LAYER TOPOLOGY

Abstract

The broad category of whorled patterns, which also comprises a curve that emanates from a point and moves outward as it rounds around the point, contains concentric objects frequently. This ignited us to think over collection of open sets forms a chain where the core open set is non empty and all the other open sets originates from it. This paves us a way to develop the new concept layer topology. Moreover, an attempt has been done to extend this definition in infinite domains like real numbers as standard layer topology, lower limit layer topology and upper limit layer topology in terms of bases. A comparative study has also done among them. Its properties and characterizations were also studied. Finally, we have given a new graphic approach to the layer topological structure. Further it was extended to associate the layer open sets with some special types of graphs such as cycle, path, and complete graph and complete bipartite graph.

Keywords: $\mathcal{L}$ -open set, $\mathcal{L}$ -closed set, core set, layer base

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I. INTRODUCTION

One important area of mathematics is topology. Topology can be described in many ways like usual definition, in terms of basis, subbasis or through some operators. In 1983, Mashhour et al. came up with an idea of supra topology by relaxing a finite intersection condition of topological spaces. In 2015, Adel. M. Al-Odhari, introduced the concept of infra topological spaces removing the arbitrary union condition of topological spaces. In 2022, [2]Amer Himza Almyaly introduced the concept of interior topology. In nature all occurring process are towards a centre and all the other objects are originated from it. Likewise, we have made an attempt to explore the notion of layer space. The concept of layer topology was initiated from a nonempty core open set which is a source of the other layer open sets. It is named so because, open sets in layer space forms some pattern which originates from a core open set. It forms an increasing chain of layer open sets that ends up with the whole set where the layer space was defined. Also there doesn't exist any disjoint layer open sets.

Graph theory, gives us a hand to have pictorial view of many real-life problems. Many researchers have connectively studied topology and graph theory. In such a way we have also linked our new concept layer topology to simple connected undirected graphs by using the concept of closed neighborhood of a vertex of a graph. Moreover some of its properties were discussed on special graphs.

II. PRELIMINARIES

In this context, we have recollected certain fundamental definitions that are required f or our work.

1. **Definition 2.1 [4]** A topology on a nonempty set X is a collection τ of subsets of X having the following properties: (i) ϕ and X are in τ. (ii) The union of the elements of any subcollection of τ is in τ. (iii) The intersection of the elements of any finite subcollection of τ is in τ. A set X for which a topology τ has been specified is called a topological space.

2. **Definition 2.2 [4]** If Y is a subset of X, the collection $\tau_Y = \{Y \cap U : U \in \tau\}$ is a topology on Y, called the subspace topology(relative topology).

3. **Definition 2.3 [2]** Let X be a nonempty set. A subclass $I_t \subseteq P(X)$ is called interior topology on X if the following are satisfied: (a) $\emptyset \notin I_t$ (b) It is closed under an arbitrary union of elements of I_t (c) It is closed under the arbitrary intersection of elements of I_t. An interior topological space is set X together with the interior topology I_t on X.

4. **Definition 2.4 [5]** A graph G = (V, E) consists of a set of objects $V = \{v_1, v_2, \cdots\}$ called vertices, and another set $E = \{e_1, e_2, \cdots\}$, whose elements are called edges, such that each edge e_k is identified with an unordered pair (v_i, v_j) of vertices.

5. **Definition 2.5 [5]** A graph G is finite if the number of vertices and the number of edges in G is finite; otherwise, it is an infinite graph.

6. **Definition 2.6 [5]** If any vertex can be reached from any other vertex in G by travelling along the edges, then G is called connected graph.

7. **Definition 2.7 [5]** The number of edges incident on a vertex v is called the degree. A vertex of degree one is called an end vertex.

8. **Definition 2.8 [5]** A walk is defined as a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it. No edge appears more than once in a walk. It is possible for a walk to begin and end at the same vertex. Such a walk is called a closed walk. A walk that is not closed is called an open walk.

9. **Definition 2.9 [5]** An open walk in which no vertex appears more than once is called a path.

10. **Definition 2.10 [5]** A closed walk in which no vertex (except the initial and the final vertex) appears more than once is called a circuit. A circuit is also called a cycle.

III. NEW RESULTS

1. **Layer topology:** In this section, we have introduced the concept of $\mathscr{L}$ - space and studied some of its properties.

Definition 3.1 Let X be a non-empty set. The collection $\mathscr{L}$ of subsets of X is called Layer topology on X if the following conditions are satisfied: (i) $\varphi \notin \mathscr{L}$ and $X \in \mathscr{L}$ (ii) If $A_1, A_2, \cdots, A_n$ ($n \in \wedge$) $\in \mathscr{L}$, then $A_1 \subset A_2 \subset \cdots \subset A_n$. Also (X, $\mathscr{L}$) is called as the layer topological space. In shortly we may denote it as $\mathscr{L}$ - space. The elements of $\mathscr{L}$ are called $\mathscr{L}$ -open sets. By condition (ii), arbitrary union of $\mathscr{L}$ -open sets is $\mathscr{L}$ -open and arbitrary intersection of $\mathscr{L}$ -open sets is $\mathscr{L}$ -open.

Example 3.2 Let X = {a, b, c, d}. Then $\mathscr{L}$ = {{a}, {a, b}, {a, b, c}, X} is a layer topology on X. Also (X, $\mathscr{L}$) is the $\mathscr{L}$ - space.

Example 3.3 Let X = C, the set of all complex numbers and $\mathscr{L}$ = {N, W, Z, Q, R, C}, where N is the set of all natural numbers, W is the set of all whole numbers, Z is the set of all integers, Q is the set of all rational numbers, R is the set of all real numbers. Clearly (X, $\mathscr{L}$) is the $\mathscr{L}$ - space.

Definition 3.4 A layer topology $\mathscr{L}$ on a set X having only two $\mathscr{L}$-open sets is called the indiscrete $\mathscr{L}$ - space.

Remark 3.5 The following example shows that indiscrete layer topology on a set X is not unique.

Example 3.6 (i) Let X = {a, b, c, d, e}. The following are some of the indiscrete layer topologies on X.
$\mathscr{L}_1$ = {{a, b, c}, X}
$\mathscr{L}_2$ = {{a, d}, X}

$\mathscr{L}_3$ = {{b}, X}
(ii) Let X = R, the set of all real numbers.
$\mathscr{L}_1$ = {Z, R}, where Z is the set of all integers
$\mathscr{L}_2$ = {Q, R}, where Q is the set of all rational numbers

Definition 3.7 A layer topology $\mathscr{L}$ on a set X having 'n' elements is said to be discrete if it has exactly 'n' number of $\mathscr{L}$-open sets. (i.e) $|\mathscr{L}| = n$.

Remark 3.8 The following example shows that the discrete layer topology on a finite set is not unique.

Example 3.9 Let X = {a, b, c, d}. The following are some of the discrete layer topologies on X.
$\mathscr{L}_1$ = {{a}, {a, d}, {a, c, d}, X}.
$\mathscr{L}_2$ = {{c}, {b, c}, {b, c, d}, X}.
$\mathscr{L}_3$ = {{d}, {a, d}, {a, b, d}, X}.
$\mathscr{L}_4$ = {{c}, {c, d}, {b, c, d}, X}.
$\mathscr{L}_5$ = {{d}, {a, d}, {a, c, d}, X}.

Definition 3.10 A subset of A of a $\mathscr{L}$ - space is said to be $\mathscr{L}$ -closed set if X-A is $\mathscr{L}$ -open.

Theorem 3.11 Let (X, $\mathscr{L}$) be a $\mathscr{L}$ - space, then the collection $\mathscr{L}^*$ of all $\mathscr{L}$ -closed sets satisfy the following:
(i) $X \notin \mathscr{L}^*$, $\varphi \in \mathscr{L}^*$
(ii) If $C_1, C_2, \cdots, C_n$ ($n \in \wedge$) $\in \mathscr{L}^*$ then $C_1 \supset C_2 \supset \cdots \supset C_n$.

Proof.
(i) $X \in \mathscr{L} \Rightarrow \varphi \in \mathscr{L}^*$ and $\varphi \notin \mathscr{L} \Rightarrow X \in \mathscr{L}^*$.
(ii) Let $C_1, C_2, \cdots, C_n \in \mathscr{L}^*$. Then $C_i = A_i^c$, i = 1, 2, · · · , n, where $A_i \in \mathscr{L}$.
Also, $A_1 \subset A_2 \subset \cdots \subset A_n \Rightarrow A_1^c \supset A_2^c \supset \cdots \subset A_n^c$. Hence $C_1 \supset C_2 \supset \cdots \supset C_n$.

Definition 3.12 Let (X, $\mathscr{L}$) be a $\mathscr{L}$ - space, then $x \in X$ is called a core point if $x \in U$, $\forall U \in \mathscr{L}$.
The set of all core points is called the core set denoted by $\mathbb{C}$.

Example 3.13 In Example 3.2, {a} is the core set and in Example 3.3, N is the core set.

Theorem 3.14 Let (X, $\mathscr{L}$) be a $\mathscr{L}$ - space, then the following are equivalent.
(i) $\mathbb{C} \subseteq X$ is the core set.
(ii) $\mathbb{C} = \cap U_i$; $\forall U_i \in \mathscr{L}$.
(iii) $\mathbb{C}$ is the minimal $\mathscr{L}$ -open set.

Proof. (i) ⇒ (ii) Let $\mathbb{C}$ be the core set. By definition $\mathbb{C} \subseteq U_i$, $\forall$ i. $\Rightarrow \mathbb{C} \subseteq \cap U_i$.
To prove the other side, let $x \in \cap U_i \Rightarrow x \in U_i$, $\forall$ i $\Rightarrow x \in \mathbb{C}$. Hence $\mathbb{C} = \cap U_i$.
(ii) ⇒ (iii) Let $\mathbb{C} = \cap U_i$; $\forall U_i \in \mathscr{L}$. Then $\mathbb{C}$ is the subset of every $\mathscr{L}$-open set. Hence $\mathbb{C}$ is the minimal $\mathscr{L}$ -open set. (iii) ⇒ (i) Let A be a minimal $\mathscr{L}$ -open set and $\mathbb{C}$ be the core set. Then $A \subseteq \mathbb{C}$. To prove $\mathbb{C} \subseteq A$. Suppose not, there exist $x \in \mathbb{C}$ such that $x \notin A$. Now $x \in$

$\mathbb{C}$ means x is the core point and $x \in U_i$, $\forall i$, where U_i is the $\mathscr{L}$-open set. Therefore $x \in A$. Which is a contradiction. Hence $A = \mathbb{C}$.

Theorem 3.15 In $\mathscr{L}$- space the core set is unique.

Proof. Let $(X, \mathscr{L})$ be a $\mathscr{L}$- space. We have $\mathbb{C} = \cap U_i$, U_i' s are the $\mathscr{L}$-open sets. Also, $\cap U_i = U_i$ for some i. Therefore, $\mathbb{C}$ is a $\mathscr{L}$-open set. Suppose $\mathbb{C}_1$ and $\mathbb{C}_2$ are two core sets in $(X, \mathscr{L})$. Now $\mathbb{C}_1 = \cap U_i = \mathbb{C}_2$. Hence the core set exists and unique.

Definition 3.16 Let $(X, \mathscr{L})$ be a $\mathscr{L}$- space, then $x \in X$ is called a border point if $x \notin U$, $\forall U \in L$ such that $U \neq X$. The set Ꞓ of all border points is called the border set.

Example 3.17 In Example 3.2, {d} is the border set and in Example3.3, the set of all purely imaginary numbers is the border set.

Remark 3.18 Border set is not a $\mathscr{L}$-open set.

Remark 3.19 Discrete layer topology has exactly one core point and one border point.

Theorem 3.20 Core set and border set of a layer topology are disjoint.

Proof. Let $(X, \mathscr{L})$ be a $\mathscr{L}$- space. Let $\mathbb{C}$ be the core set and Ꞓ the border set.
Suppose $\mathbb{C} \cap$ Ꞓ $\neq \varphi$. Let $x \in \mathbb{C} \cap$ Ꞓ. Then $x \in \mathbb{C}$ and $x \in$ Ꞓ. Now from the definition of border set if $x \in$ Ꞓ then $x \notin U$, $\forall U \in \mathscr{L}$ such that $U \neq X$. Therefore $x \notin \mathbb{C}$. Which is a contradiction. Hence $\mathbb{C} \cap$ Ꞓ $= \varphi$.

Theorem 3.21 No two distinct $\mathscr{L}$-open sets have the same number of elements.
Proof. Let A and B be distinct $\mathscr{L}$-open sets. From the definition it is clear that, either $A \subset B$ or $B \subset A$. Hence they have different number of elements.

Theorem 3.22 Let X be a non-empty set with 'n' elements and $\mathscr{L}$ be a layer topology on X. Then $\mathscr{L}$ has a maximum of 'n' number of $\mathscr{L}$-open sets.
Proof. Suppose $\mathscr{L}$ has a maximum of 'n + 1' $\mathscr{L}$-open sets. Let $\mathscr{L} = \{A_1, A_2, A_3, \cdots, A_n, X\}$ and $|A_1| = 1, |A_2| = 2, \cdots |A_n| = n$. Also $|X| = n$. $\therefore |A_n| = |X|$. Which is a contradiction, by theorem 3.14.

Definition 3.23 Let $\mathscr{L}$ and $\mathscr{L}$' be two layer topologies on X. If $\mathscr{L}' \supset \mathscr{L}$, then $\mathscr{L}$' is said to be stronger than $\mathscr{L}$; if $\mathscr{L}$' properly contains $\mathscr{L}$, then $\mathscr{L}$' is strictly stronger than $\mathscr{L}$. Also it can be say that $\mathscr{L}$ is weaker than $\mathscr{L}$'; if $\mathscr{L}$ properly contained in $\mathscr{L}$', then $\mathscr{L}$ is strictly weaker than $\mathscr{L}$'. So, $\mathscr{L}$ is comparable with $\mathscr{L}$' if either $\mathscr{L} \supset \mathscr{L}'$ or $\mathscr{L}' \supset \mathscr{L}$.

Example 3.24 Let X = {a, b, c, d}. The layer topologies $\mathscr{L}_1 = \{\{a\}, \{a, d\}, X\}$ and $\mathscr{L}_2 = \{\{a\}, \{a, d\}, \{a, c, d\}, X\}$ are comparable such that $\mathscr{L}_1 \subset \mathscr{L}_2$.

2. **Relative Layer Topology:** In this section, we have discussed the condition for the subspace layer topology.

Theorem 4.1 Let $(X, \mathscr{L})$ be a $\mathscr{L}$ - space and $\mathbb{C} \subseteq X$ be the core set and let $Y \subseteq X$ such that $Y \cap \mathbb{C} \neq \varphi$ then the collection $\mathscr{L}_Y = \{U \cap Y : \forall U \in \mathscr{L}\}$ is a layer topology on Y .

Proof. (i) Let $Y \subseteq X$, then $X \cap Y = Y \in \mathscr{L}_Y$.

(ii) Let $A_1, A_2, \ldots A_n \in \mathscr{L}_Y$.

Then $A_1 = U_1 \cap Y$, $A_2 = U_2 \cap Y$, …, $A_n = U_n \cap Y$, where $U_1, U_2, \ldots, U_n \in \mathscr{L}$, such that $U_1 \subset U_2 \subset \ldots, U_n$.

Now, $(U_1 \cap Y) \subset (U_2 \cap Y) \subset \ldots \subset (U_n \cap Y) \Rightarrow A_1 \subset A_2 \subset \cdot\cdot\cdot \subset A_n$.

(iii) Suppose $\varphi \in \mathscr{L}_Y$. Then there exist some $A \in \mathscr{L}$ such that $A \cap Y = \varphi$.

$\Rightarrow (\mathbb{C} \cap A) \cap (\mathbb{C} \cap Y) = \mathbb{C} \cap \varphi$, where $\mathbb{C}$ is the core set of $(X, \mathscr{L})$.

$\Rightarrow \mathbb{C} \cap (\mathbb{C} \cap Y) = \varphi$.

$\Rightarrow \mathbb{C} \cap Y = \varphi$.

Which is a contradiction. Hence $\varphi \notin \mathscr{L}_Y$.

Definition 4.2 The collection $\mathscr{L}_Y$ is called relative layer topology on $Y \subseteq X$ and $(Y, \mathscr{L}_Y)$ is called relative $\mathscr{L}$ - space.

Example 4.3 Let X = {a, b, c, d} and $\mathscr{L}$ = {{b}, {b, d}, {a, b, d}, X} be the layer topology on X.

Consider Y = {b, c, d}, then the relative layer topology $\mathscr{L}_Y$ = {{b}, {b, d}, Y}.

3. **Layer topology in ℝ:** In this section, we have defined a base for the layer topology and some special types of layer topologies generated by the layer base in real line ℝ.

Definition 5.1 Let $(X, \mathscr{L})$ be a $\mathscr{L}$ - space and $\mathscr{B} \subseteq \mathscr{L}$. Then $\mathscr{B}$ is called a base for a layer topology $\mathscr{L}$ if every $\mathscr{L}$-open set $U \in \mathscr{L}$ is a union of members of $\mathscr{B}$. Equivalently, $\mathscr{B}$ is a layer base for $\mathscr{L}$ if for any $x \in U \in \mathscr{L}$, there exist $B \in \mathscr{B}$ such that $x \in B \subseteq U$.

Lemma 5.2 Let $\mathscr{L}$ and $\mathscr{L}$ ' be layer topologies on X, with its bases $\mathscr{B}$ and $\mathscr{B}$' respectively. Then

$\mathscr{L}' \supset \mathscr{L}$ if and only if for every x in X and each basis element $B \in \mathscr{B}$ containing x, there exist $B' \in \mathscr{B}'$ such that $x \in B' \subseteq B$.

Proof. Assume for every $x \in X$ and each $B \in \mathscr{B}$ containing x, there exist $B' \in \mathscr{B}'$ such that $x \in B' \subseteq B$.

Consider $A \in \mathscr{L}$. Let $x \in A$. Since $\mathscr{B}$ is a base for $\mathscr{L}$, there exist B in $\mathscr{B}$ such that $x \in B \subseteq A$. By our assumption, there exist $B' \in \mathscr{B}'$ such that $x \in B' \subseteq B$. Then $x \in B' \subseteq A$. Therefore $A \in \mathscr{L}'$. Hence $\mathscr{L}' \supset \mathscr{L}$.

Conversely, assume $\mathscr{L}' \supset \mathscr{L}$. Let $x \in X$ and $B \in \mathscr{B}$ containing x. Now $B \in \mathscr{L}$ and $\mathscr{L} \subset \mathscr{L}'$. Therefore $B \in \mathscr{L}'$. Since $\mathscr{L}'$ is induced by $\mathscr{B}'$, there exist $B' \in \mathscr{B}'$ such that $x \in B' \subseteq B$.

Definition 5.3 Let $\mathscr{B}_{\mathbb{R}} = \{(-n, n) / n \in \mathbb{R}\}$, then the layer topology induced by $\mathscr{B}_{\mathbb{R}}$ is called the standard layer topology on $\mathbb{R}$, represented by $\mathscr{L}_{\mathbb{R}}$.

Definition 5.4 Let $\mathscr{B}_L = \{[-a, a) / a \in \mathbb{R}\}$, then the layer topology induced by $\mathscr{B}_L$ is called the lower limit layer topology on $\mathbb{R}$, represented by $\mathscr{L}_L$.

Definition 5.5 Let $\mathscr{B}_U = \{(-b, b] / b \in \mathbb{R}\}$, then the layer topology induced by $\mathscr{B}_U$ is called the upper limit layer topology on $\mathbb{R}$, represented by $\mathscr{L}_U$.

Lemma 5.6 The layer topologies of $\mathscr{L}_L$ and $\mathscr{L}_U$ strictly stronger than the standard layer topology $\mathscr{L}_{\mathbb{R}}$.

Proof. Let $\mathscr{B}_{\mathbb{R}}$, $\mathscr{B}_L$ and $\mathscr{B}_U$ be the bases for the layer topologies $\mathscr{L}_{\mathbb{R}}$, $\mathscr{L}_L$ and $\mathscr{L}_U$ respectively.

Consider (-n, n) $\in \mathscr{B}_{\mathbb{R}}$ with x $\in$ (-n, n), then there exist [-x-ε, x+ε) $\in \mathscr{B}_L$ such that x $\in$ [-x-ε, x+ε) $\subset$ (-n, n), for any small ε. On the other side, consider [-a, a) $\in \mathscr{B}_L$, here -a $\in$ [-a, a), but there does not exist any element of $\mathscr{B}_{\mathbb{R}}$, that contains -a and lies in [-a, a). Hence $\mathscr{L}_L \supset \mathscr{L}_{\mathbb{R}}$.

Suppose, if we take (-n, n) $\in \mathscr{B}_{\mathbb{R}}$ with x $\in$ (-n, n), then there exist (-x-ε, x+ε] $\in \mathscr{B}_U$ such that x $\in$ (-x-ε, x+ε] $\subset$ (-n, n), for any small ε. On the other side, consider (-b, b] $\in \mathscr{B}_U$, here b $\in$ (-b, b], but there does not exist any element of $\mathscr{B}_{\mathbb{R}}$, that contains b and lies in (-b, b]. Hence $\mathscr{L}_U \supset \mathscr{L}_{\mathbb{R}}$.

Lemma 5.7 The layer topologies of $\mathscr{L}_L$ and $\mathscr{L}_U$ are not comparable.

Proof. Consider [-a, a) $\in \mathscr{B}_L$, here -a $\in$ [-a, a), but there does not exist any element of $\mathscr{B}_U$, that contains -a and lies in [-a, a). Hence $\mathscr{L}_U \not\supset \mathscr{L}_L$. On the other side, consider (-b, b] $\in \mathscr{B}_U$, here b $\in$ (-b, b], but there does not exist any element of $\mathscr{B}_U$, that contains b and lies in (-b, b]. Hence $\mathscr{L}_L \not\supset \mathscr{L}_U$.

Definition 5.8 For a fixed x $\in \mathbb{R}$ or y $\in \mathbb{R}$, if $\mathscr{B}_1$ = {(x, y) / x < y}, then the layer topology induced by $\mathscr{B}_1$ is called the open ray layer topology on $\mathbb{R}$.

Definition 5.9 For a fixed x $\in \mathbb{R}$ or y $\in \mathbb{R}$, if $\mathscr{B}_2$ = {[x, y] / x < y}, then the layer topology induced by $\mathscr{B}_2$ is called the closed ray layer topology on $\mathbb{R}$.

4. **Layer topology on graphs**: In this section, we have defined layer topology for each non empty proper subset of the vertex set V of a simple connected graph G.

Definition 6.1 Let G = (V, E) be a graph. The closed neighbourhood of the sub set A of V is defined by, N[A] = A ∪ {v $\in$ V-A : uv $\in$ E, ∀ u $\in$ A}.

Definition 6.2 Let G = (V, E) be a graph with 'n' vertices. Then the layer topology corresponding to a non-empty proper subset A of V is defined as $\mathscr{L}_A$ = {A, A_1, A_2, …, A_k}, where $A_i = N[A_{i-1}]$, i = 2, 3, …, k; k < n; with A_k = V.

Example 6.3 Consider a graph in figure 1, with vertex set V = {a, b, c, d}.

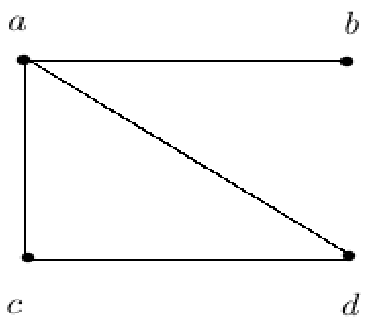

Figure 1

$\mathscr{L}_{\{b\}}$ = {{b}, {a, b}, V};
$\mathscr{L}_{\{a,d\}}$ = {{a, d}, V}.

Theorem 6.4 The layer topology corresponding to every vertex in a complete graph is indiscrete.

Proof. Let V = {v_1, v_2, …, v_n} be the vertex set a complete graph K_n. Consider $v_i \in$ V. Since v_i is adjacent to every other vertex, the layer topology corresponding to a vertex $v_i \in$ V is $\mathscr{L}_{\{vi\}}$ = {{v_i}, V}.

Theorem 6.5 The layer topology corresponding to every vertex in a complete bipartite graph has exactly three $\mathscr{L}$-open sets. (i.e) $|\mathscr{L}_{\{v\}}| = 3$, ∀ v ϵ V.

Proof. Let V be the vertex set of a complete bipartite graph $K_{m,\ n}$. Let V can be partitioned into V_1 and V_2 such that $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \varphi$. Let V_1 = {x_1, x_2, …, x_m} and V_2 = {y_1, y_2, …, y_n}. Let $x_i \in V_1$; i = 1,2,…, m, then the layer topology $\mathscr{L}_{\{xi\}}$ = {{x_i}, {x_i, y_1, y_2, …, y_n}, V}. Now, $y_j \in V_2$; j = 1,2,…, n, then the layer topology $\mathscr{L}_{\{yi\}}$ = {{y_j}, {y_j, x_1, x_2, …, x_m}, V}. Hence for every vertex v ϵ $\mathscr{L}$, $|\mathscr{L}_{\{v\}}| = 3$.

Theorem 6.6 The layer topology corresponding to the end vertices of a path graph P_n is discrete.

Proof. Let V = {v_1, v_2, …, v_n} be the vertex set a path graph P_n. Consider $v_1 \in$ V. Then $\mathscr{L}_{\{v1\}}$ = {{A_1, A_2, …, A_n}, where A_1 = {v_1}, A_2 = {v_1, v_2}, … A_n = V, is a discrete layer topology. Similar argument for $v_n \in$ V.

Theorem 6.7 Let G = (V, E) be a cycle graph C_n ; $n \geq 3$, and $\mathscr{L}_{\{v\}}$ be a layer topology corresponding to a vertex v ϵ V, then $|\mathscr{L}_{\{v\}}| = \begin{cases} \frac{(n+1)}{2} \ if\ n\ is\ odd \\ \frac{n}{2} + 1\ if\ n\ is\ even \end{cases}$

Proof. Let V = {v_1, v_2, …, v_n}. Consider $v_i \in$ V. Since G is a cycle, for i = 1; $v_{i-1} = v_n$, v_{i-2} = v_{n-1} and so on.
Also, for i = n; $v_{i+1} = v_1$, $v_{i+2} = v_2$ and so on.
Case(i). Suppose ‘n’ is odd.
Let A_1 = {v_i}. Then A_2 = N[A_1] = {v_{i-1}, v_i, v_{i+1}}, A_3 = N[A_2] ={v_{i-2}, v_{i-1}, v_i, v_{i+1}, v_{i+2}}, …, $A_{\frac{(n+1)}{2}}$ = V.
Then $\mathscr{L}_{\{vi\}}$ = {A_1, A_2, …, $A_{\frac{(n+1)}{2}}$} is a layer topology, with $|\mathscr{L}_{\{vi\}}| = \frac{(n+1)}{2}$.
Case(ii). Suppose ‘n’ is even.
Let A_1 = {v_i}. Then A_2 = N[A_1] = {v_{i-1}, v_i, v_{i+1}}, A_3 = N[A_2] = {v_{i-2}, v_{i-1}, v_i, v_{i+1}, v_{i+2}}, …, $A_{(\frac{n}{2}+1)}$= V.
Then $\mathscr{L}_{\{vi\}}$ = { A_1, A_2, …, $A_{(\frac{n}{2}+1)}$} is a layer topology, with $|\mathscr{L}_{\{vi\}}| = \frac{n}{2} + 1$.

IV. CONCLUSION

In this paper, we have explored the concept of layer topology and studied its basic properties and characterisations. We have also induced layer topological structure from graphs. In addition, we have also analysed and enumerated the layer open sets for some special types of graphs. In future, we will extend this notion to compactness and separation axioms. Also, this idea may be extended to determine various parametres of a given graph which may lead us to give some real life applications.

REFERENCES

[1] Adem Kilicman, Khalid Abdulkalek, Topological spaces associated with simple graphs, Journal of Mathematical Analysis, Vol.9, Issue 4 (2018), 44-52.
[2] Amer Himza Almyaly, Interior topology: A new approach in topology, International Journal of Advances in Applied Sciences (IJAAS) Vol. 11, No. 4, December 2022, pp. 367~372.
[3] Anabel E.Gamorez, Caen Grace S.Nianga, Sergio R.Canoy Jr., Topologies Induced by Neighbourhoods of a graph under some Binary Operation, European Journal of Pure and Applied Mathematics, Vol.12, No.3, 2019, 749-755.
[4] Munkres, Topology Updated Second Edition, Pearson India Education Services Pvt. Ltd, 2022.
[5] Narsingh Deo, Graph Theory with applications to Engineering and Computer Science, Dover Publications, Inc, Mineola, New York, 2016.
[6] M. Shokry, Y.Y. Yousif, Closure Operators on Graphs, Australian Journal of Basic and Applied Sciences, 5(11): 1856-1864, 2011.