes a spreadsheet component that enables you to organize and analyze data. You can enter data sets, perform statistical calculations, create charts, and apply mathematical functions to the data.

6. **Dynamic Interactivity:** One of the key features of Geogebra is its dynamic nature. You can manipulate objects and parameters using sliders, checkboxes, and input fields, allowing you to observe how changes affect the mathematical models in real time.

7. **Scripting:** Geogebra supports scripting with its own programming language called Geogebra /Script. It allows you to create custom tools, automate tasks, and develop interactive applications within the software.

8. **Integration with Other Tools:** Geogebra seamlessly integrates different mathematical representations, including geometric, algebraic, and numeric views. You can switch between these views to explore mathematical concepts from various perspectives.

9. **Community and Resources:** Geogebra has a large and active user community. You can find a wealth of resources, including tutorials, lesson plans, and user-created applets, on the Geogebra website and forums.

These are just some of the preliminary features of Geogebra. The software offers many advanced capabilities and options for customization, making it a valuable tool for mathematical exploration, visualization, and teaching.

II. GENERAL FOUNDATION FOR LEARNING GEOGEBRA

Given accessibility through social media websites such as Twitter and YouTube, as well as Facebook, adolescents today have become acclimated to media with visual culture. Additionally, there are plenty of different communication categories utilised, which includes as text, music, graphics, animation, video, and augmented reality. A completely distinct world of technology has also emerged as the consequence of the Internet's rapid expansion and progress, along with the public's enhanced access to it. Students are consequently more probable to embrace the knowledge offered to them in this manner. Throughout their educational process. Students are deterred towards acquiring an idea when it fails to be explained in a relevant and comprehensible manner, particularly when it comes to mathematical concepts wherever a wide range of concerns require for quite a bit of inventiveness.

The primary hurdles in math education are:

- Premises lacks adequate representation;
- Algebraic graph remain rigid in the conventional approach for demonstrating mathematics, that involves creating the graph on an expanse of sheet; &
- static entities are incompatible an adequate generalisation of a particular theory.
- The National Council of Teachers of Mathematics (NCTM), the biggest union of math professionals in the entire world, involved innovation. In each of its six fundamental tenets for the teaching of mathematics in educational institutions.

Technological innovation performs a vital part during the instruction and acquisition of mathematical concepts; influences the mathematical subject content presented and promotes the learning process for students. Considering each of the aforementioned, GeoGebra is a fantastic tool that can be used for educators and students to teach mathematical concepts. Since every component in GeoGebra becomes constantly students have access to observe as it evolves as the problem characteristics are tweaked. Every component, especially points, sections, circles, and lines, are capable of being manipulated in any orientation in geometric inventions. It strengthens the visual appeal of the layouts. Furthermore, every component are able to implemented through the command prompt or via point-and-click method.

III. GRAPHICAL USER INTERFACE

Featuring components for both geometry and algebra, GeoGebra's website is simple to operate. It's possible to flexibly tweak it to correspond with your specifications as well as the possibility of problems.

We can consider an assortment of perspectives, including the

- algebraic view,
- geometric view,
- spreadsheetview,
- CAS(ComputerAlgebraSystem)view,
- protocol design view,
- command line.

IV. ALGEBRAIC INPUT, FUNCTIONS AND VISUALIZATION OF GRAPHS

1. **ParametersofaLinearEquation:** In a linear equation, there are two main parameters: the slope and the y-intercept. The general form of a linear equation is: y indicates the variable that is dependent (which is frequently as the vertical axis), and its mathematical form is y = m x + b.

   - The independent parameter can be expressed by x, which are generally the direction that is horizontal.

     The point that defines the y-intercept is expressed by b, and the gradient of the resulting line is depicted by m.

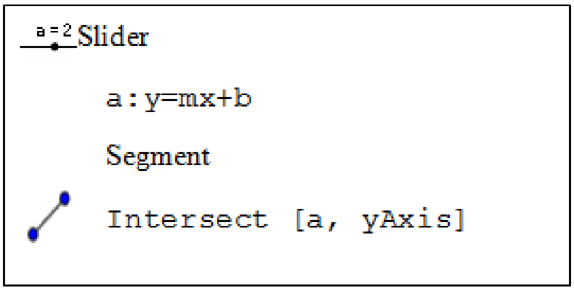
     Slope (m): slope determines a steepness or inclination of the line. It shows how quickly the independent variable (x) and the variable that depends (y) are evolving. A straight line featuring aelevatedslope upward, whereas one of them with an antagonistic gradient towards the ground. Each slope's concept change in y divided by the change in x between any two points on the line. Mathematically, it can be calculated as,

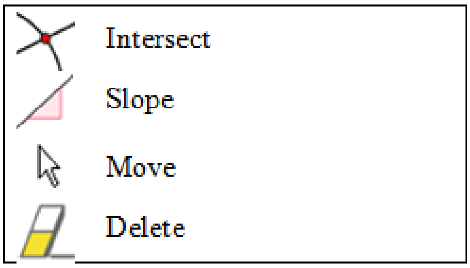
     m = (y2 - y1) / (x2 - x1)

     where (x1, y1) and (x2, y2) are either or both of the line components.

   - Y-intercept (b): The line closest junction alongside the y-axis is termed as the y-intercept. It depicts what transpires with y whenever x is identical to zero.

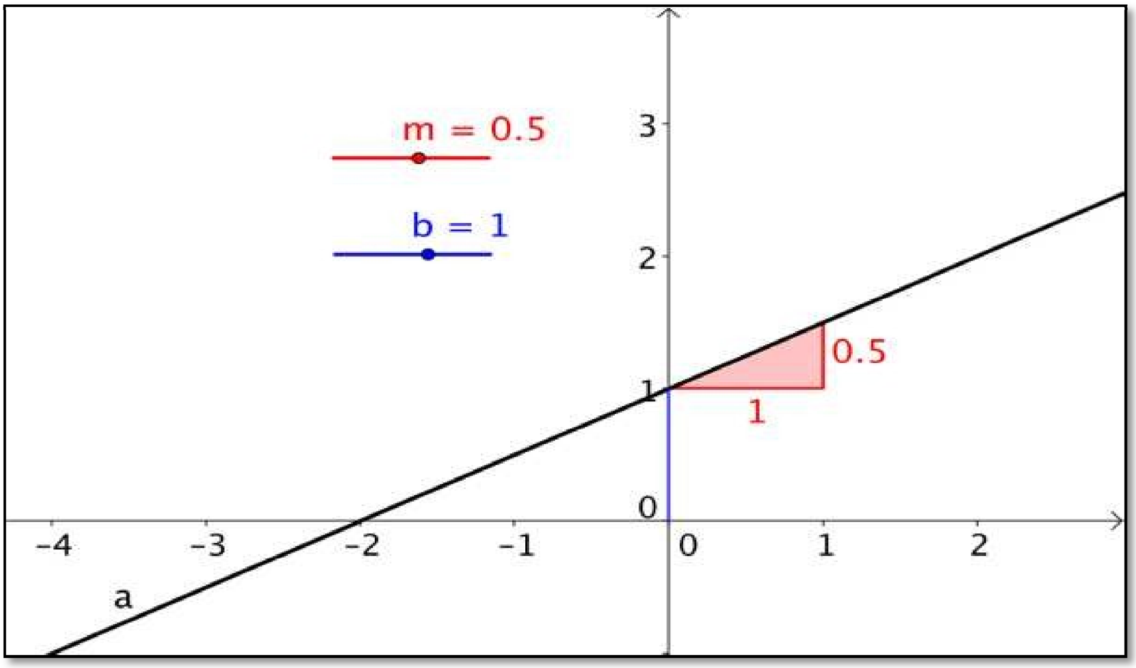
   - In the equation y = mx + b, the y-intercept is the value of b. Geometrically, the y-intercept is the value of y when x = 0.





1.OpenanewGeoGebrawindow.
2. SwitchtoPerspectives–Algebra& Graphics.
3. Add scales between m and b through the slider's initial configuration.
4. Ent y=mx+b.
5 w the position A wherever path a and the yaxis cross. Apply the Intersect[a, y axis] t que.

6. Construct the destination point B at the initial position.
7. Draw a line segment interconnecting the two corresponding points.
8. Createtheslope(triangle)oftheline.
9. Make use of the Stylebar to customise the method by which your construction appear.



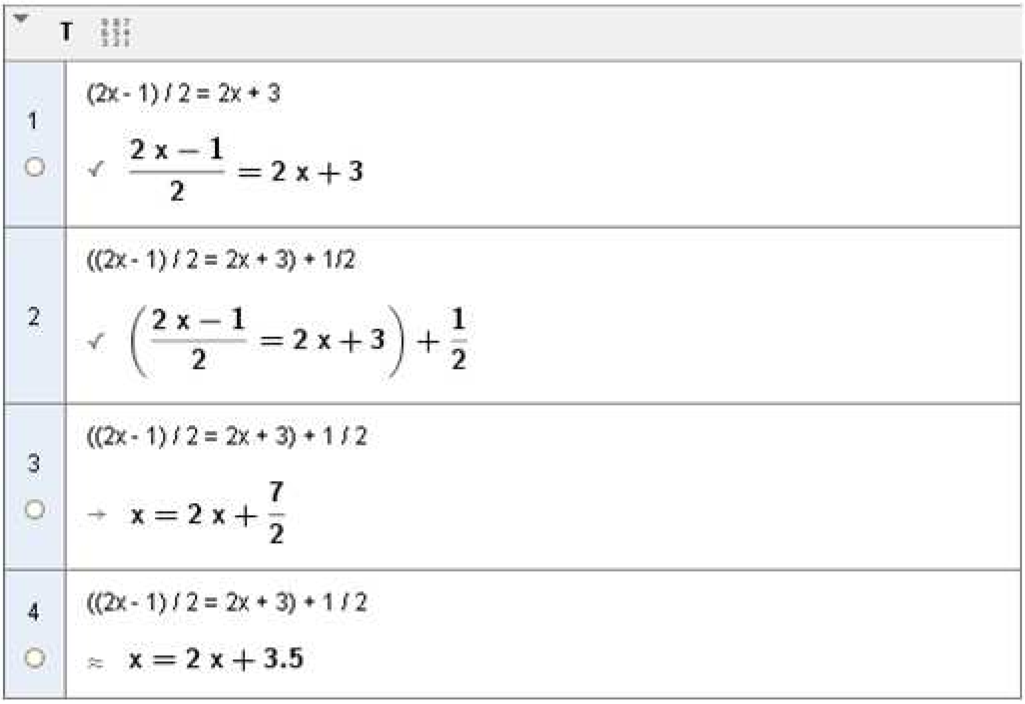
V. MANIPULATINGEQUATIONS

1 In the first row, insert the equation (2x - 1) / 2 = 2x + 3. If you desire to circumvent automatic reduction in complexity, make use of the Conserve Input tool.

2 To insert the initial result into the subsequent row, implement the space key. Utilise the evaluate tool to mathematically, calculate the outcome, explicitly presenting a reasonable number as an integer fraction.

3 Click on the second row's output in order to move it to the fourth row, which is the presently determined row. The rational number can be obtained in decimal notation after the result generated by employing the Mathematical tool has been mathematically determined quantitatively.

VI. INTERSECTINGPOLYNOMIALFUNCTIONS



To find the intersection points of two polynomial equations algebraically, you can set the two equations equal to each other and solve for the variables.

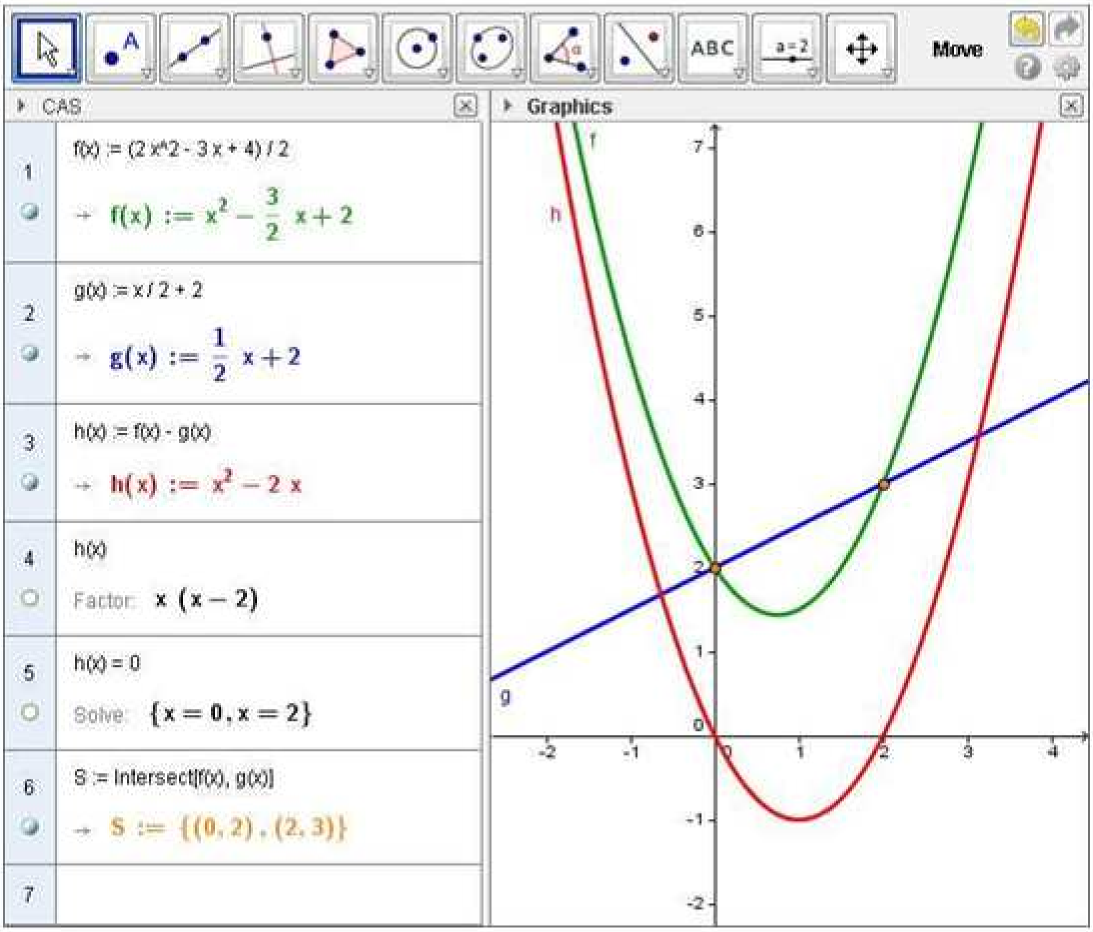
1 Given two polynomial equations, let's call them f(x) and g(x).
2 Set f(x) equal to g(x): f(x) = g(x).
3 Rearrange the equation to have zero on one side: f(x) - g(x) = 0.
4 Now you have a new equation in the form h(x) = 0, where h(x) = f(x) - g(x).
5 Solve the equation h(x) = 0 to find the values of x where the two polynomial functions intersect.

Depending on the degree of the polynomials and the complexity of the equation, you may need to use different techniques to solve for x. These techniques may include factoring, the quadratic formula, synthetic division, or numerical methods such as Newton's method or the bisection method. If the polynomials have a degree higher than 4, finding exact solutions may be difficult, on numerical approximations.

Example:
Intersecttheparabolaf(x):=($2x^2$–3x+4)/2withtheline g(x):=x/2+2.

- The function f can be expressed as f(x):= (2x-3x+4)/2.
- State that the function g(x):= x/2+2
- Declare the function h is defined as h(x):= f(x) - g(x).
- Add the tool Factor and enter h(x) in the fourth row. The origins of h may be read off instantly.

- Insert h(x) = 0 and select the Calculated tool to derive the intersection points x coordinates.
- Select the command Intersect for identifying the intersection of the spots.
- Intersect[f(x), g(x)] generates S.
- Modify line and colour in the visualisation section.



VII. SOLVINGSYSTEMOFEQUATIONS

Resolving arithmetic equationinvolves finding values& variables that satisfy all the equations simultaneously. There are different methods to solve systems of equations, including substitution, elimination, and matrix methods. Here's a brief explanation of these approaches.

1 **Substitution Method**: In this method, the purposereduce the system to a single equation with one variable, which can then be solved. The steps involved are:
   - Solve one equation for a variable (e.g., y = 2x + 3).
   - Substitute the expression obtained in step a into the other equations, replacing the variable.
   - Simplify and solve the resulting equations for the remaining variables.
   - Substitute the values found in step c back into any of the original equations to find

the remaining variables.

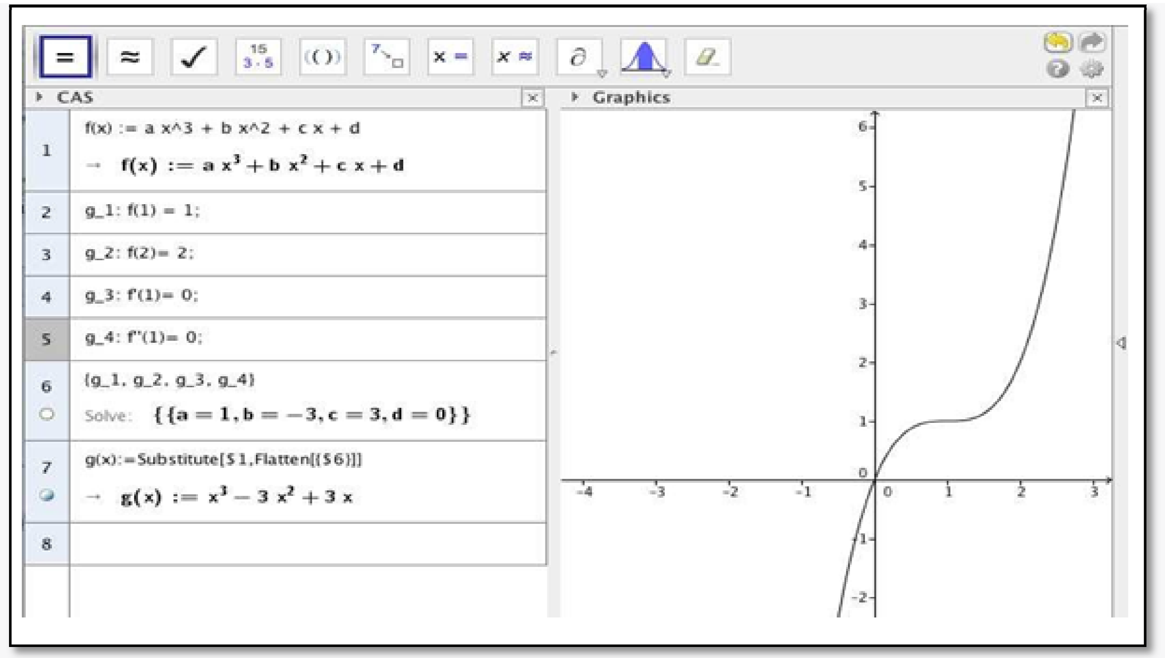
2 **Elimination Method:** In this method, you manipulate the equations by adding or subtracting them to eliminate one of the variables. The steps involved are:

- Multiply the equations, if necessary, to make the coefficients of one variable the same (or multiples of each other).
- To enable for eliminating a single parameter, add or subtract the mathematical equations.
- Choose a single parameter to solve the equation which ensues.
- In order to identify the rest of the variables, return to the value from step c from any of the earlier equations.

Matrix Method (Gaussian Elimination): In this approach, the mathematical equations are represented as matrices, and the matrix is then simplified using row operations.

The involved steps are:

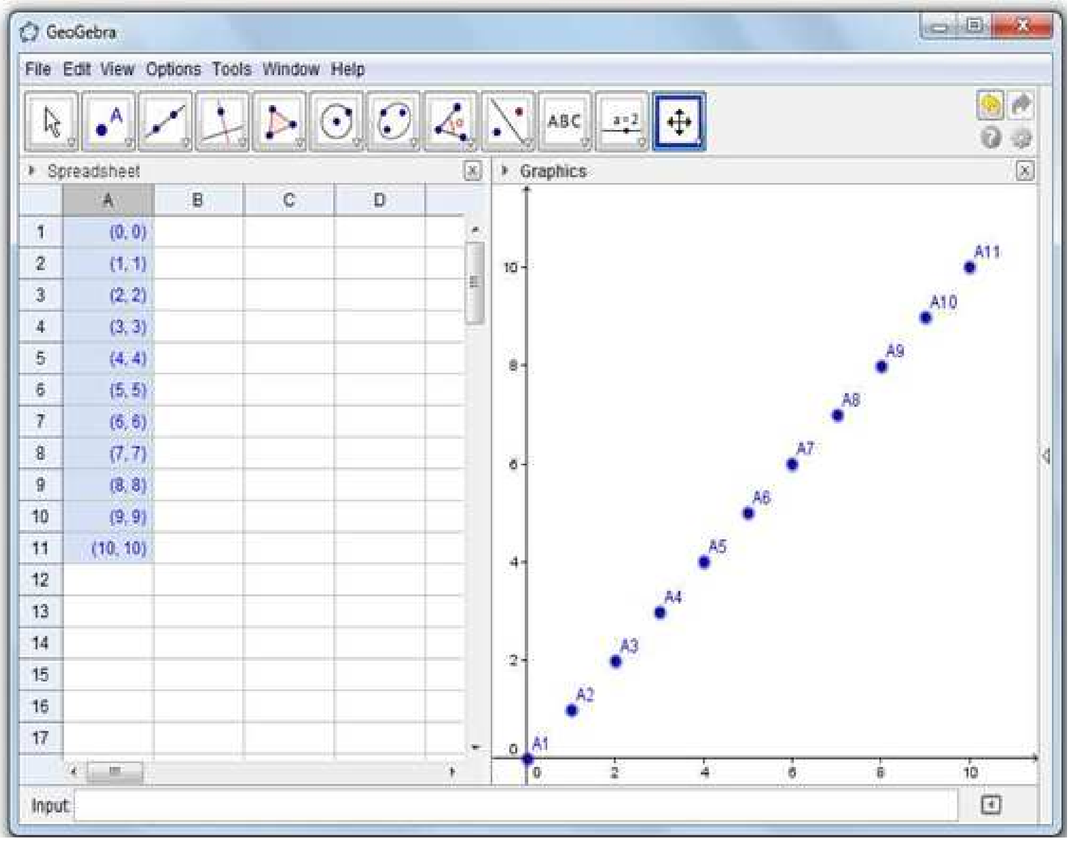
- Write the augmented matrix, which combines the system of variables constants and coefficients.
- b. Convert the matrix into reduced or row-echelon formate using row operations.
- c. Convert the resulting matrix into an equation system and find the variables.
- Find a polynomial function of degree three with the saddle point (1,1) and the point (2,2), for instance.



VIII. SPREADSHEETCELLSINPUT

- Establish an entirely new GeoGebra tab.
- Set the view to Spreadsheet– & Graphical Viewpoint.
- Select the option titled View to access the input bar.

1		Enable the icon . Drag the coordinate system origin to the lower left corner of the Graphics View and then move it.
2	(0,0)	Click on the cell labelled A1 in the Spreadsheet View as well as input the pointcoordinates (0,0).
3	(1,1)	Click on the cell labelled A1 in the Spreadsheet View as well as input thepointcoordinates (1,1).
4		Display both points identifiers in the Visuals Perspective.
5		Generally replicate the input object measurements in various cells in column A by carrying out each of the following steps: (1) Tohighlightened cells A1 and A2; (2) Click on the tiny circle in the bottom-right corner of the highlightened cell range; and (3) Keep the cursor down while dragging your cursor to cell A11.
6	 	To alter the Graphics View's visible area to render every single point visible, use the Move Graphics View, Zoom In, and Zoom Out tools.



IX. MEAN, MEDIAN, MODE AND HISTOGRAM IN SPREADSHEET VIEW

In Geogebra, we can determine the mean, median, and mode using various spreadsheet functionalities. Here's how you can calculate these measures in Geogebra's spreadsheet view:

1. mean

- Open Geogebra and create a new spreadsheet view.
- Enter the values for which you want to calculate the mean in a column, such as Column A.
- In an empty cell, use the formula "=mean(A1:A)" where "A1:A" represents the range of cells containing the values you want to calculate the mean for.
- The result will be displayed as the mean value of the numbers.

2. Median

- In an empty column, such as Column B, copy the values from Column A.
- Sort the values in ascending order by selecting the cells in Column B and choosing the "Sort Ascending" option from the toolbar or right-click menu.
- In an empty cell, use the formula "=median(B1:B)" where "B1:B" represents the range of cells containing the sorted values.
- The result will be displayed as the median value of the numbers.

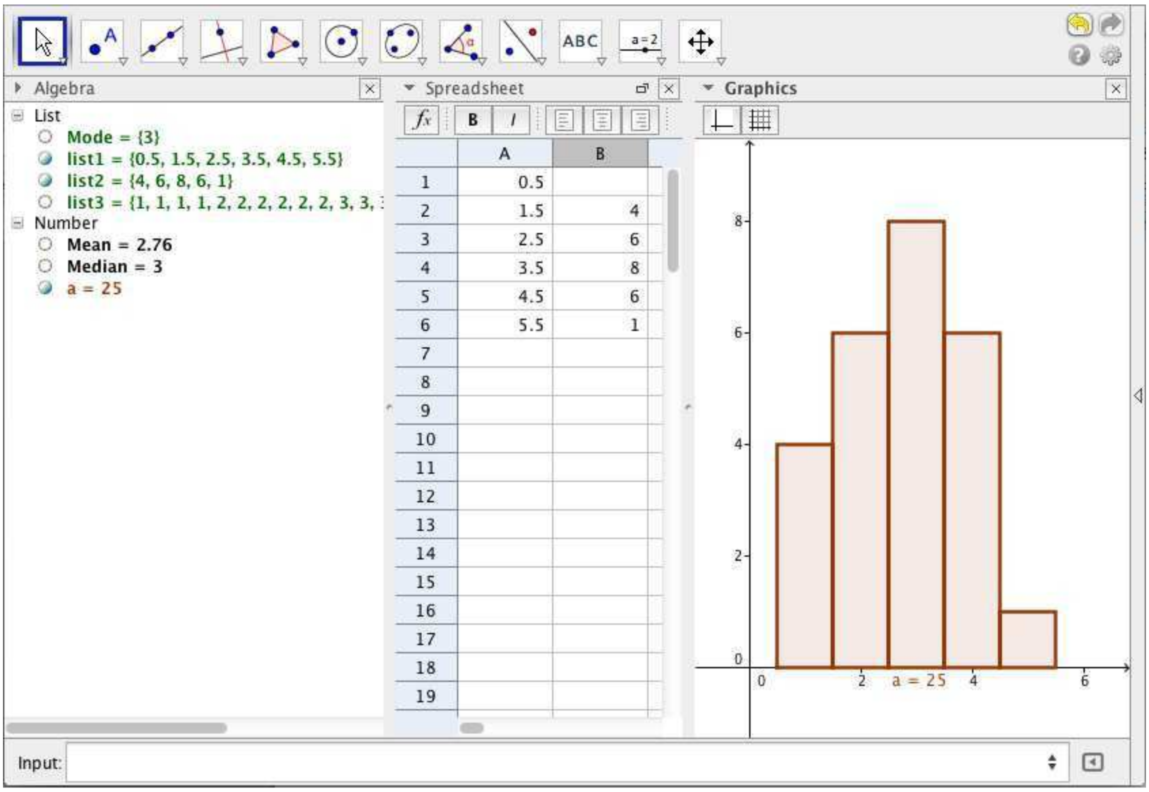
3. Mode

- In an empty column, such as Column C, copy the values from Column A.
- In an empty cell, use the formula "=mode(C1:C)" where "C1:C" represents the range

of cells containing the values for which you want to find the mode.

- The result will be displayed as the mode value(s) of the numbers. Note that if there are multiple modes, only the first mode will be returned.

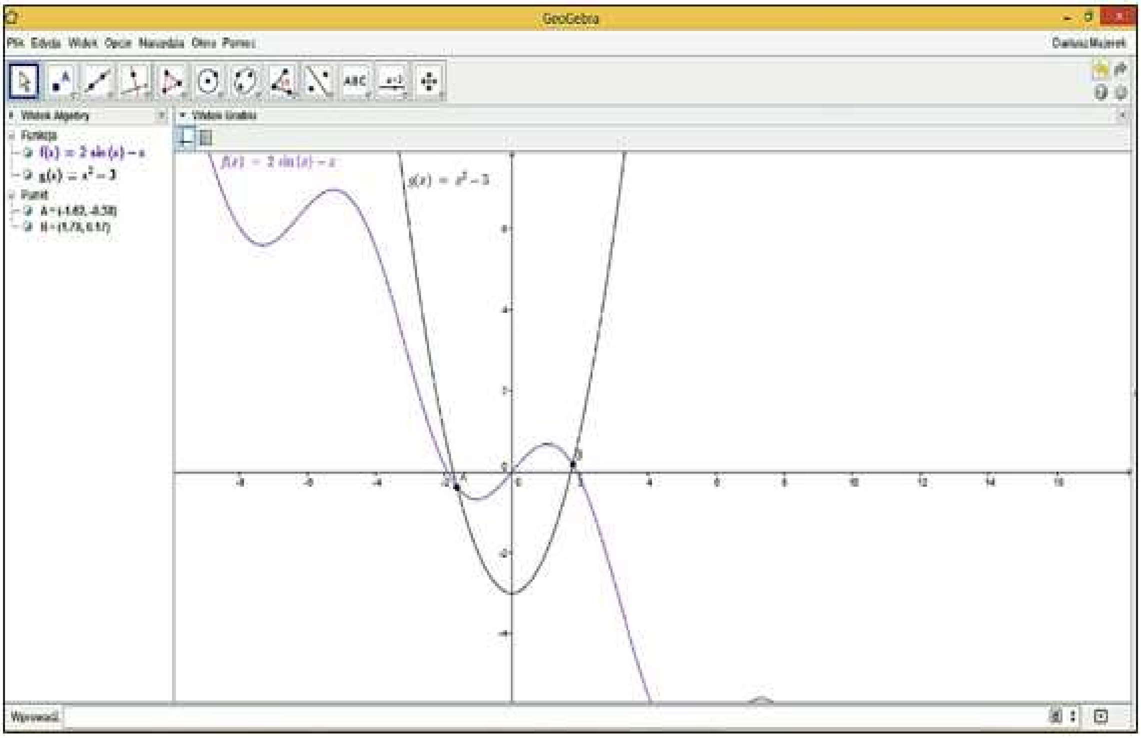
4. In graphics view, visualize the histogram.

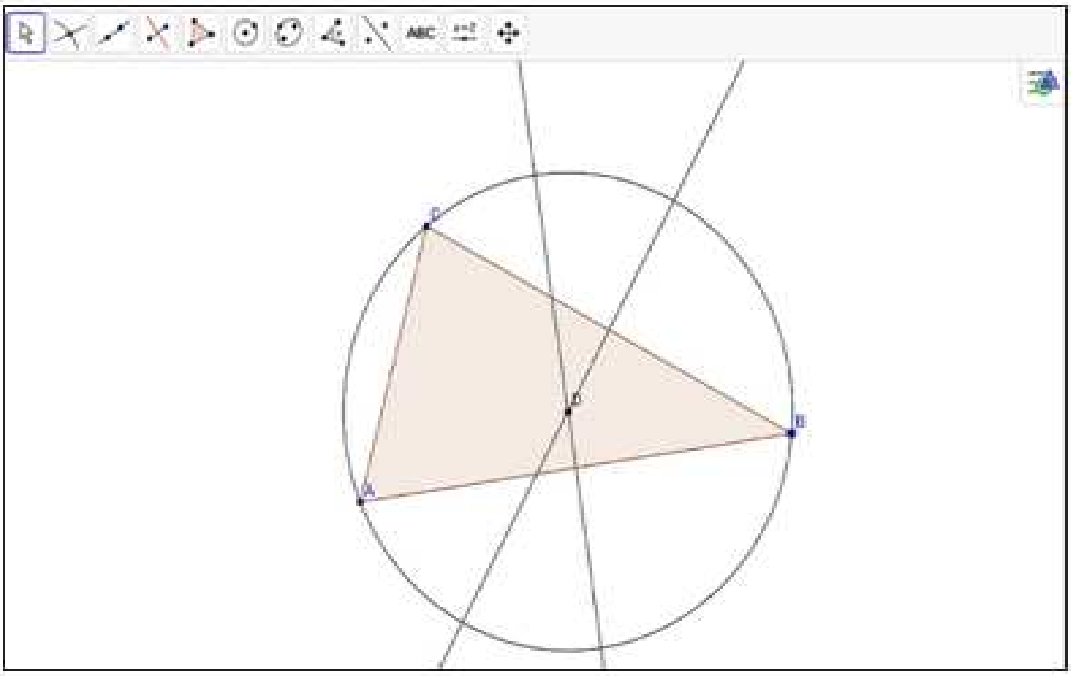


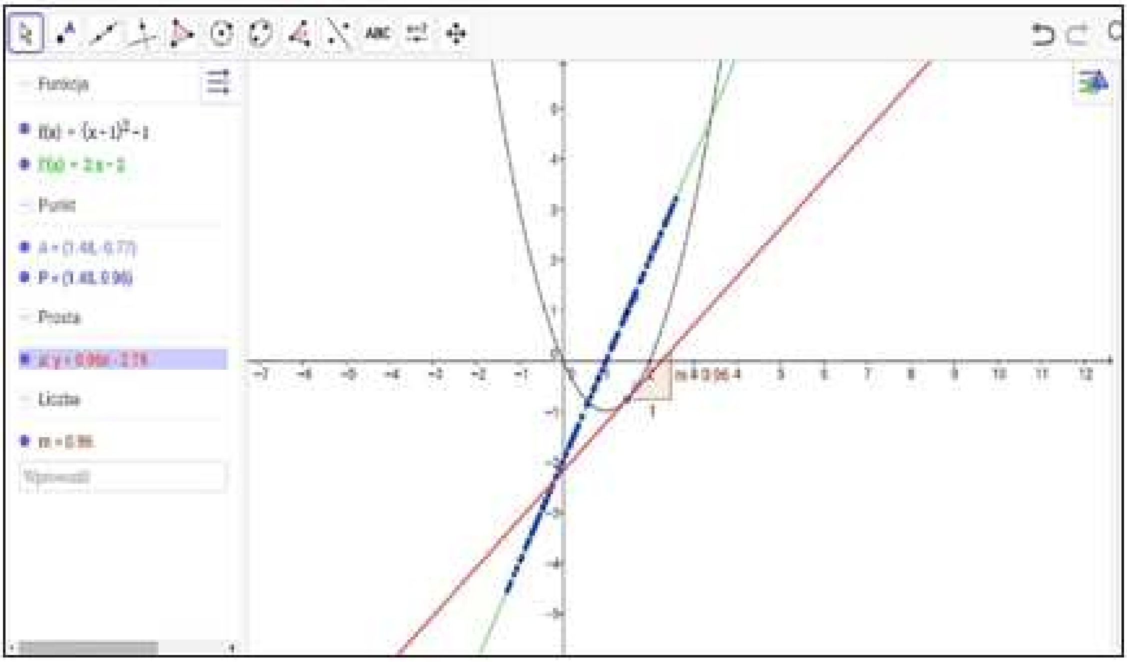
X. CIRCLE AND TANGENT LINE

To construct a circle described on the triangle .There must be a bisector interception at the centre of the circle. The area measured from the point where two bisectors connect one of the vertex equates to the circle's radius. These are the steps involved in constructing a circle. Create two bisectors of any two sides,

- Draw any triangle ABC, then create a circle with a centre in D and a diameter in DA.Learners may comprehend the concept of the descendant of an equation through the use of the following illustration. Obviously, if we alter the formula, all points and lines will change as soon as possible. Students can observe this tendency for many functions.


GeoGebra
f(x) = 2 sin(x) − x
g(x) = x² − 3
Wprowadź

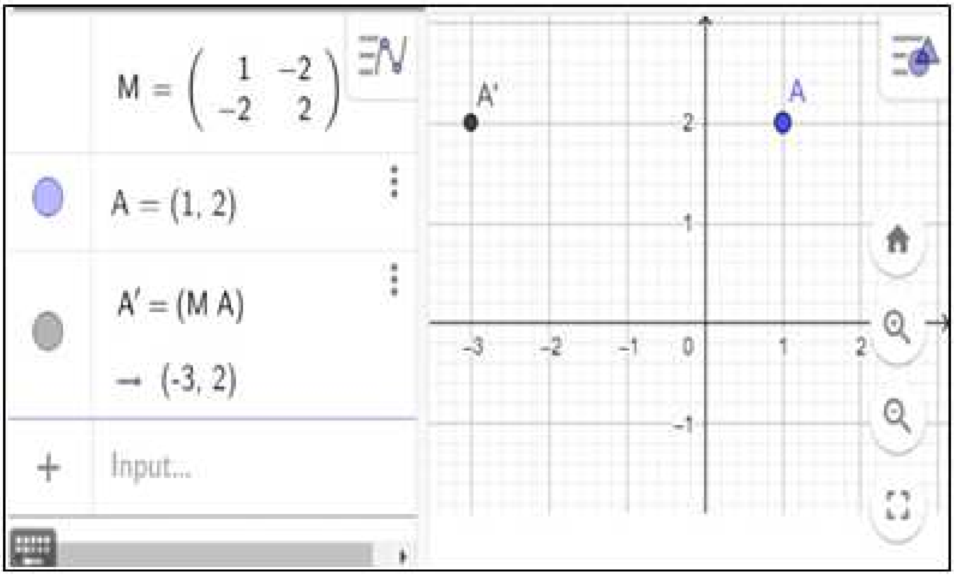

ABC
A
B
C
D



XI. MATRICES IN GOEGEBRA

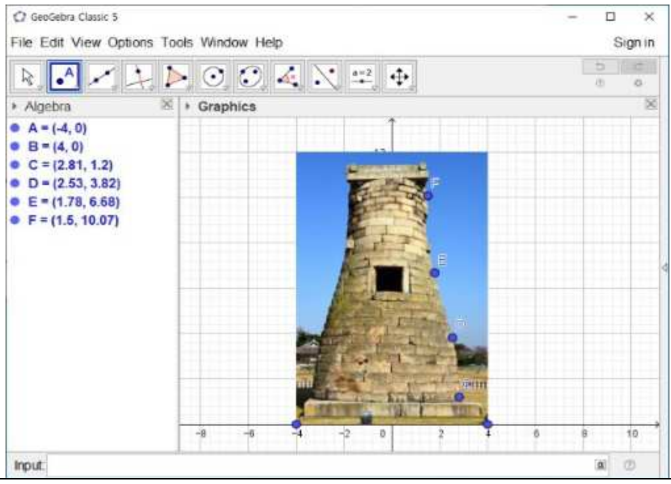
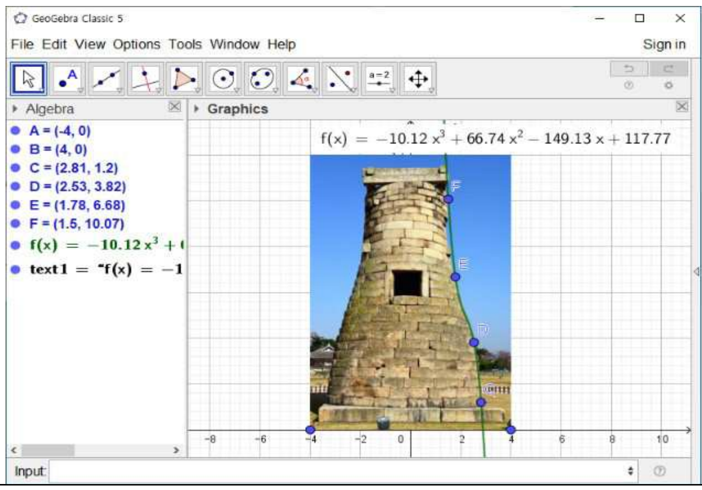
Create an matricesin algebraic view.

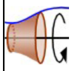
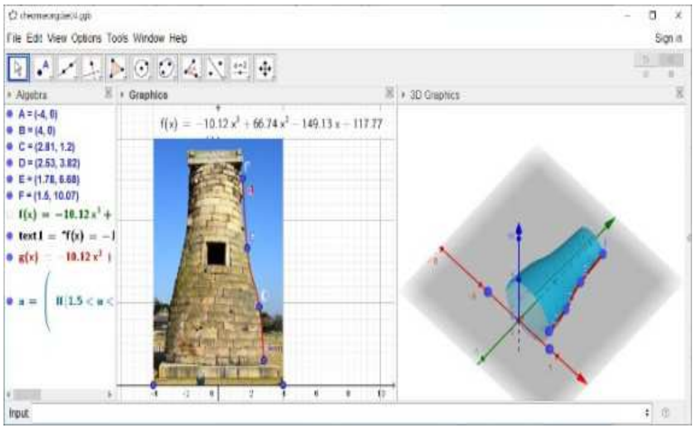
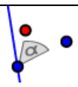
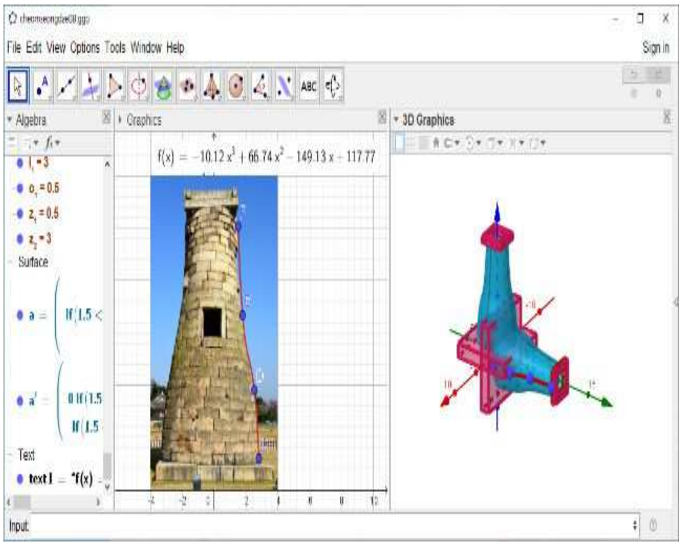
- Enter the formula A'=M*A to calculate the image of a point, A, beneath a change in value, M.
- Input poly1 under the modification, M, to find the shape's image.: **applymatrix[m,poly1].**



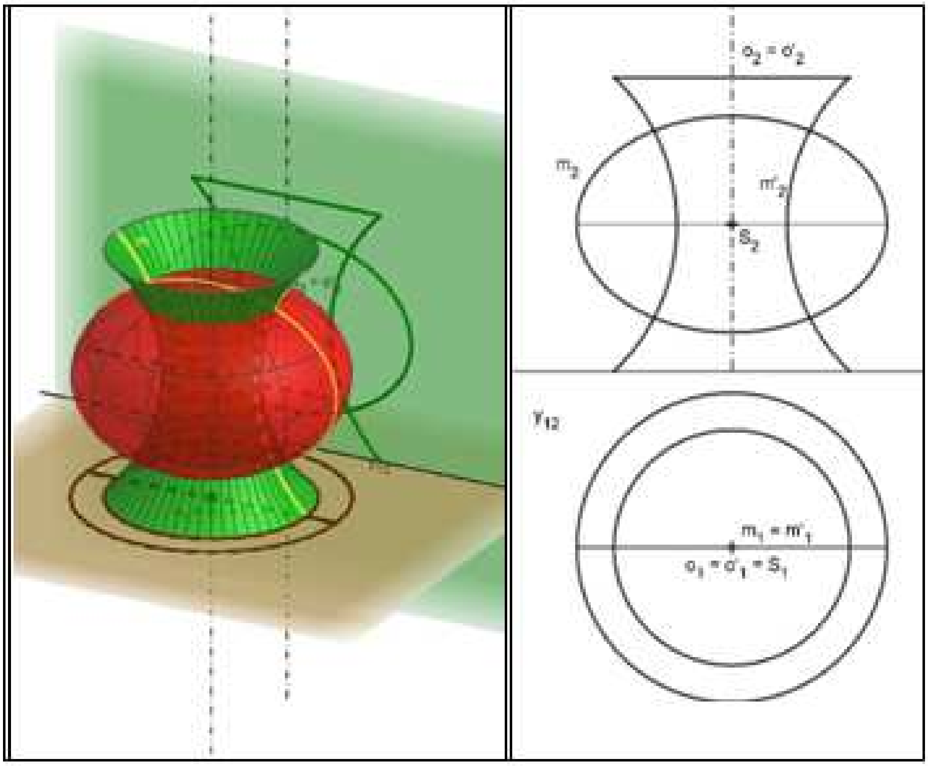
XII. APPLETSINGEOGEBRA

GeoGebra was chosen for process because Sketch-up and CAD are used to carry out extremely advanced modelling but make it impossible for students to use or utilise knowledge. GeoGebra is capable of handling a variety of Dimensions in 2D and 3D as well as mathematical objects in a number of academic fields.Furthermore, models made with GeoGebra may export STL files for 3D printing. The number of formats, such as algebraic equations or 2D/3D views, may be made easier for pupils by using GeoGebra. By using the extrude tool, any GeoGebra polygon can be transformed from a 2D polygon to a 3D polygon. Users get access to an object's various angles and cross sections thanks to GeoGebra. We want to promote students' use of spatial thinking by allowing them to articulate the same models in several ways, especially when transformations are involved. Students can use transformations to engage in speculative modelling. If students use GeoGebra to create the Temple of Dendera or Cheomseongdae, they can engage in augmented reality (AR) exploration or 3D printing. This section details the procedures and resources used to analyse the Cheomseongdae structure and represent it in GeoGebra. Without the bottom and top, Cheomseongdae can be visualised as a rotating body. Thus, the teachers and their students rotating the building's side curve about its central axis.

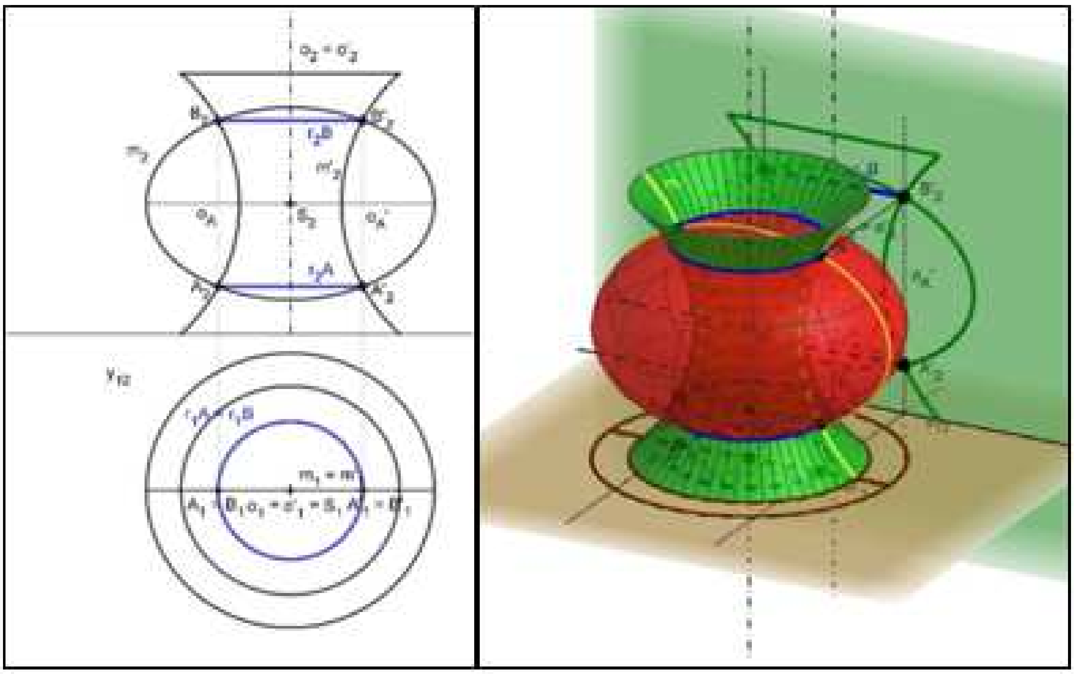
S.no	input	visualization	Instructions
1			Create notations along the curved side of Cheomseongdae.
2	**Polynomial()**		To get a curve composed of polynomials that traverses every point on the screen (Follow that, by adjusting the function's operational range.)

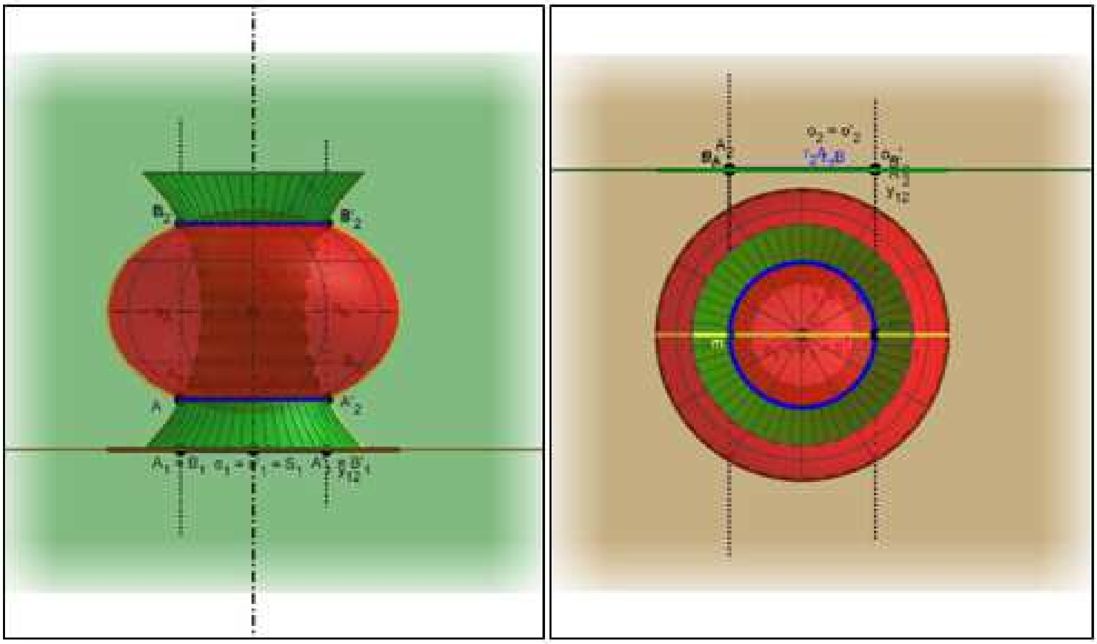
3			The Surfaces aspect of Transformation can be used to generate an irregular surface.
4			Place the Cheomseongdae in the XY plane by using the Rotation Across Lines .

For clarity, certain GeoGebra applets for resolving constructive issues are made up of a 3D window, auxiliary window, and a 2D graphic. This shows a geographical solution to the issue, which is built in the 2D graphic using the Monge projection (the chosen projection method). The problem's assignment, for illustrating the problems construction and solution) are all located in the auxiliary window. Each problem is built step by step using the slider. Additionally, each step of the solution is explained. The perspectives on the models of the surfaces of revolution.

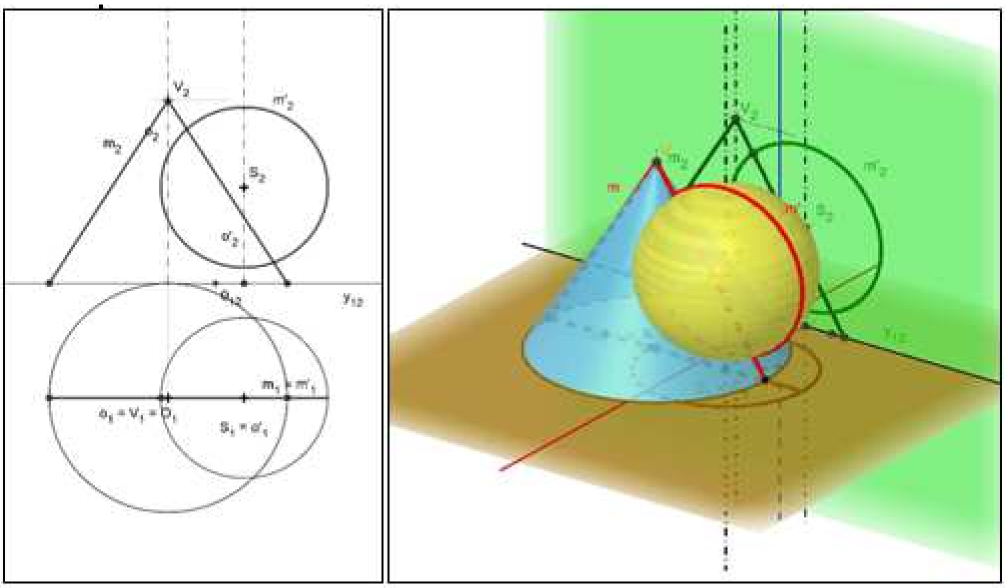


The building of the intersection of two rotation surfaces with converging axes is the first applet (problem). When the axes are in this orientation, the crossing circles of the surfaces are shown from above as circles and from the front as line segments. Students learn from that the primary meridian intersections are located.





The surfaces in the following problem have parallel axes. The intersecting surfaces of the axis of revolution and again parallel to the vertical reference plane. The principal meridians of the surfaces cross at the points of the intersection curve as a result. It is readily visible in the 3D window.



Parallel circles are formed where the surfaces of revolution intersect the adjacent planes; these points of intersection are the curves at which the aforementioned surfaces of revolution are said to intersect. In the type of problem mentioned, the auxiliary surfaces that are helpful for determining the precise points of the curve of intersection are perpendicular planes to the axes of both surfaces

XIII. CONCLUSION

In conclusion, GeoGebra constitutes the most widely used instruments since it can help most people develop the requisite level of geometric ability and spatial imagination. The majority of students find it helpful to view the 3D objects of the intersecting surfaces of revolution in the GeoGebra software 3D window. Students are better able to understand the foundations of mapping specific points of curves of intersection of two revolution surfaces in the Monge projection.

XIV. ACKNOWLEDGEMENT

We would like to express our gratitude to the creators and developers of geogebra for their outstanding work in providing a powerful and versatile software tool for mathematics education. Geogebrahas significantly enriched the teaching and learning of algebra by integrating dynamic geometry, algebraic geometry, algebraic manipulation and graphing capabilities into a single platform.

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