# USING FUZZY CONTROL CHARTS FOR PREVALENCE OF VITAMIN DEFICIENCY AMONG INDIAN CHILDREN: FINDINGS FROM A NATIONAL CROSS-SECTIONAL SURVEY

**Abstract**

To sequentially monitor the average and variability in a national cross-sectional survey, fuzzy $\overline{x}$ and s charts are commonly employed techniques that require the data to be represented as real numbers. When applying these methods, it becomes evident that constructing fuzzy control charts offers enhanced flexibility, greater mathematical convenience, and more rational outcomes compared to traditional quality control chart methods. In this study, we have introduced fundamental concepts and key insights related to the use of control charts within a fuzzy framework for the purpose of monitoring sequential variations in both average and variability. Our specific focus has been on assessing vitamin deficiency among children aged 1-5 years in India by analyzing data obtained from the most recent nutritional survey.

**Keywords:** Fuzzy control charts, vitamin deficiency, cross-sectional survey, and Indian children

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## I. INTRODUCTION

UNICEF reports that approximately one-third of children do not receive the necessary Vitamin A supplementation. According to WHO guidelines, roughly 190 million preschool children suffer from Vitamin A deficiency, with the majority residing in the African and Southeast Asian regions. The World Bank has asserted that Vitamin A supplementation is a cost-effective measure to enhance the survival of preschool children. Vitamin A deficiency is linked to around 65,000 early childhood deaths annually due to malaria, measles, and other infections. A randomized controlled study conducted in South India demonstrated that children given small weekly doses of vitamin A had a risk of death less than half that of the control group. Moraditadi [1] explored the use of fuzzy control charts with triangular and trapezoidal fuzzy numbers, implemented through MATLAB software, to address uncertainties in process parameters and data modes. The study indicated that fuzzy control was more sensitive compared to traditional control charts. Chen and Yu [2] applied a fuzzy approach instead of traditional control charts, using fuzzy rules to construct corresponding fuzzy zone control charts, which showed superior performance to classical control charts. Tadi and Darestani [3] investigated the fuzzy moving range control chart when data were transformed into trapezoidal fuzzy numbers using fuzzy mode and fuzzy rule approaches. The results were categorized as in control, out of control, rather in control, and rather out of control. Sogandi et al. [4] developed a novel fuzzy control chart for monitoring attribute quality characteristics based on the α-level fuzzy midrange approach. The performance and comparative results of the proposed fuzzy control chart were evaluated in terms of average run length (ARL) using Monte Carlo simulations.

The structure of this study is as follows. The essential elements of conventional ($\bar{x}$ and s) control charts are presented in Section 3, which serves as the basis for our comprehensive process of creating fuzzy $\bar{x}$ and s control charts, which is shown in Section 4. The results and graphical presentation are shown in the following section 5, respectively.

## II. DATA AND METHODS

Data collected as part of the Comprehensive National Nutrition Survey (CNNS) in 2016-18 are used for this study. This paper proposed traditional and fuzzy control charts.

## III. REVIEW OF $\bar{x}$ AND S TRADITIONAL CONTROL CHARTS

A process mean can be observed using an x-chart, but process variability can be observed using an R-chart or s-chart. Typically, an x-chart is used in conjunction with an R-chart or an s-chart. In particular, when the sample size is small (less than ten), the x chart and R chart should be used; otherwise, these charts should be utilized [14]. Because this study focuses on samples with varied sizes, s and $\bar{x}$ charts are taken into account..

Let $p_i$ be the sample size of the q samples discussed. Let $x_{ij}$ be the value of the quality characteristic in the sample $i^{th}$ at the observation $j^{th}$. The mean of the $i^{th}$ sample, determined by $\overline{x_i}$ and the grand mean of the samples, determined by $\bar{x}$ are denoted by

$$\overline{x_i} = \frac{1}{p_i}\sum_{j=1}^{n_i} x_{ij} \tag{1}$$

$$\bar{x} = \frac{1}{\sum_{i=1}^{q} p_i} \quad (2)$$

Let $\sigma_i$and $\bar{\sigma}$ be the standard deviation of the $i^{th}$ sample and the qsamples, respectively. They are obtained by

$$\sigma_i = \sqrt{\frac{1}{p_i - 1} \sum_{j=1}^{p_i} (x_{ij} - \bar{x_i})^2} \quad (3)$$

$$\bar{\sigma} = \sqrt{\frac{1}{\sum_{i=1}^{q} p_i - q} \sum_{i=1}^{q} (p_i - 1)\sigma_i^2} \quad (4)$$

The centerline (CL), upper control limit (UCL), and lower control limit (LCL) of the x¯chart are specified by the estimator of x¯and s¯

$$UCL_{x_i} = \bar{x} + K\frac{\bar{\sigma}}{c_{4_i}\sqrt{p_i}} \quad (5)$$

$$CL_{x_i} = \bar{x} \quad (6)$$

$$LCL_{x_i} = \bar{x} - KK\frac{\bar{\sigma}}{c_{4_i}\sqrt{p_i}} \quad (7)$$

Here, $c_{4_i}$is a constant denoted by sample size $p_i$ as shown [18]. Then, k be the number of units of standard deviation and it usually used [1].Similarly, the control limits for s chart are denoted by

$$UCL_{\sigma_i} = (1 + \frac{K}{c_{4_i}}\sqrt{1 - c_{4_i}^2})\bar{\sigma} \quad (8)$$

$$CL_{\sigma_i} = \bar{\sigma} \quad (9)$$

$$LCL_{\sigma_i} = (1 - \frac{K}{c_{4_i}}\sqrt{1 - c_{4_i}^2})\bar{\sigma} \quad (10)$$

Because each sample has a unique value of C_4, the control limits of the control charts derived by equations (5) to (10) fluctuate as seen in figure 1. All of x¯_i. and σ_i can be plotted against the variable control limits to identify any out-of-control signals.

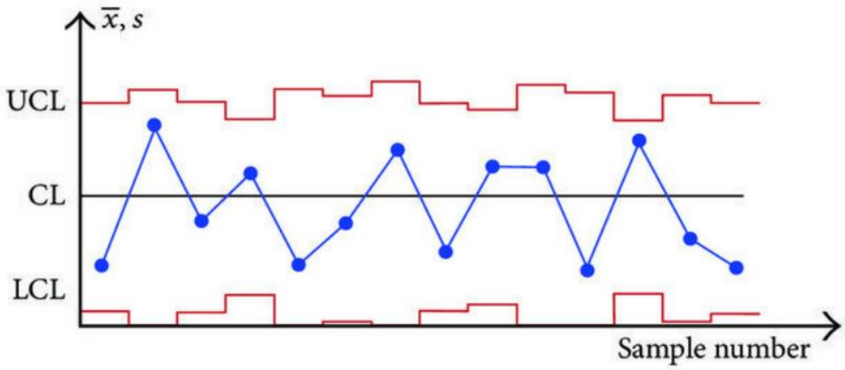



**Figure 1:** $\bar{x}$ charts and s chart with variable sample size

## IV. CONSTRUCTION OF FUZZY $\bar{x}$ AND S CONTROL CHART

This section shows how our introduced fuzzy x and s chart are applied in order to monitor the underlying process average and variability.Let $\bar{x}_{i1}, \bar{x}_{i2}, \ldots, \bar{x}_{ipi}$ (i= $\overline{1,q}$) be fuzzy observation (or) fuzzy real numbers discussed in [13]. Lower and Upper control limits discussed based on these fuzzy data. For any given $\gamma$ € [0,1], Corresponding real valued data $(\bar{x}ij)^L_\gamma$ and $(\bar{x}ij)^U_\gamma$ for i= $(\overline{1,q})$ and j= $(\overline{1,pi})$. In order to abtain the evaluate of the fuzzy control limits for $(\bar{x}ij)^L_\gamma$ and $(\bar{x}ij)^U_\gamma$.

$$\bar{x}^U_{i,\gamma} = \frac{1}{q_i}\sum_{j=1}^{qi}(\bar{x}_{ij})^U_\gamma$$

$$\bar{\bar{x}}^U_\gamma = \frac{1}{\sum_{i=1}^{p} q_i}\sum_{i=1}^{p} q_i\, \bar{x}^U_{i,\gamma}$$

$$\bar{x}^L_{i,\gamma} = \frac{1}{q_i}\sum_{j=1}^{q_i}(\bar{x}_{ij})^L_\gamma$$

$$\bar{\bar{x}}^L_\gamma = \frac{1}{\sum_{i=1}^{p} q_i}\sum_{i=1}^{p} q_i\, \bar{x}^L_{i,\gamma}$$

$$S^U_{i,\gamma} = \sqrt{\frac{1}{q_{i-1}}\sum_{j=1}^{q_i}[(\bar{x}_{ij})^U_{i,\gamma} - \bar{x}^U_{i,\gamma}]^2} \tag{11}$$

$$\bar{S}^L_\gamma = \sqrt{\frac{1}{\sum_{i=1}^{p} q_i - p_i}\sum_{i=1}^{p}[(q_i - 1(\bar{S}^U_\gamma)^2]}$$

$$\bar{S}^L_{i,\gamma} = \sqrt{\frac{1}{q_i-1}\sum_{j=1}^{q_1}[(\bar{x}_{ij})^L_{i,\gamma} - \bar{x}^L_{i,\gamma}]^2}$$

$$\bar{S}^L_\gamma = \sqrt{\frac{1}{\sum_{i=1}^{p} q_i - p}\sum_{i=1}^{p}[(q_i - 1)(S^L_{i,\gamma})^2]}$$

1. **Fuzzy $\bar{x}$ control charts:** Applied with the fuzzy outcomes in (11), the parameters for the fuzzy $\bar{x}$ chart in 5, 6 and 7 are obtained by

$$U^U_{\bar{x}_{i,\gamma}} = (ucl_{\bar{x}_i})^U_\gamma = \bar{\bar{x}}^U_\gamma + K\,\frac{\bar{S}^U_\gamma}{C_{4_i}\sqrt{q_i}}$$

$$C^U_{x_{i,\gamma}} = (cl_{\bar{x}_i})^U_\gamma = \bar{x}^U_\gamma$$

$$l^U_{x_{i,\gamma}} = (lcll_{\bar{x}_i})^U_\gamma = \bar{\bar{x}}^U_\gamma - K\,\frac{\bar{S}^U_\gamma}{C_{4_i}\sqrt{q_i}}$$

$$u^L_{x_{i,\gamma}} = (ucl_{\bar{x}})^L_\gamma = \bar{\bar{x}}^L_\gamma + K\,\frac{\bar{S}^L_\gamma}{C_{4_i}\sqrt{q_i}}$$

$$C^L_{\bar{x}_{i,\gamma}} = (cl_{\bar{x}_i})^L_\gamma = \bar{\bar{x}}^L_\gamma$$

$$l^L_{\bar{x}_{i,\gamma}} = (lcl_{\bar{x}_i})^L_\gamma = \bar{\bar{x}}^L_\gamma - K\,\frac{\bar{S}^L_\gamma}{C_{4_i}\sqrt{q_i}}$$

- **Construction of Fuzzy Upper Control Limit $\tilde{u}_{\bar{x}}$:** Based on the above results, let us consider the closed interval $A_{i,\gamma}$ which is defined as follows:

$$A_{i,\gamma} = [\min\{u^L_{\bar{x}_{i,\gamma}}, u^U_{\bar{x}_{i,\gamma}}\}, \max\{u^L_{\bar{x}_{i,\gamma}}, u^U_{\bar{x}_{i,\gamma}}\}] \tag{12}$$
$$= [l_{i,\gamma}, u_{i,\gamma}],$$

where

$$l_{i,\gamma} = \min\{u^L_{\bar{x}_{i,\gamma}}, u^U_{\bar{x}_{i,\gamma}}\} \tag{13}$$
$$l_{i,\gamma} = \max\{u^L_{\bar{x}_{i,\gamma}}, u^U_{\bar{x}_{i,\gamma}}\}$$

For the membership function of the fuzzy upper control limit, the fuzzy numbers of the control-limits can be defined as

$$\xi_{\tilde{u}_{\bar{x}_i}}(\text{c}) = \sup \gamma \,.\, 1_{\tilde{A}_{i,\gamma}} \gamma \epsilon [0,1] \tag{14}$$

Since each $\tilde{x}_{ij}$ is a fuzzy real number, $(\tilde{x}_{ij})_\gamma^L$ and $(\tilde{x}_{ij})_\gamma^U$ are continuous with respect to $\gamma$ on [0,1], saying that $\bar{\bar{x}}_\gamma^L$ , $\bar{\bar{x}}_\gamma^U$ , $\bar{S}_\gamma^L$ , and $\bar{s}_\gamma^U$ are continuous with respect to $\gamma$ on [0,1]. Under these facts, the $\gamma$ - level set $(\tilde{u}_{\bar{x}_i})_\gamma$ of fuzzy upper control limit $\tilde{u}_{\bar{x}_i}$ can be written as

$$(\tilde{u}_{\bar{x}_i})_\gamma = \{\text{c}: \xi_{\tilde{u}_{\bar{x}_i}}(\text{c}) \geq \gamma\} = [\quad \min l_{i,\gamma}, \max u_{i,\gamma}] \gamma \leq \beta \leq 1 \gamma \leq \beta \leq 1 \tag{15}$$

$$= [(\tilde{u}_{\bar{x}_i})_\gamma^L \,,\, (\tilde{u}_{\bar{x}_i})_\gamma^U]$$

Where $l_{i,\gamma}$ and $u_{i,\gamma}$ are shown in (13).
From (10), the relationship between $(\tilde{u}_{\bar{x}_i})_\gamma^L$ and $u_{\bar{x}_{i,\gamma}}^L$ , $u_{\bar{x}_{i,\gamma}}^U$ is found as

$$(\tilde{u}_{\bar{x}_i})_\gamma^L = \min \text{l}\,(\beta) \quad \min \quad \min \{u_{\bar{x}_{i,\beta}}^L u_{\bar{x}_{i,\beta}}^U\} \tag{16}$$
$$\gamma \leq \beta \leq 1 \gamma \leq \beta \leq 1$$

Similarly, the relationship between $(\tilde{u}_{\bar{x}_i})_\gamma^U$ and $u_{\bar{x}_{i,\gamma}}^L$, $u_{\bar{x}_{i,\gamma}}^U$ is found as

$$(\tilde{u}_{\bar{x}_i})_\gamma^U = \max \text{u}\,(\beta) = \max \quad \max \{\, u_{\bar{x}_{i,\beta}}^L \,,\, u_{\bar{x}_{i,\beta}}^U\} \tag{17}$$

- **Construction of Fuzzy Lower Control Limits $\tilde{l}_{\bar{x}}$ :** The endpoints of the fuzzy lower control limit's γ-level closed interval were determined using the same process. $\tilde{l}_{\bar{x}}$ are determined by

$$(\tilde{l}_{\bar{x}})_\gamma = [\min \min \{l_{\bar{x}_{i,\beta}}^L, l_{\bar{x}_{i,\beta}}^L, l_{\bar{x}_{i,\beta}}^U\}, \max \max \{l_{\bar{x}_{i,\beta}}^U \,,\, l_{\bar{x}_{i,\beta}}^U\}] \quad 18)$$

2. **Fuzzy s Control Chart:** Applied with the fuzzy outcomes in equation (11), the parameters for the fuzzy s chart in (8,9 and 10) are obtained by

$$u_{s_{i,\gamma}}^U = (\text{ucl}_{s_i})_\gamma^U = (1+ \frac{K\sqrt{1-c_{4_i}^2}}{c_{4_i}})\bar{S}_\gamma^U$$
$$C_{s_{i,\gamma}}^U = (\text{cl}_{s_i})_\gamma^U = \bar{S}_\gamma^U$$
$$l_{s_{i,\gamma}}^U = (\text{lcl}_{s_i})_\gamma^U = (1- \frac{K\sqrt{1-c_{4_i}^2}}{c_{4_i}})\bar{S}_\gamma^U$$
$$u_{s_{i,\gamma}}^L = (\text{ucl}_{s_i})_\gamma^L = (1+ \frac{K\sqrt{1-c_{4_i}^2}}{c_{4_i}})\bar{S}_\gamma^L$$
$$C_{s_{i,\gamma}}^L = (\text{cl}_{s_i})_\gamma^L = \bar{S}_\gamma^L$$
$$l_{s_{i,\gamma}}^L = (\text{lcl}_{s_i})_\gamma^L = (1- \frac{K\sqrt{1-c_{4_i}^2}}{c_{4_i}})\bar{S}_\gamma^L$$

The construction of the fuzzy control limits for fuzzy s charts is done the same as that for fuzzy $\bar{x}$ chart. The results as follows:

- The endpoints of the $\gamma$ - level closed intervals $(\tilde{u}_{s_i})_\gamma$ = [$(\tilde{u}_{s_i})_\gamma^L$, $(\tilde{u}_{s_i})_\gamma^U$ ] of fuzzy upper control limit $\tilde{u}_{s_i}$are determined by

$$(\tilde{u}_{s_i})_\gamma^L = \min_{\gamma \le \beta \le 1} \min \{u_{s_{i,\beta}}^L, u_{s_{i,\beta}}^U\} \quad (19)$$

$$(\tilde{u}_{s_i})_\gamma^U = \max \max \{u_{s_{i,\beta}}^U, u_{s_{i,\beta}}^U\} \quad (20)$$

- The endpoints of the $\gamma$ - level closed interval $(l_{s_i})_\gamma$ =[$(l_{s_i})_\gamma^L$, $(\tilde{l}_{s_i})_\gamma^U$] of fuzzy upper control limit $\bar{l}_{s_i}$ are definedas follows:

$$(\tilde{l}_{s_i})_\gamma^L = \min_{\gamma \le \beta \le 1} \min \{l_{s_{i,\beta}}^L, l_{s_{i,\beta}}^U\} \quad (21)$$

$$(\tilde{l}_{s_i})_\gamma^U = \max_{\gamma \le \beta \le 1} \max\{l_{s_{i,\beta}}^U, l_{s_{i,\beta}}^U\} \quad (22)$$

Fuzzy Average $\tilde{\bar{x}}$ and Standard Deviations of the Fuzzy Mean ̃. To determine if the fuzzy average would$\tilde{\bar{x}}_i$ and fuzzy mean standard deviation $\tilde{s}_i$have membership functions that are under the fuzzy control limits, we must first compute them using the sample - statistics fuzzy numbers (SSFN_s). Using an analogous process for the CLFN_s, the α-level closed interval's endpoints$(\tilde{\bar{x}}_i)_\alpha$ = [ $(\tilde{\bar{x}}_i)_\alpha^L$ , $(\tilde{\bar{x}}_i)_\alpha^U$ of fuzzy average $\tilde{\bar{x}}_i$ are obtained by

$$(\tilde{\bar{x}}_i)_\alpha^L = \min_{\alpha \le \beta \le 1n} \min \{\bar{X}_{i,\beta}^L , \bar{x}_{i,\beta}^U\} \quad (23)$$

$$(\tilde{\bar{x}}_i)_\alpha^U = \max_{\alpha \le \beta \le 1} \max \{ \bar{x}_{i,\beta}^L , \bar{x}_{i,\beta}^U \} \quad (24)$$

And, the endpoints of the α – level closed interval $(\tilde{s}_i)_\alpha$ = [$(\tilde{S}_i)_\alpha^L$ , $(\bar{S}_i)_\alpha^U$ of fuzzy mean standard deviation $\bar{S}_i$are

(($\bar{S}_i)_\alpha^L$ = min min $\{S_{i,\beta}^L , S_{i,\beta}^U\}$.

## V. PROPOSED CLASSIFICATION MECHANISM

The strategy of Yu and Dat [17]adds an additional normal triangular fuzzy number $\tilde{0}$ = (0,0,0), which is used as a radical number to complement our comparison, making it more efficiently simplified. In particular, take into account a generalized normal fuzzy Numbers $\bar{a}_i$= $(b_i, c_i, d_i, e_i)$ whose membership functions are defined by

$$\xi_{\alpha_i}(x) = \begin{cases} f_{\overline{\gamma}_i}^L(x) & \text{if } x \in [b_i, c_i] \\ 1 & \text{if } x \in [c_i, d_i] \\ f_{\overline{\gamma}_i}^R(x) & \text{if } x \in [d_i, e_i] \\ 0 & \text{others} \\ 1 & \end{cases}$$

Let $g_{\gamma_i}^R$ (y) and $g_{\gamma_i}^R$(y) be the inverse functions of the $f_{\overline{\gamma}_i}^L$ (x) and $f^R_{\tilde{\alpha}_i}$ (x), respectively. Then, the left and right integral values of $\tilde{a}_i$ to $\tilde{0}$, respectively, denoted by L$V_i$ and R$V_i$, are calculated by L$V_i = \int_0^1 g^L_{\gamma i}$ (y) dy

$$RV_i = \int_0^1 g^R_{\gamma i}\,(y)\,dy \tag{26}$$

By incorporating optimism level $\beta$ an index presenting subjective attitude of decision – maker ($\beta \in [0,1]$), with L$V_i$ and R$V_i$ our proposed ranking index Of $\tilde{\gamma}_i$,, denoted by S$V_i^\beta$, is defined by

$$SV_i^\beta = \beta RV_i + (1-\beta)\,LV_i \tag{27}$$

1. **Monitoring Process Variability:** To keep an eye on process variability, we must contrast the fuzzy data that has been gathered. $\tilde{S}_i$ with its fuzzy control limits $\tilde{u}_i$ and $\tilde{l}_{s_i}$

   Hence, the following process is recommended in light of the ranking index in (21:)

- Calculate S$V^\beta_{\tilde{u}_{s_i}}$, S$V^\beta_{\tilde{s}_i}$ and S$V^\beta_{\tilde{l}_{s_i}}$ for $\tilde{u}_{s_i}$ , $\tilde{s}_i$ , and $\tilde{l}_{s_i}$, respectively.
- For $\tilde{s}_i$ ,we calculate its standard deviation, denoted by S$d^\beta_s$ across the m sample by the following Formulas

$$\overline{SV}^\beta_{\tilde{s}} = \frac{1}{m}\sum_{i=1}^m SV^\beta_{\tilde{s}_i}$$

$$Sd^\beta_s = \sqrt{\frac{1}{m-1}\sum_{i=1}^m (SV^\beta_{\tilde{s}_i} - \overline{SV}^\beta_{\tilde{s}})^2} \tag{28}$$

- For $u_{s_i}$, from the obtained S$V^\beta_{u_{s_i}}$ and S$d^\beta_s$, we establish two relevant control limits for $u_{s_i}$, from the obtained S$V^\beta_{\tilde{u}_{s_i}}$ and S$d^\beta_s$ , we establish two relevant control limits for $\tilde{u}_{s_i}$ as

$$SV^{U,\beta}_{\tilde{u}_{s_i}} = SV^\beta_{\tilde{u}_{s_i}} + Sd^\beta_s$$

$$SV^{L,\beta}_{\tilde{u}_{s_i}} = SV^\beta_{\tilde{u}_{s_i}} - Sd^\beta_s \tag{29}$$

- For $\tilde{l}_{s_i}$ , from the obtained S$V^\beta_{\tilde{l}_{s_i}}$ and S$d^\beta_s$, we establish two relevant control limits for $\tilde{l}_{s_i}$ as Follows $SV^{U,\beta}_{\tilde{l}_{s_i}} = SV^\beta_{\tilde{l}_{s_i}} + Sd^\beta_s$

$$SV^{L,\beta}_{\tilde{l}_{s_i}} = SV^\beta_{\tilde{l}_{s_i}} - Sd^\beta_s \tag{30}$$

- We initially reorder the variables before suggesting the manufacturing process's classification. of S$V^{U,\beta}_{\tilde{u}_{s_i}}$ , S$V^\beta_{\tilde{u}_{s_i}}$, S$V^{U,\beta}_{\tilde{l}_{s_i}}$, S$V^\beta_{\tilde{l}_{s_i}}$, and S$V^{L,\beta}_{l_{s_i}}$ in a descending

  Order represented by six critical values $S^\beta_i$ (i=$\overline{1,6}$ ) where $S^\beta_1 > S^\beta_2 > S^\beta_3 > S^\beta_4 > S^\beta_5 > S^\beta_6$.
- A manufacturing process can be categorized using the following guidelines based on the six important values:

- If the following scenario materializes, the process is in control at the optimism level β:

$$S^\beta_4 < SV^\beta_{\tilde{s}_i} < S^\beta_3 \tag{31}$$

- If any of the following scenarios materializes, the process is more in control at the optimism level β:

$S_3^{\beta} \leq \mathrm{SV}_{\tilde{s}_i}^{\beta} \leq S_2^{\beta}$
$S_5^{\beta} \leq \mathrm{SV}_{\tilde{s}_i}^{\beta} \leq S_4^{\beta}$

- If any of the following scenarios occurs, the process is somewhat out of control at the optimism level β:
  $S_2^{\beta} < \mathrm{SV}_{\tilde{s}_i}^{\beta} < S_1^{\beta}$
  $S_6^{\beta} < \mathrm{SV}_{\tilde{s}_i}^{\beta} < S_5^{\beta}$

- If any of the following occurs, the process is out of control at the optimism level β: Complexity
  $\mathrm{SV}_{\tilde{s}_i}^{\beta} \geq S_1^{\beta}$
  $\mathrm{SV}_{\tilde{s}_i}^{\beta} \leq S_6^{\beta}$

2. **Monitoring Process Average:** To keep track of the process average, we must compare the fuzzy data that has been gathered with its fuzzy control boundaries.$\tilde{u}_{\bar{x}_i}$ and $l_{\bar{x}_i}$. Similarly, we suggest the following procedure:

Step 1: Calculate $\mathrm{SV}_{\tilde{u}_{\bar{x}_i}}^{\beta}$ , $\mathrm{SV}_{\tilde{\bar{X}}_I}^{\beta}$, and $\mathrm{SV}_{\tilde{l}_{\bar{x}_i}}^{\beta}$ for $\tilde{u}_{\bar{x}_i}$, $\tilde{\bar{x}}_i$, and $\tilde{l}_{\bar{x}_i}$ , respectively

Step 2: For$\tilde{\bar{x}}_i$, we calculate its standard deviation, denoted by

$$\overline{SV}_{\tilde{\bar{x}}}^{\beta} = \frac{1}{m}\sum_{1=0}^{m} SV_{\tilde{\bar{x}}_i}^{\beta}$$

$$\mathrm{Sd}_{\bar{x}}^{\beta} = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(SV_{\tilde{\bar{x}}_i}^{\beta} - \overline{SV}_{\tilde{\bar{x}}}^{\beta})^2} \tag{32}$$

Step 3: For $\tilde{u}_{\bar{x}_i}$ from the obtained $\mathrm{SV}_{\tilde{u}_{\bar{x}_i}}^{\beta}$and $\mathrm{S}d_{\bar{x}}^{\beta}$, we establish two relevant control limits for $\tilde{u}_{\bar{x}_i}$ as follow:

$$\text{And } \mathrm{SV}_{\tilde{u}_{\bar{x}_i}}^{U,\beta} = \mathrm{SV}_{\tilde{u}_{\bar{x}_i}}^{\beta} + \mathrm{Sd}_{\bar{x}}^{\beta} \tag{33}$$

Step 4: For $\tilde{l}_{\bar{x}_i}$ , from the obtained $\mathrm{SV}_{\tilde{l}_{\bar{x}_i}}^{\beta}$ and $\mathrm{S}d_{\bar{x}}^{\beta}$, we establish two relevant control limits for $\tilde{l}_{\bar{x}_i}$as follows:

$$\text{And } \mathrm{SV}_{\tilde{l}_{\bar{x}_i}}^{U,\beta} = \mathrm{SV}_{\tilde{l}_{\bar{x}_i}}^{\beta} + \mathrm{S}d_{\bar{x}}^{\beta} \tag{34}$$

Step 5: We initially reorder the numbers before suggesting the manufacturing process's classification. of $\mathrm{SV}_{\tilde{u}_{\bar{x}_i}}^{U,\beta}$, $\mathrm{SV}_{\tilde{u}_{\bar{x}_i}}^{\beta}$, $\mathrm{SV}_{\tilde{u}_{\bar{x}_i}}^{L,\beta}$, $\mathrm{SV}_{\tilde{l}_{\bar{x}_i}}^{U,\beta}$, $\mathrm{SV}_{\tilde{l}_{\bar{x}_i}}^{\beta}$, and $\mathrm{SV}_{\tilde{l}_{\bar{x}_i}}^{L,\beta}$ in a descending order represented by six critical values

$S_i^{\beta}$ (i=$\overline{1,6}$) where $S_1^{\beta} > S_2^{\beta} > S_3^{\beta} > S_4^{\beta} > S_5^{\beta} > S_6^{\beta}$.

3. **The process can be classified based on the following steps:**

- If the following scenario materializes, the process is in control at the optimism level β:
  $S_4^{\beta} < \mathrm{SV}_{\tilde{\bar{x}}_i}^{\beta} < S_3^{\beta}$

- If any of the following scenarios materializes, the process is more in control at the optimism level β:
  $S_3^{\beta} < SV_{\tilde{\tilde{X}}_I}^{\beta} < S_2^{\beta}$
  $S_5^{\beta} < SV_{\tilde{\tilde{X}}_I}^{\beta} < S_4^{\beta}$

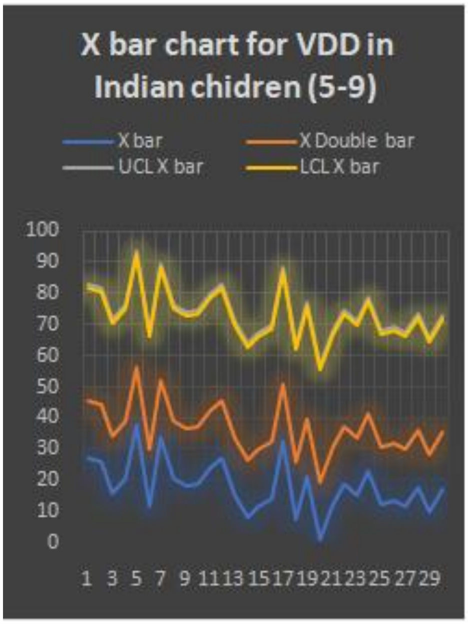
- If any of the following occurs, the process is somewhat out of control at the optimism level β:
  C1) $S_2^{\beta} < SV_{\tilde{\tilde{x}}_i}^{\beta} < S_1^{\beta}$
  C2) $S_6^{\beta} < SV_{\tilde{\tilde{x}}_i}^{\beta} < S_5^{\beta}$

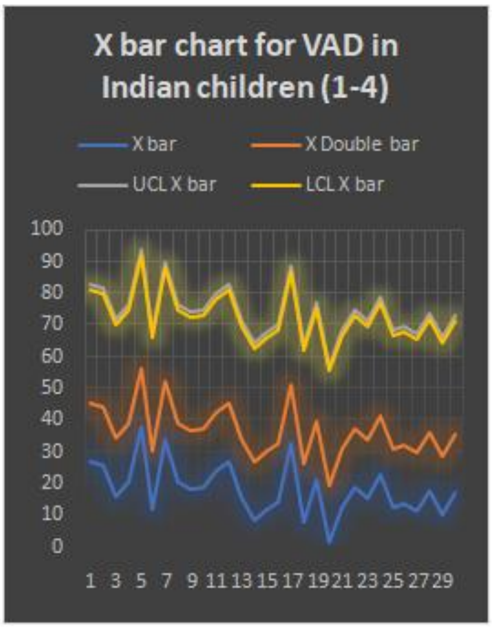
- If any of the following occurs, the process is out of control at the optimism level β:
  $SV_{\tilde{\tilde{x}}_i}^{\beta} \geq S_1^{\beta}$
  $SV_{\tilde{\tilde{x}}_i}^{\beta} \leq S_6^{\beta}$

  The fuzzy control charts displayed in the results below are now being evaluated using the previously mentioned categorization process.

## VI. ANALYSIS AND RESULTS

Control charts are defined as a type of graphical analysis tool that indicates a signal when a product is outside of permissible boundaries and as a method of defining whether a product should remain inside those limitations. Thirty samples were selected at random from a current turning process for our study. The samples' sizes vary because of the random sampling. Fuzzy control charts and fuzzy control limits can be computed using the data. The following figures show the plotted fuzzy x¯and s control chart with k=3 for the 30 samples.



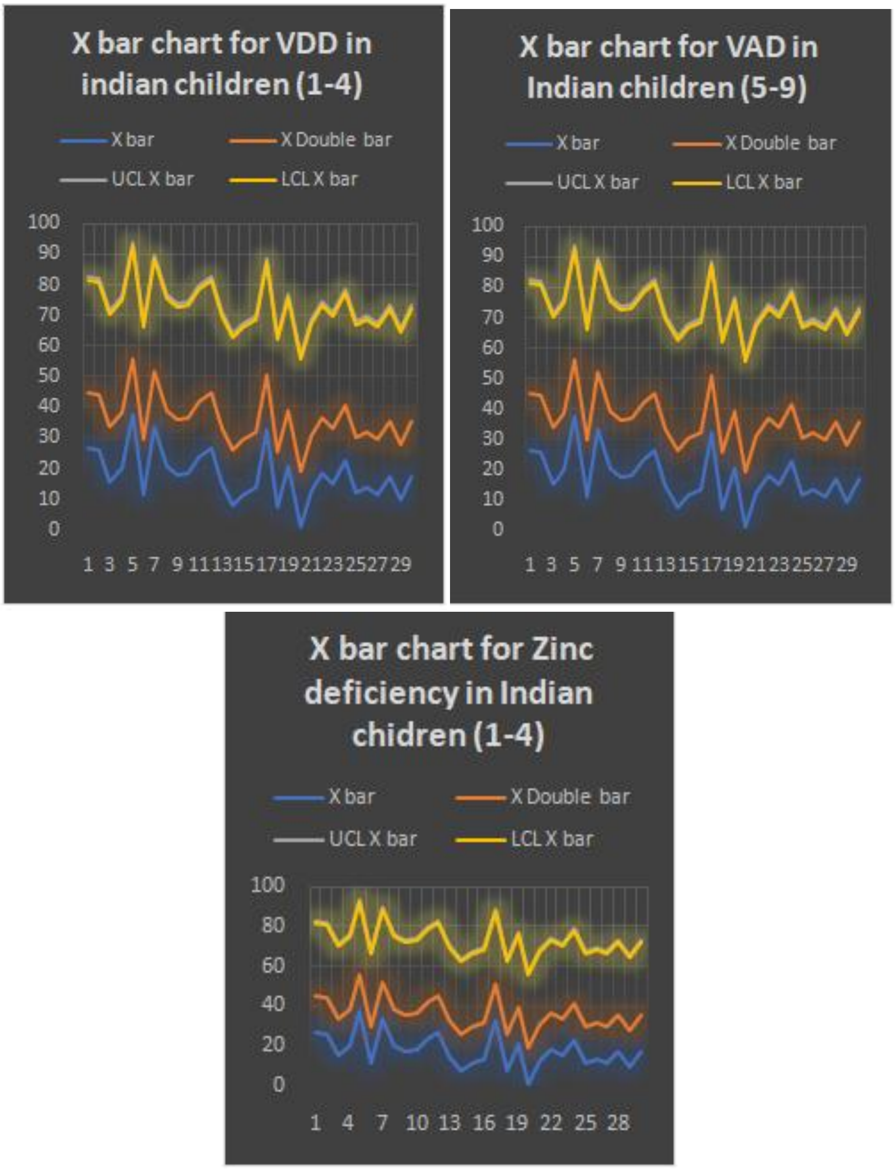



**Figure 2:** X bar charts for vitamin deficiency among Indian children.

**Table 1: Process variability classification**

| Sample Number | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 | Status | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 17.8 | 32.5 | 18.9 | 21.9 | 43.4 | | In | In | In | In | In |
| 2 | 26.1 | 27.6 | 6.2 | 24.2 | 45.5 | | In | In | In | In | In |
| 3 | 5.9 | 4.6 | 41.1 | 11.4 | 15.4 | | In | In | In | In | In |
| 4 | 8.7 | 22.9 | 21.4 | 12.9 | 36.0 | | In | In | In | In | In |
| 5 | 17.2 | 52.1 | 21.0 | 22.9 | 76.1 | | In | In | In | In | In |
| 6 | 0 | 25.2 | 9.1 | 1.0 | 23.9 | | R-In | R-In | R-In | R-In | R-In |
| 7 | 14.3 | 46.4 | 22.4 | 23.1 | 62.0 | | In | In | In | In | In |
| 8 | 26.6 | 10.5 | 18.6 | 29.3 | 18.5 | | In | In | In | In | In |
| 9 | 27.1 | 7.7 | 22.3 | 13.4 | 19.7 | | In | In | In | In | In |
| 10 | 17.1 | 13.2 | 22.1 | 28.7 | 12.3 | | In | In | In | In | In |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 23.5 | 22.7 | 19.7 | 28.3 | 25.8 | | In | In | In | In | In |
| 12 | 43.2 | 19.2 | 28.4 | 24.3 | 20.0 | | In | In | In | In | In |
| 13 | 19.8 | 6.7 | 18.7 | 18.3 | 12.4 | | In | In | In | In | In |
| 14 | 5.0 | 7.0 | 15.2 | 3.9 | 10.2 | | In | In | In | In | In |
| 15 | 14.8 | 7.3 | 8.4 | 14.7 | 14.3 | | In | In | In | In | In |
| 16 | 21.4 | 1.1 | 27.1 | 16.6 | 4.0 | | In | In | In | In | In |
| 17 | 17.1 | 41.2 | 26.6 | 22.6 | 55.5 | | In | In | In | In | In |
| 18 | 6.3 | 2.1 | 14.3 | 9.9 | 5.8 | | In | In | In | In | In |
| 19 | 39.2 | 5.5 | 4.6 | 46.8 | 9.0 | | In | In | In | In | In |
| 20 | 0 | 2.2 | 0 | 0 | 4.1 | | R-In | R-In | R-In | R-In | R-In |
| 21 | 2.7 | 14.2 | 22.4 | 5.4 | 18.3 | | R-In | R-In | R-In | R-In | R-In |
| 22 | 20.6 | 15.2 | 17.1 | 26.1 | 14.7 | | In | In | In | In | In |
| 23 | 2.4 | 18.2 | 25.6 | 7.3 | 23.3 | | In | In | In | In | In |
| 24 | 14.6 | 25.2 | 19.8 | 26.4 | 29.0 | | In | In | In | In | In |
| 25 | 9.4 | 12.8 | 12.3 | 8.9 | 17.6 | | In | In | In | In | In |
| 26 | 20.8 | 4.9 | 10.0 | 22.8 | 10.3 | | In | In | In | In | In |
| 27 | 9.6 | 4.8 | 20.1 | 14.5 | 8.5 | | In | In | In | In | In |
| 28 | 17.1 | 11.8 | 9.0 | 26.5 | 22.6 | | In | In | In | In | In |
| 29 | 13.1 | 1.4 | 20.1 | 10.0 | 5.4 | | In | In | In | In | In |
| 30 | 26.5 | 9.6 | 10.1 | 35.0 | 5.5 | | In | In | In | In | In |

**Table 2: Process average classification**

| Sample Number | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 | Status | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 35.57 | 65 | 37.8 | 43.8 | 86.8 | | In | In | In | In | In |
| 2 | 52.18 | 55.2 | 12.4 | 48.4 | 91 | | In | In | In | In | In |
| 3 | 11.8 | 9.2 | 82.2 | 22.8 | 30.8 | | R-In | R-In | In | In | In |
| 4 | 17.4 | 45.8 | 42.8 | 25.8 | 72 | | In | In | In | In | In |
| 5 | 34.4 | 104.2 | 42 | 45.8 | 152.2 | | In | In | In | In | In |
| 6 | 0 | 50.4 | 18.2 | 2 | 47.8 | | R-In | R-In | R-In | R-In | R-In |
| 7 | 28.6 | 92.8 | 44.8 | 46.2 | 124 | | In | In | In | In | In |
| 8 | 53.2 | 21 | 37.2 | 58.6 | 37 | | In | In | In | In | In |
| 9 | 54.2 | 15.4 | 44.6 | 26.8 | 39.4 | | In | In | In | In | In |
| 10 | 34.2 | 26.4 | 44.2 | 57.4 | 24.6 | | In | In | In | In | In |
| 11 | 47 | 45.4 | 39.4 | 56.6 | 51.6 | | In | In | In | In | In |
| 12 | 86.4 | 38.4 | 56.8 | 48.6 | 40 | | In | In | In | In | In |
| 13 | 39.6 | 13.4 | 37.4 | 36.6 | 24.8 | | In | In | In | In | In |
| 14 | 10 | 14 | 30.4 | 7.8 | 20.4 | | R-In | R-In | In | In | In |
| 15 | 29.6 | 14.6 | 16.8 | 29.4 | 28.6 | | In | In | In | In | In |
| 16 | 42.8 | 2.2 | 54.2 | 33.2 | 8 | | In | In | In | In | In |
| 17 | 34.2 | 82.4 | 53.2 | 45.2 | 111 | | In | In | In | In | In |
| 18 | 12.6 | 4.2 | 28.6 | 19.8 | 11.6 | | In | In | In | In | In |
| 19 | 78.4 | 11 | 9.2 | 93.6 | 18 | | In | In | In | In | In |
| 20 | 0 | 4.4 | 0 | 0 | 8.2 | | R-In | R-In | R-In | R-In | R-In |
| 21 | 5.4 | 28.4 | 44.8 | 10.8 | 36.6 | | R-Out | R-In | R-In | R-In | R-In |
| 22 | 41.2 | 30.4 | 34.2 | 52.2 | 29.4 | | In | In | In | In | In |
| 23 | 4.8 | 36.4 | 51.2 | 14.6 | 46.6 | | R-Out | In | In | In | In |
| 24 | 29.2 | 50.4 | 39.6 | 52.8 | 58 | | In | In | In | In | In |
| 25 | 18.8 | 25.6 | 24.6 | 17.8 | 35.2 | | In | In | In | In | In |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 26 | 41.6 | 9.8 | 20 | 45.6 | 20.6 | | In | In | In | In | In |
| 27 | 19.2 | 9.6 | 40.2 | 29 | 17 | | In | In | In | In | In |
| 28 | 34.2 | 23.6 | 18 | 53 | 45.2 | | In | In | In | In | In |
| 29 | 26.2 | 2.8 | 40.2 | 20 | 10.8 | | In | In | In | In | In |
| 30 | 53 | 19.2 | 20.2 | 70 | 11 | | In | In | In | In | In |

The current process can be classified according to the rules we proposed for the process average and variability, which are shown in Tables 1 and 2, where we also provide a brief presentation of the numerical results at each of the five optimism levels (0.1, 0.3, 0.5, 0.7, and 0.9). Although the process variability of the sixth, twentieth, and twenty-first samples is regarded as acceptable (rather in control), the numerical data in Table 1 show that the process variability is currently under control. Because the process's control limitations depend on its variability, the in-control process variability is widely regarded as the first and most crucial criterion to further analyze the process under the □ chart. Furthermore, eliminating assignable causes of variability does not reveal any systematic pattern on the □ chart (Figure 1).We examined the 30 samples based on Table 2, and five worrying signals, including the samples from the 14th, 20th, 21st, and 23rd, are found. A pessimistic decision maker (at optimism level 0.1) thinks that the 15th and 20th samples are somewhat out-of-control (R-Out), whereas the 19th, 23rd, and 25th samples are thought to be rather in-control (R-In).

## VII. DISCUSSION AND CONCLUSION

The control limits of the fuzzy □ and □ control charts that we suggested in this study are derived from the resolution identity results in the well-known fuzzy set theory. Conventional control charts are only useful for real-valued data in the monitoring process, where they can be used to classify the process as either in control or out of control. However, fuzzy data refers to data that is not exactly obtained due to specific measurement issues. Consequently, the classic controlcharts seem to be inappropriate in the fuzzy environment. As a result, we suggested fuzzy □ and □ control charts in this study, whose control limits are derived from the resolution identity results in the well-known fuzzy set theory. Furthermore, we created comprehensive assessment rules by streamlining a newly suggested ranking approach based on the left and right integral value in order to monitor the process based on these fuzzy control charts.

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