A MATHEMATICAL MODEL OF WAVY PROPOGATION OF FOOD UNDER THE IMPACT OF WALL PROPERTIES AND HEAT TRANSFER

Abstract

Present paper deals with the flow of food bolus through oesophagus under the influence of elastic, wavy walls and heat transfer. Viscous fluid represents food bolus and wavy propagation is shown using cosine wave expression. Analytical approach is used to solve the governing equations by assuming Reynolds number to be low and the wavelength to be long. The closed form solutions are Velocities in both directions, Stream function and Temperature. Investigations are made on velocity, stream function and temperature under the influence of physical parameters like viscous damping force parameter, rigidity, Grashof number, stiffness of the wall, and thermal conductivity. Many interesting facts were observed graphically. One of the important points to be noted is that increase in heat source parameter results in increase of right bolus size. Increase in size of the bolus may lead to formation of bubble in oesophagus which in turn lead to critical health issues. This paper stands as a base for young researchers who are interested in the field of fluid flow with elastic walls.

Keywords: Peristaltic transport, Oesophagus, Food bolus, Viscous fluid.

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I. INTRODUCTION

Peristaltic movement is a form of material transport by a wave propagation of area expansion or contraction. This movement of peristalsis has important applications in understanding many transport processes through physiological systems under peristaltic motion. In the living body peristalsis is involved in transport of many biofluids between organ to organ. (Hewson 1774 and Hall et al. 1965) and in small blood vessels (Nicoll 1956). Burns and Parks (1967), Hanin (1968) and Shapiro et al. (1969) and several others proved that peristaltic movement in biological ducts comes in the form of sinusoidal wave. Many biofluids present in living organisms are more likely to be considered as non-Newtonian fluids. There are several authors Devi and Devanathan (1975); Shukla and Gupta (1982); Srivastava and Srivastava (1984); Usha and Rao(1995), Vajravelu et al. (2005a, 2005b); Hayat and Ali(2008); Hayat et al. (2010a, 2010b) etc who investigated analytically on the flow of non-Newtonian fluid in either tubes or channels with wavy effects in to consideration.

An important fact to be noted is that the movement of food in oesophagus is due to peristalsis. Even if a person is upside down the food enters to stomach through mouth due to its peristaltic movement. Oesophagus is a long muscular tube begins at the neck opposite to the long border of cricoids cartilage and continues from the lower end of the pharynx to the cardiac orifice of the stomach. The swallowing of the food bolus happens due to contraction of the oesophagal wall periodically. Retrograde motion can be noticed due to the process is imbalanced. The influence of wall properties on the Poiseuille flow under peristalsis is studied by Mittra and Prasad (1973). Mishra and Pandey (2001) analyzed a mathematical model for the oesophagal swallowing of a food-bolus. Flow of viscous fluid under peristalsis and wall properties was concentrated by Mokhtar and Haroun (2008) and many interesting facts were observed. Radhakrishnamacharya and Srinivasulu (2007) investigated on heat transfer effects on the peristaltic motion of incompressible fluid with elasticity.

Heat transfer in living organisms is observed while transformation of biofluids between the organs. This bioheat transfer can occur due to acidity, blockages, usage of drugs, usage of electrical gadgets, etc. Many living beings in day-today life’s are under many health issues faced due to radiations caused by mobiles, laptops, earphones etc. In this regard study on heat transfer on living organisms is an essential concept to be noted and further investigation are required. Srinivas et al. (2009) concentrated on wavy movement of fluid flow in channel with slip and MHD.

With this literature survey it is understood that analysis on flow of food bolus through oesophagus with wavy walls and heat transfer is one of hot topics of current research area. Closed form technique is considered to solve the governing equations of the incompressible viscous fluid in a channel. Impact of various parameters on the solutions can be observed through graphical representation, which were plotted using MATLAB software.

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II. MATHEMATICAL FORMULATION

Consider peristaltic movement of an incompressible viscous fluid in a channel with elastic walls induced by cosine wave trains transmitting with constant speed c along the channel walls.

$$\tilde{h}\left(\tilde{\xi}, \tilde{t}\right) = a - \tilde{\phi}\cos^2\frac{\pi}{\lambda}\left(\tilde{\xi} - c\tilde{t}\right) \tag{1}$$

where a is half width of the channel, c is wave velocity, λ is wave length, $\tilde{h}$ is transverse vibration of the wall, $\tilde{t}$ is time, $\tilde{\phi}$ is amplitude of the wave, and $\tilde{\xi}$ is axial coordinate.

The equations governing the flow of Food bolus in a channel with temperature is given by

$$\rho\left[\frac{\partial}{\partial\tilde{t}} + \tilde{u}\frac{\partial}{\partial\tilde{\xi}} + \tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right]\tilde{u} = -\frac{\partial\tilde{p}}{\partial\tilde{\xi}} + \mu\left[\frac{\partial^2\tilde{u}}{\partial\tilde{\xi}^2} + \frac{\partial^2\tilde{u}}{\partial\tilde{\eta}^2}\right] + \rho g\alpha(T - T_0) \tag{2}$$

$$\rho\left[\frac{\partial}{\partial\tilde{t}} + \tilde{u}\frac{\partial}{\partial\tilde{\xi}} + \tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right]\tilde{v} = -\frac{\partial\tilde{p}}{\partial\tilde{\eta}} + \mu\left[\frac{\partial^2\tilde{v}}{\partial\tilde{\xi}^2} + \frac{\partial^2\tilde{v}}{\partial\tilde{\eta}^2}\right] \tag{3}$$

$$\frac{\partial\tilde{u}}{\partial\tilde{\xi}} + \frac{\partial\tilde{v}}{\partial\tilde{\eta}} = 0 \tag{4}$$

$$\rho c_p\left[\frac{\partial}{\partial\tilde{t}} + \tilde{u}\frac{\partial}{\partial\tilde{\xi}} + \tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right]T = K\left[\frac{\partial^2 T}{\partial\tilde{\xi}^2} + \frac{\partial^2 T}{\partial\tilde{\eta}^2}\right] + \Phi \tag{5}$$

where Φ is constant heat addition/absorption, K is thermal conductivity, c_p is specific heat at constant pressure, T is temperature, α is coefficient of linear thermal expansion of fluid, g is acceleration due to gravity, μ is fluid viscosity, $\tilde{p}$ is pressure, , $\tilde{\eta}$ is transverse coordinate, $\tilde{v}$ is transverse velocity, $\tilde{u}$ is axial velocity, and ρ is fluid density.

Temperature defined at centre of the channel and at the wall of the channel are as below

$$T = T_0 \text{ at } \tilde{\eta} = 0, T = T_1 \; at \; \tilde{\eta} = \tilde{h} \tag{6}$$

where T_1 is the temperature on the wall of peristaltic channel and T_0 is the temperature at centre is line.

The equation governing the motion of the elastic wall can be considered as

$$L^*\left(\tilde{h}\right) = \tilde{p} - \tilde{p}_0$$

where L^* is an operator, which is used to represent the motion of stretched membrane with viscosity damping forces such that

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$$L^* = -\tau \frac{\partial^2}{\partial \xi^2} + m_1 \frac{\partial^2}{\partial t^2} + C \frac{\partial}{\partial t}$$

Continuity of stress at $\eta = h$ and using momentum equation, yield

$$\frac{\partial}{\partial \tilde{\xi}} L^*(\tilde{h}) = \frac{\partial \tilde{p}}{\partial \tilde{\xi}} = \mu \left[\frac{\partial^2 \tilde{u}}{\partial \tilde{\xi}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{\eta}^2}\right] + \rho g \alpha (T - T_0) - \rho \left[\frac{\partial}{\partial \tilde{t}} + \tilde{u} \frac{\partial}{\partial \tilde{\xi}} \tilde{v} \frac{\partial}{\partial \tilde{\eta}}\right] \tilde{u} \tag{7}$$

Here τ is the elastic tension in the membrane, m_1 is the mass per unit area, C is the coefficient of viscous damping forces.

Introducing the following non – dimensional quantities,

$$\xi = \frac{\tilde{\xi}}{\lambda}, \eta = \frac{\tilde{\eta}}{a}, t = \frac{c\tilde{t}}{\lambda}, u = \frac{\tilde{u}}{c}, v = \frac{\tilde{v}}{c\delta}, \delta = \frac{a}{\lambda}, h = \frac{\tilde{h}}{a}, \phi = \frac{\tilde{\phi}}{a}, P = \frac{\tilde{p}a^2}{\mu c \lambda}, \psi = \frac{\tilde{\psi}}{ac},$$

$$Q = \frac{\tilde{Q}}{ac}, Re = \frac{\rho c a \delta}{\mu}, Gr = \frac{g\rho\alpha\, a^2 (T_1 - T_0)}{\mu^2 c}, \theta = \frac{T - T_0}{T_1 - T_0}, \beta = \frac{a^2 \Phi}{k(T_1 - T_0)}, \ \mathrm{Pr} = \frac{\mu c_p}{k} \tag{8}$$

where Pr is Prandtl number, β is dimensionless heat source/sink parameter, θ is dimensionless temperature, Gr is Grashof number, Re is Reynolds number, $\tilde{Q}$ is volume flow rate, $\tilde{\psi}$ is stream function and δ is wave number.

in equations (1 – 8), we finally get

$$h(\xi, t) = 1 - \phi \cos^2 \pi(\xi - t) \tag{9}$$

$$Re \left[\frac{\partial}{\partial t} + u \frac{\partial}{\partial \xi} + v \frac{\partial}{\partial \eta}\right] u = -\frac{\partial p}{\partial \tilde{\xi}} + \left[\delta^2 \frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2}\right] + Gr\theta \tag{10}$$

$$Re\delta^3 \left[\frac{\partial}{\partial t} + u \frac{\partial}{\partial \xi} + v \frac{\partial}{\partial \eta}\right] v = -\frac{\partial p}{\partial \eta} + \delta^4 \frac{\partial^2 v}{\partial \xi^2} + \delta^2 \frac{\partial^2 u}{\partial \eta^2} \tag{11}$$

$$\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta} = 0 \tag{12}$$

$$\frac{Re\ Pr}{(T_1 - T_0)} \left[\frac{\partial}{\partial t} + u \frac{\partial}{\partial \xi} + v \frac{\partial}{\partial \eta}\right] [\theta(T_1 - T_0) + T_0] = \delta^2 \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} + \beta \tag{13}$$

$$\delta^2 \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} + Gr\theta - Re \left[\frac{\partial}{\partial t} + u \frac{\partial}{\partial \xi} + v \frac{\partial}{\partial \eta}\right] u = \left[E_1 \frac{\partial^3}{\partial \xi^3} + E_2 \frac{\partial^3}{\partial \xi \partial t^2} + E_3 \frac{\partial^2}{\partial \xi \partial t}\right] (h) \tag{14}$$

$$\theta = 0\ at\ \eta = 0, \theta = 1\ at\ \eta = h \tag{15}$$

III. CLOSED FORM SOLUTION

Assuming Reynolds number to be low and the wavelength to be long, equations (9 - 15) becomes

$$h(\xi, t) = 1 - \phi \cos^2 \pi(\xi - t) \tag{16}$$

$$\frac{\partial P}{\partial \xi} = \frac{\partial^2 u}{\partial \eta^2} + Gr\theta \tag{17}$$

$$\frac{\partial P}{\partial \eta} = 0 \tag{18}$$

$$\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta} = 0 \tag{19}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} + \beta = 0 \tag{20}$$

$$\theta = 0 \; at \; \eta = 0, \theta = 1 \; at \; \eta = h \tag{21}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} + Gr\theta = E_1 \frac{\partial^3 h}{\partial \xi^3} + E_2 \frac{\partial^3 h}{\partial \xi \partial t^2} + E_3 \frac{\partial^2 h}{\partial \xi \partial t} \tag{22}$$

The boundary conditions will become

$$\left.\frac{\partial u}{\partial \eta}\right|_{\eta=0} = 0 \tag{23}$$

$$u|_{\eta=h} = 0 \tag{24}$$

$$v|_{\eta=0} = 0 \tag{25}$$

From (17) we get

$$\frac{\partial^3 u}{\partial \eta^3} + Gr \frac{\partial \theta}{\partial \eta} = 0 \tag{26}$$

Equation (22) gives $\frac{\partial^2 u}{\partial \eta^2} + Gr\theta = E_1 \frac{\partial^3 h}{\partial \xi^3} + E_2 \frac{\partial^3 h}{\partial \xi \partial t^2} + E_3 \frac{\partial^2 h}{\partial \xi \partial t}$ (27)

The closed form solution for equations (17) and (20) with the boundary conditions (21),(23) and (24) is

$$\theta = \frac{\eta}{h} + \frac{\beta}{2}[h\eta - \eta^2] \tag{28}$$

$$\begin{aligned} u = \pi^2\phi\left(\eta^2 - h^2\right)\Big\{&E_3\big(\sin^2\pi(\xi - t) - \cos^2\pi(\xi - t)\big) \\ &- (E_1 + E_2)\{4\pi\cos\pi(\xi - t)\sin\pi(\xi - t)\}\Big\} \\ &+ \frac{Gr}{6}\left[\frac{\beta}{4}\left[h^4 - 2\eta^3 h + \eta^4\right] - \frac{1}{h}\left[\eta^3 - h^3\right]\right] \end{aligned} \tag{29}$$

Using equation (29) and the boundary condition (25), we get transverse velocity as

$$\begin{aligned} v = -\pi^2\phi\eta\left(\frac{\eta^3}{3} - h^2\right)&\{4\pi E_3\sin4\pi(\xi - t) - (E_1 + E_2)4\pi^2\cos2\pi(\xi - t)\} \\ &- 2\pi^2\phi h\eta\frac{\partial h}{\partial \xi}\{E_3\cos4\pi(\xi - t) + (E_1 + E_2)2\pi\sin2\pi(\xi - t)\} \\ &- \frac{Gr}{6}\left[\frac{\beta}{4}\left[4h^4\eta - \frac{2\eta^4}{4}\right] - \left[\frac{\eta^3}{3h^2} + 2h\eta\right]\right]\frac{\partial h}{\partial \xi} \end{aligned} \tag{30}$$

Stream function can be obtained by integrating eqn (29) and using the condition $\psi = 0$ at $\eta = 0$. It is given by

$$\psi = \pi^2\phi\eta\left(\frac{\eta^3}{3} - h^2\right)\Big\{E_3\left(\sin^2\pi(\xi - t) - \cos^2\pi(\xi - t)\right) - (E_1 + E_2)\,\{4\pi\cos\pi(\xi - t)\sin\pi(\xi - t)\}\Big\} + \frac{Gr}{6}\left[\frac{\beta}{4}\left[h^4\eta - \frac{\eta^4 h}{2} + \frac{\eta^5}{5}\right] - \frac{1}{h}\left[\frac{\eta^4}{4} - h^3\eta\right]\right] \quad (31)$$

IV. RESULTS AND DISCUSSIONS

The governing equations are solve in closed form and impact of physical parameters over velocity, temperature and stream function are noticed with the help of graphs plotted using MATLAB in Fig(1)-(11). From Fig.(1) and (2) it is noticed that velocity increases with raise in thermal conductivity β and Grashof number Gr in the oesophagus.

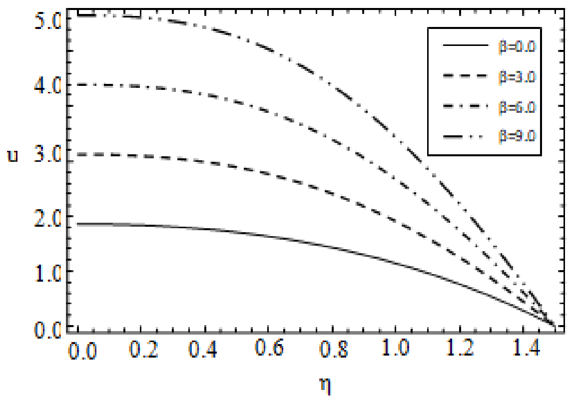

Figure 1: Velocity Profiles for different β $(E_1 = 0.7, E_2 = 0.5, E_3 = 0.1$ $t = 0.5, Gr = 2, \eta = 0.5)$

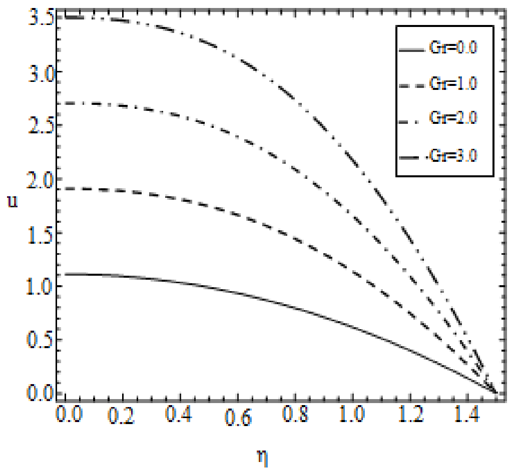

Figure 2: Velocity Profiles for different Gr $(E_1 = 0.7, E_2 = 0.5, E_3 = 0.1, t = 0.5,$ $\eta = 0.5, \beta = 2)$

In the presence of stiffness $(E_2 \neq 0)$ and viscous damping force $(E_3 \neq 0)$Fig. (3) shows the impact of rigidity parameter. Graphs tells that raise in rigidity parameter leads to raise in velocity. In the presence of E_1 *and* E_3 impact of E_2 on velocity is observed in Fig. (4), which shows that velocity and E_2 travel in the same direction.

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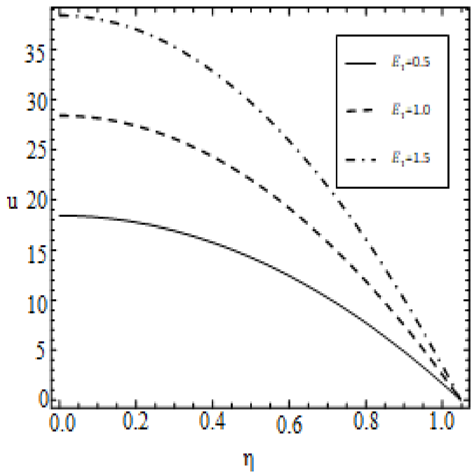


Figure 3: Effect of E_1 on velocity u $(E_2 = 0.5, E_3 = 0.5, Gr = 2, \beta = 2, t = 0.1, x = 0.5)$

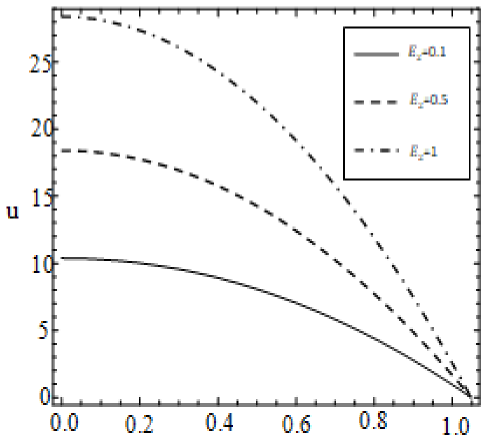


Figure 4: Effect of E_2 on velocity u $(E_1 = 0.5, E_3 = 0.5, Gr = 2, \beta = 2, t = 0.1, x = 0.5)$

Fig. (5) speaks about the variation of E_3on velocity by fixing other parameters. It says that velocity and E_3 travel in opposite direction.

Impact of thermal conductivity β on velocity is depicted in Fig. (6). From this figure it is clear that velocity and thermal conductivity pass parallel.

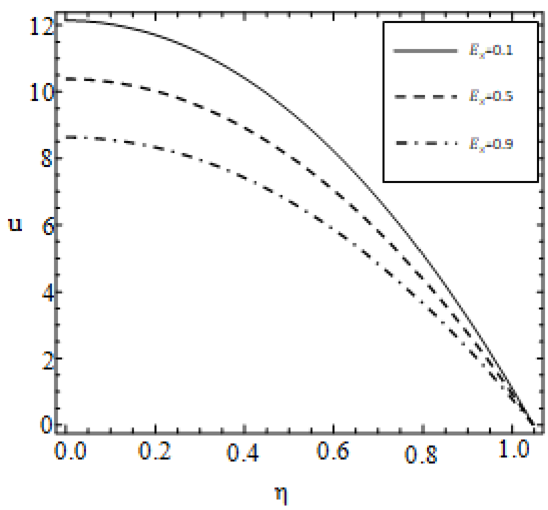


Figure 5: Effect of E_3 on velocity u (E_1=0.5,E_2=0.5,Gr=2,β=2, t=0.1,x=0.5)

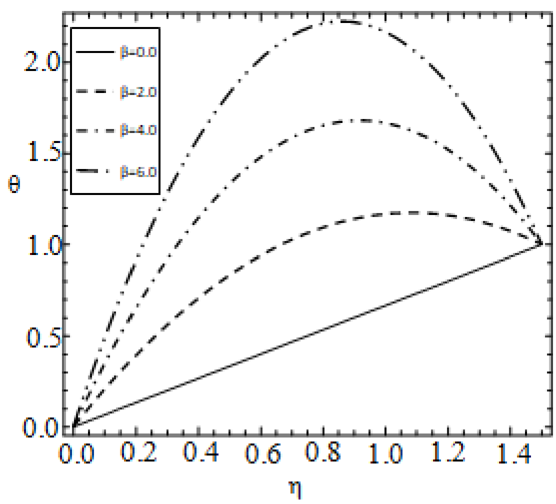


Figure 6: Temperature profiles for different β

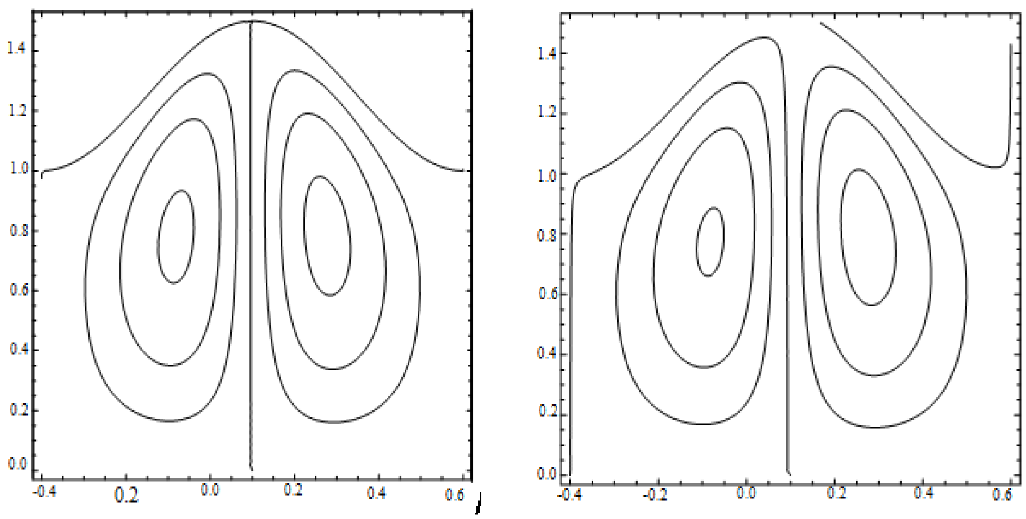
Peristaltic movement of the elastic wall creates interest to study on trapping, where in wave frame the streamlines split to trap a bolus. Influence of thermal conductivity is studied on trapping through Fig. (7). It is noticed that increase in thermal conductivity will increase the size of the bolus in right side of the channel and decrease the size of the bolus in left side of the channel.

Fig. (8) reflects the impact of Grashof number on trapping. As Gr increases the size of the bolus decreases in left of channel and increases in right of channel. The effect of E_1 on trapping can be seen in Fig.(9). We notice that the size of the bolus increases with increase in E_1. Fig. (10) shows the influence of E_2 on trapping. One can observe that the size of the trapped bolus decreases with increase in E_2, stiffness of the wall. The effect of E_3 on trapping is shown in Fig. (11). It is concluded that increase in E_3 result in decrease in size of the left bolus and increase in right bolus. Furthermore, it is observed that more trapped bolus appears with increase in E_3.

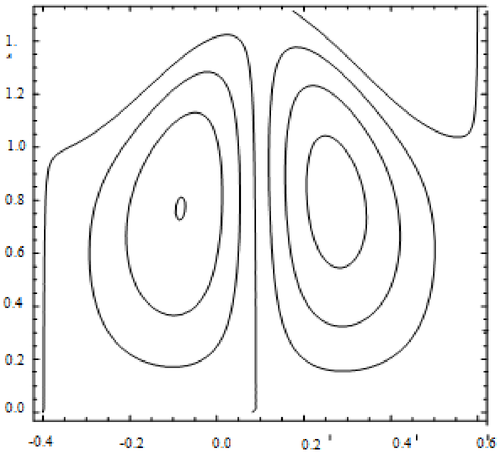
V. CONCLUSIONS

By analysing viscous fluid flow in a channel with elastic walls and heat transfer the following conclusions are made.

1. The axial velocity increases with the increase in β,Gr,E_1,E_2. Further, the axial velocity decreases with increase in E_3.
2. The coefficient of temperature increases with increasing values of thermal conductivity.
3. The volume of the trapped bolus increases with increase in E_1. Moreover, more trapped bolus appears with increase in E_1.

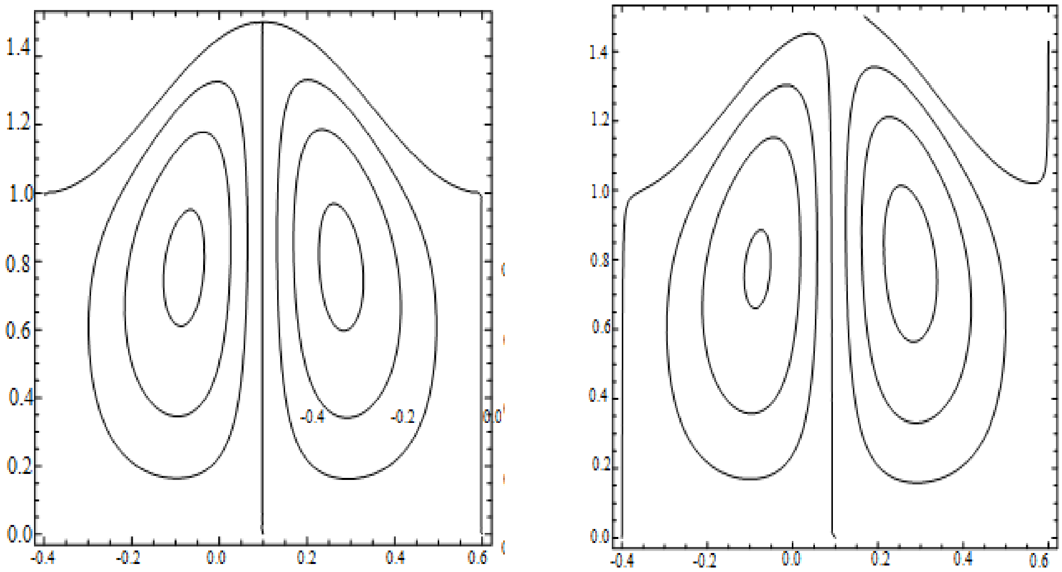


(a) (b)

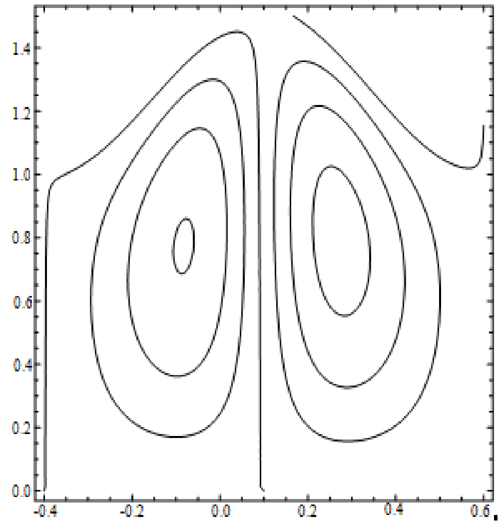


(c)

Figure 7 : Effect of β on Trapping (a) β=0 (b) β=4 (c) β=8
(E_1=0.7,E_2=0.5,E_3=1,t=0.1,Gr=2)

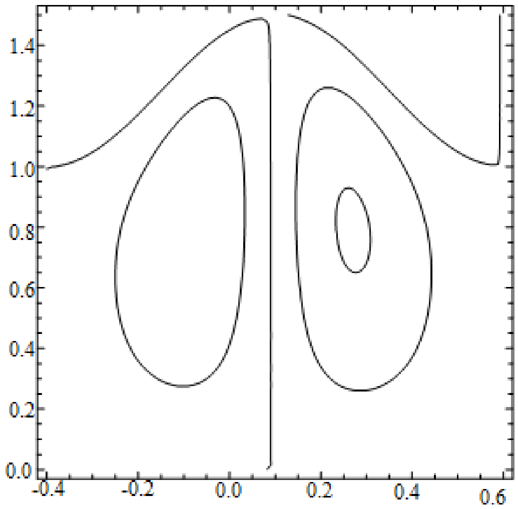


(a) (b)

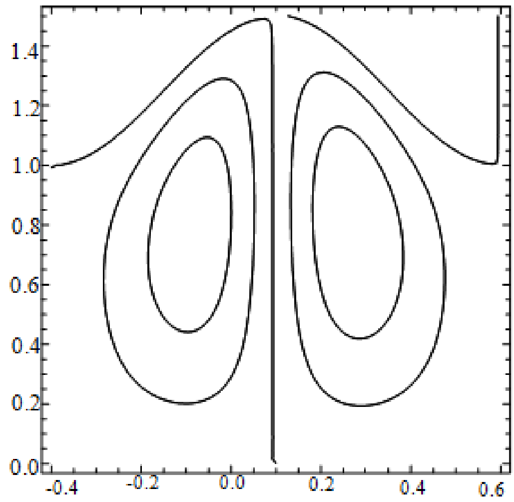


(c)

Figure 8: Effect of Gr on Trapping (a) Gr=0 (b) Gr=2 (c) Gr=4 ($E_1=0.7, E_2=0.5, E_3=1, t=0.1, \beta=2$)



(a)



(b)

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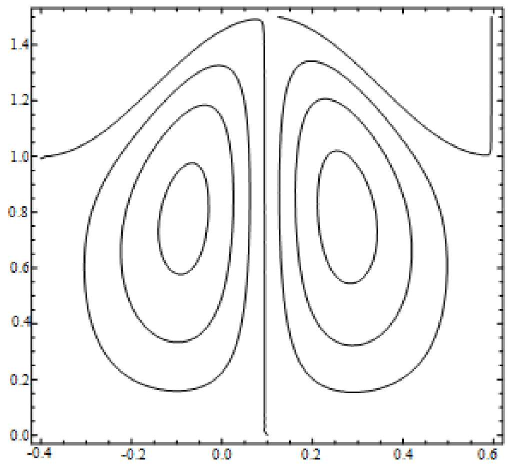
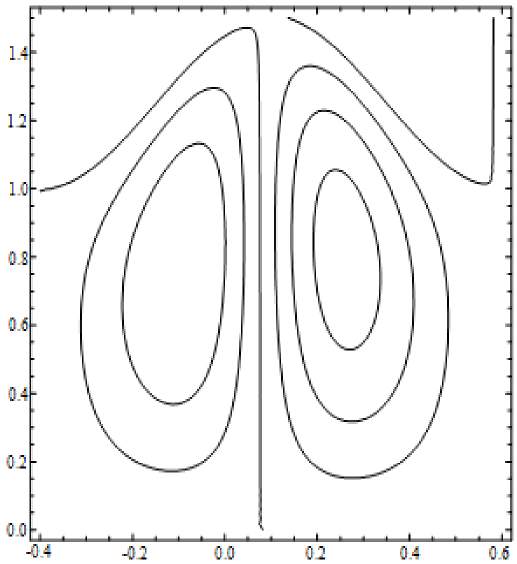
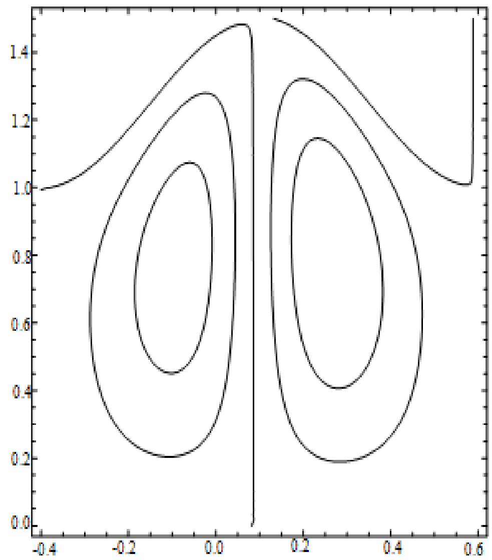


(c)

Figure 9 : Effect of E_1 on Trapping (a) E_1=1 (b) E_1=1.5 (c) E_1=2 (Gr=2,E_2=0.5,E_3=0.5,t=0.1,β=2)



(a)



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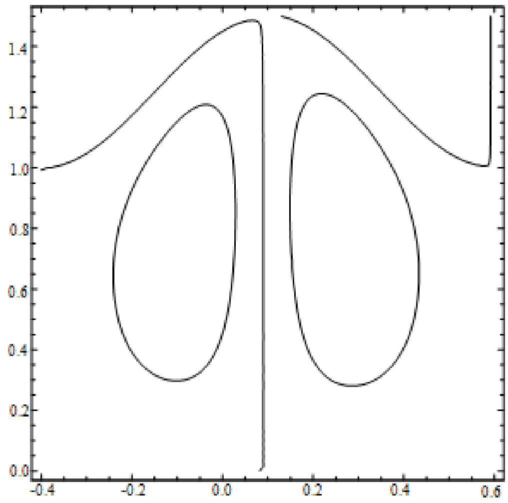
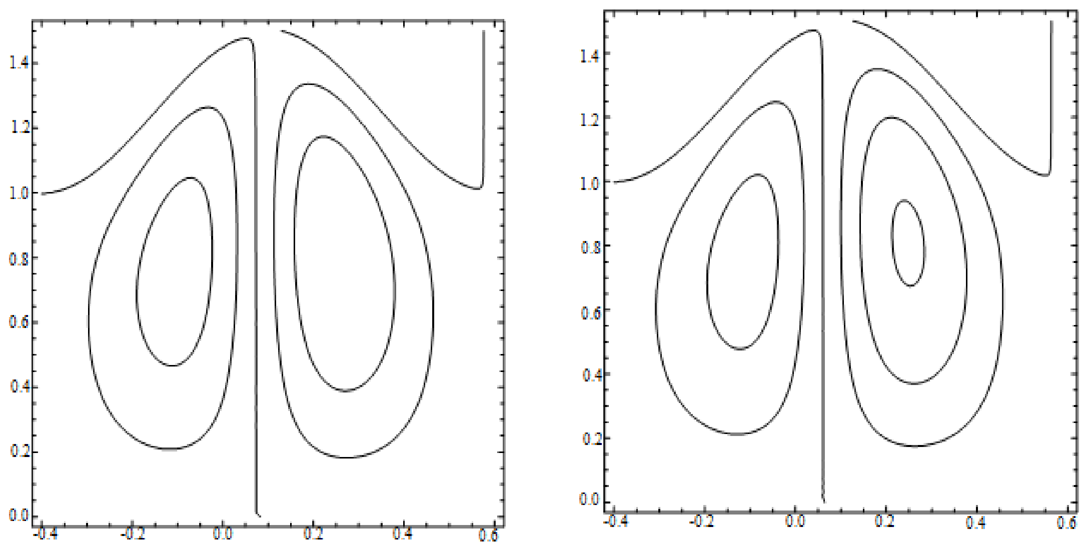
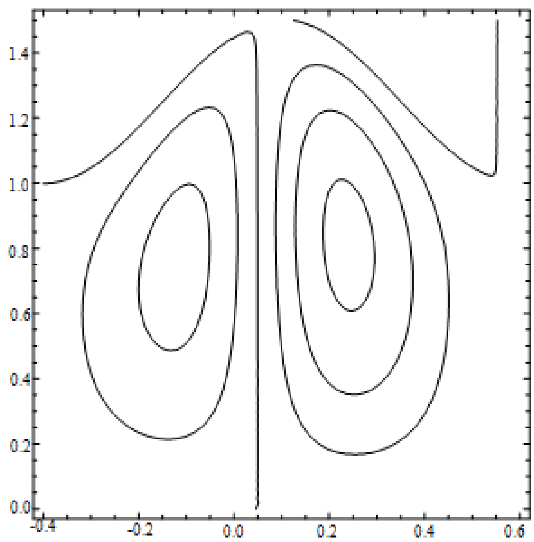


(c)

Figure 10: Effect of E_2 on Trapping (a) E_2=0.1 (b) E_2=0.5 (c) E_2=0.9 (Gr=2,E_1=0.5,E_3=0.5,t=0.1,β=2)



(a) (b)



(c)

Figure 11: Effect of E_3 on Trapping (a)E_3=1 (b) E_3=1.5 (c) E_3=2 (Gr=2,E_1=0.5,E_2=0.5,t=0.1,β=2)

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