H_k CORDIAL LABELING OF PATH, STAR AND CYCLE GRAPHS

Abstract

In this paper we investigate H_k cordial labeling of star, path, cycle and use operation such as subdivision, super subdivision and H - super subdivision on it, i.e. $S(Pn)$, $SS(Pn)$, $HSS(Pn)$, $S(K1,n)$, $SS(K1,n)$, $HSS(K1,n)$, $S(Cn)$, $SS(Cn)$, $HSS(Cn)$.

Keywords: H cordial labeling, H_k cordial labeling, Subdivision, Super subdivision, H-Super subdivision of graphs..

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I. INTRODUCTION

In the present work we contemplate a finite graph which is connected and undirected. We refer to a dynamic survey of graph labeling by Gallian (2020) for detailed survey on graph labeling. For all other standard terminology and notations we refer to Gross and Yellen [4]. A labeling of a graph $G = (V, E)$ is a mapping that carries vertices, edges or both to the set of labels (usually to the positive or non-negative integers).

A graph $G = (V, E)$ is said to beHcordial graph if there exists a mapping$f$ from edge set to $\{-1, 1\}$such that induced mapping$f^*$from vertex set to $\{-k, k\}$defined by $f^*(v) = \sum_{e\in I(v)} f(e)$, where $I(v)$ is the set of all edges incident to vertex v,satisfies the cordiality conditions $|e_f(1) - e_f(-1)| \leq 1$ and $|v_{f^*}(k) - v_{f^*}(-k)| \leq 1$. Map f is called Hcordial labeling of G. By extending the concept a graphis $H_k$cordial graph if there exists a mapping$f$ from edge set to$\{\pm 1, \pm 2, \ldots, \pm k\}$such that the induced mapping f^* from vertex set to$\{\pm 1, \pm 2, \ldots, \pm k\}$ defined by $f^*(v) = \sum_{e\in I(v)} f(e)$, where $I(v)$ is the set of all edges incident to vertex v,satisfies the cordiality conditions$|e_f(i) - e_f(-i)| \leq 1$ and$|v_{f^*}(i) -$ vf∗(−i)≤1 for 1≤i≤k . Map f is calledHkcordial labeling of G and graph G is called $H_k$cordial graph[5].

Barycentric subdivision of graph G is denoted as$S(G)$, obtained by subdividing every edge of graph G. [10]. The super subdivision of any graph G denoted by $SS(G)$ is obtained fromgraph by replacing every edge of graph by complete bipartite graph $K_{2,m}$ (where m is positive integer)[8].

II. MAIN RESULT

Theorem 2.1 The star graph $K_{1,n}$ $(n \geq 2)$is H_2cordial if n is even.

Proof: Let $V(K_{1,n}) = \{u_0, u_i : 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{u_0u_i : 1 \leq i \leq n\}$, where $u_0$is apex vertex.
Consider a function $f: E(K_{1,n}) \rightarrow \{-2, -1, 1, 2\}$ defined as

$$f(u_0u_1) = -2$$

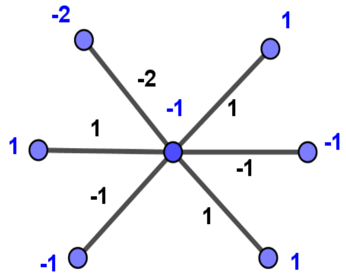
$f(u_0u_i) = (-1)^i \;;\; 2 \leq i \leq n.$

Table 1

$n \geq 2$	Edge Conditions	Vertex Conditions
n is even	$e_f(1) = \frac{n}{2}, e_f(-1) = \frac{n-2}{2}$ $e_f(2) = 0, e_f(-2) = 1$	$v_{f^*}(1) = \frac{n}{2} = v_{f^*}(-1)$ $v_{f^*}(2) = 0, v_{f^*}(-2) = 1$

For $i = 1,2$.The $K_{1,n}$ satisfies the condition $|e_f(i) - e_f(-i)| \leq 1$ and$|v_f(i) - v_f(-i)| \leq 1$.
Hence, $K_{1,n}$is H_2cordial.

Illustration 2.2 Figure shows that $K_{1,6}$is H_2cordial graph.



Theorem 2.3 Star graph $K_{1,n}$($n \geq 2$)is H_3cordial.

Proof: Let $V(K_{1,n}) = \{u_0, u_i : 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{u_0u_i : 1 \leq i \leq n\}$, where $u_0$is apex vertex.

Type1:n is even,$K_{1,n}$ is H_2cordial from Theorem 2.1. Hence it is also admits H_3cordial labeling.

Type2:n is odd.

Consider a function $f: E(K_{1,n}) \to \{-3, -2, -1, 1, 2, 3\}$ defined as

$$f(u_0u_1) = -2$$

$$f(u_0u_1) = -2$$

$$f(u_0u_2) = 3$$

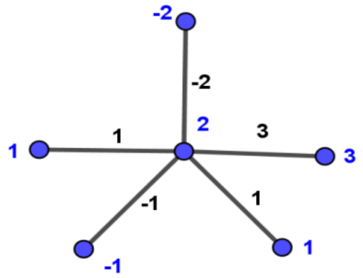
$$f(u_0u_i) = (-1)^{i+1} \; ; \; 3 \leq i \leq n$$

Table 2

$n \geq 2$	Edge Conditions	Vertex Conditions
n is odd	$e_f(1) = \frac{n-1}{2}, e_f(-1) = \frac{n-3}{2}$ $e_f(2) = 0, e_f(-2) = 1$ $e_f(3) = 1, e_f(-3) = 0$	$v_{f^*}(1) = \frac{n-1}{2}, v_{f^*}(-1) = \frac{n-3}{2}$ $v_{f^*}(2) = 1 = v_{f^*}(-2)$ $v_{f^*}(3) = 1, v_{f^*}(-3) = 0$

For $i = 1,2,3$, the graph satisfies the condition $|e_f(i) - e_f(-i)| \leq 1$ and$|v_f(i) - v_f(-i)| \leq 1$.
Hence, $K_{1,n}$is H_3- cordial.

Illustration 2.4$K_{1,5}$is H_3cordialas shown in Figure.



Theorem 2.5 The barycentric subdivision graph of a star ($S(K_{1,n})$ $(n \geq 2$)) is H_2cordial if n is odd.

Proof: Let $V(\,S(K_{1,n})) = \{u_i, u'_i, u_0; 1 \leq i \leq n\}andE(\,S(K_{1,n})) = \{u_0u'_i, u_iu'_i\,; 1 \leq i \leq n.$

Consider a function $f: E(\,S(K_{1,n})) \to \{-2,-1,1,2\}$ defined as
$f(u_0u'_1) = -2$

$f(u_0u'_i) = (-1)^{i+1};\ 2 \leq i \leq n$

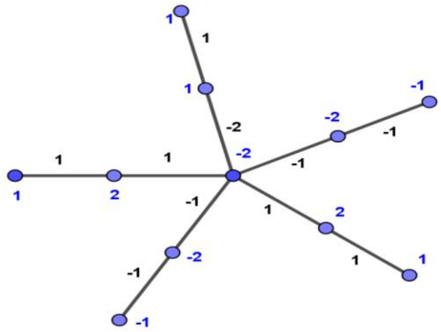
$f(u_iu'_i) = (-1)^{i+1}\,;\ 1 \leq i \leq n$

Table 3

$n \geq 2$	Edge Conditions	Vertex Conditions
n is odd	$e_f(1) = n, e_f(-1) = n+1$ $e_f(2) = 0, e_f(-2) = 1$	$v_{f^*}(1) = \frac{n+1}{2} = v_{f^*}(-1)$ $v_{f^*}(2) = \frac{n-1}{2}, v_{f^*}(-2) = \frac{n+1}{2}$

Hence, $S(K_{1,n})$ is H_2cordial.

Illustration 2.6$S(K_{1,5})$ is H_2cordialas shown in Figure.



Theorem 2.7 The barycentric subdivision graph of a star $S(K_{1,n})$ $(n \geq 2)$ is H_3cordial.

Proof: Let$V(S(K_{1,n})) = \{u_i, u'_i, u_0; 1 \leq i \leq n\}$and $E(S(K_{1,n})) = \{u_0u'_i, u_iu'_i ; 1 \leq i \leq n\}$.

Type 1:n is odd.
$S(K_{1,n})$ isH_2cordial from Theorem 2.5, it is also admitsH_3cordial.

Type 2:n is even.
Consider a function $f: E(S(K_{1,n})) \to \{-2, -1, 1, 2\}$ defined as

$f(u_0u'_1) = 2$

$f(u_0u'_i) = (-1)^i;\ 2 \leq i \leq n$

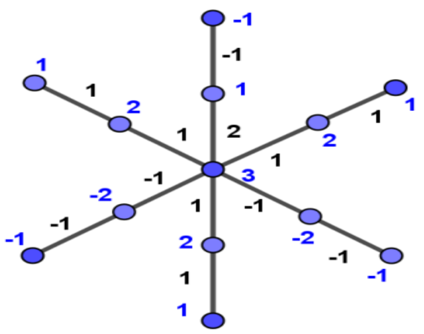
$f(u_iu'_i) = (-1)^i;\ 1 \leq i \leq n$

Table 4

$n \geq 2$	Edge Conditions	Vertex Conditions
n is even	$e_f(1) = n, e_f(-1) = n-1$ $e_f(2) = 1, e_f(-2) = 0$	$v_{f^*}(1) = \frac{n+2}{2}, v_{f^*}(-1) = \frac{n}{2}$ $v_{f^*}(2) = \frac{n}{2}, v_{f^*}(-2) = \frac{n-2}{2}$ $v_{f^*}(3) = 1, v_{f^*}(-3) = 0$

Hence, $S(K_{1,n})$ is H_3cordial.

Illustration2.8H_3cordial labeling of $S(K_{1,6})$ is demonstrated in Figure.



Theorem 2.9 Super subdivision of star graph $SS(K_{1,n})(n \geq 2)$ is H_3cordial.

Proof: Let $V(SS(K_{1,n})) = \{u_i, u_{ij}, u_0; 1 \leq i \leq n, 1 \leq j \leq m\}andE(SS(K_{1,n})) = \{u_0u_{ij}, u_{ij}u_i; 1 \leq i \leq n, 1 \leq j \leq m\}$, where $u_0$is apex vertex.

Consider a function $f: E(SS(K_{1,n})) \to \{-2, -1, 1, 2\}$ defined as

Type 1: m is evenand n is odd

$f(u_0u_{11}) = -2$

$f(u_0u_{12}) = 1$

$f(u_{11}u_1) = -1$

$f(u_{12}u_1) = 2$

$f(u_0u_{ij}) = f(u_{ij}\,u_i) = (-1)^i$; $2 \le i \le n, j = 1,2$

$f(u_0u_{ij}) = f(u_{ij}\,u_i) = (-1)^j$; $3 \le j \le m, 1 \le i \le n.$

Table 5

$n \ge 2$	Edge Conditions	Vertex Conditions
m even n odd	$e_f(1) = mn - 1 = e_f(-1)$ $e_f(2) = 1 = e_f(-2)$	$v_{f^*}(1) = 1 = v_{f^*}(-1)$ $v_{f^*}(2) = \frac{(m+1)n-3}{2} = v_{f^*}(-2)$ $v_{f^*}(3) = 1 = v_{f^*}(-3)$

Type 2: m and n both are even

$f(u_0u_{11}) = -2$

$f(u_0u_{12}) = f(u_{11}u_1) = f(u_{12}u_1) = -1$

$f(u_0u_{ij}) = f(u_{ij}\,u_i) = (-1)^i$; $2 \le i \le n,\ j = 1,2$

$f(u_0u_{ij}) = f(u_{ij}\,u_i) = (-1)^j$; $3 \le j \le m,\ 1 \le i \le n$

Table 6

$n \ge 2$	Edge Conditions	Vertex Conditions
m even n even	$e_f(1) = mn,\ e_f(-1) = mn - 1$ $e_f(2) = 1, e_f(-2) = 0$	$v_{f^*}(1) = 1, v_{f^*}(-1) = 0$ $v_{f^*}(2) = \frac{n(m+1)}{2}$ $v_{f^*}(-2) = \frac{n(m+1)-2}{2}$ $v_{f^*}(3) = 1, v_{f^*}(-3) = 0$

Type 3: m and n both are odd

$f(u_0u_{11}) = -2$

$f(u_{11}u_1) = 1$

$f(u_0u_{i1}) = f(u_{i1}u_i) = (-1)^{i+1}$; $2 \le i \le n$

$f(u_0u_{ij}) = f(u_{ij}u_i) = (-1)^j$; $2 \le j \le m, 1 \le i \le n$

Table 7

$n \ge 2$	Edge Conditions	Vertex Conditions
m odd n odd	$e_f(1) = mn, e_f(-1) = mn - 1$ $e_f(2) = 0, e_f(-2) = 1$	$v_{f^*}(1) = \frac{n+1}{2} = v_{f^*}(-1)$ $v_{f^*}(2) = \frac{mn-1}{2}, v_{f^*}(-2) = \frac{mn-1}{2}$

Type 4: m is odd and n is even

$f(u_0u_{11}) = 2$

$f(u_{11}u_1) = -1$

$f(u_0u_{i1}) = f(u_{i1}u_i) = (-1)^i$; $2 \le i \le n$

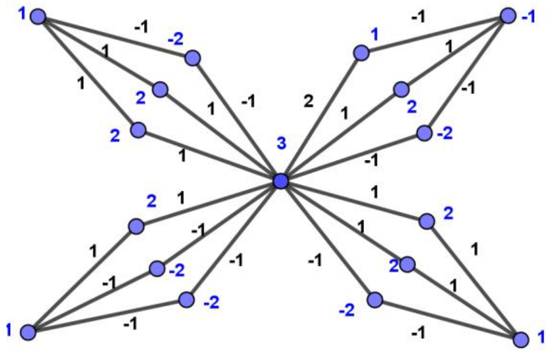
$f(u_0u_{ij}) = f(u_{ij}u_i) = (-1)^j$; $2 \le j \le m, 1 \le i \le n$

Table 8

$n \ge 2$	Edge Conditions	Vertex Conditions
m odd n even	$e_f(1) = mn, e_f(-1) = mn - 1$ $e_f(2) = 1, e_f(-2) = 0$	$v_{f^*}(1) = \frac{n+2}{2}, v_{f^*}(-1) = \frac{n}{2}$ $v_{f^*}(2) = \frac{mn}{2}, v_{f^*}(-2) = \frac{mn-2}{2}$ $v_{f^*}(3) = 1, v_{f^*}(-3) = 0$

Hence, $SS(K_{1,n})$ is H_3cordial.

Illustration 2.10 $SS(K_{1,4})$ with$m = 3$ is H_3cordial shown in Figure.



Theorem 2.11 The H -super subdivision of path $HSS(K_{1,n})(n \geq 2)H_3$cordial.

Proof: Let $V\left(HSS(K_{1,n})\right) = \{u_i, u_{ij}, u_0; 1 \leq i \leq n, 1 \leq j \leq 4\}$and $E\left(HSS(K_{1,n})\right) = \{u_0u_{i1}, u_iu_{i3}, u_{i1}u_{i3}, u_{i1}u_{i2}, u_{i3}u_{i4}; 1 \leq i \leq n\}$, where $u_0$is apex vertex.

Type 1:n is odd, consider a function $f: E\left(HSS(K_{1,n})\right) \rightarrow \{-1,1\}$ defined as

$f(u_0u_{11}) = f(u_{11}u_{12}) = 1$

$f(u_1u_{13}) = f(u_{13}u_{14}) = -1$

$f(u_{i1}u_{i3}) = (-1)^i; 1 \leq i \leq n$

$f(u_0u_{i1}) = f(u_{i1}u_{i2}) = f(u_iu_{i3}) = f(u_{i3}u_{i4}) = (-1)^{i+1}; 2 \leq i \leq n.$

Table 9

$n \geq 2$	Edge Conditions	Vertex Conditions
n is odd	$e_f(1) = \frac{5n-1}{2}, e_f(-1) = \frac{5n+1}{2}$	$v_{f^*}(1) = \frac{5n+1}{2}, v_{f^*}(-1) = \frac{5n-1}{2}$ $v_{f^*}(3) = 0, v_{f^*}(-3) = 1$

Hence f satisfies the conditions of H_3 cordial labeling in this Type and hence the graph under consideration is H_3cordial graph, when n is odd.

Type 2: n is even, consider a function $f: E \rightarrow \{-2, -1, 1, 2\}$ defined as

$f(u_0u_{11}) = 2$

$f(u_0u_{21}) = -1$
$f(u_iu_{i3}) = (-1)^{i+1}; i = 1,2$

$f(u_{i1}u_{i2}) = f(u_{i3}u_{i4}) = (-1)^i; i = 1,2$

$f(u_{i1}u_{i3}) = (-1)^{i+1}; 1 \leq i \leq n$

$f(u_0u_{i1}) = f(u_{i1}u_{i2}) = f(u_iu_{i3}) = f(u_{i3}u_{i4}) = (-1)^i; 3 \leq i \leq n.$

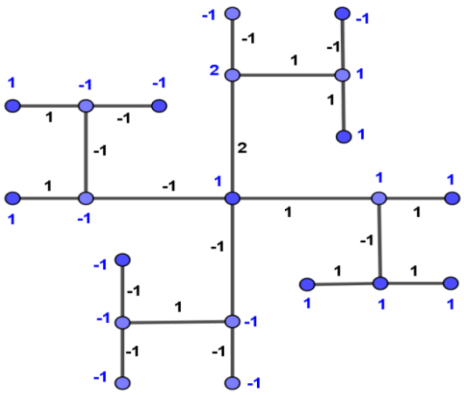
Table 10

$n \geq 2$	Edge Conditions	Vertex Conditions
n is even	$e_f(1) = \frac{5n-2}{2}, e_f(-1) = \frac{5n}{2}$ $e_f(2) = 1, e_f(-2) = 0$	$v_{f^*}(1) = \frac{5n}{2} = v_{f^*}(-1)$ $v_{f^*}(2) = 1, v_{f^*}(-2) = 0$

In this Type$f$ satisfies the conditions of H_2cordiallabeling and hence the graph under consideration is H_2cordial graph, when n is even.

Hence, $HSS(K_{1,n})$ is H_3cordial as per above Types.

Illustration2.12 $HSS(K_{1,4})$ is H_2cordial shown in Figure.



Theorem 2.13 Path $P_n(n \geq 3)$ is H_3cordial.

Proof: Let P_n be the path $u_1, u_2, \ldots, u_n$.

Consider a function $f: E(P_n) \rightarrow \{-2, -1, 1, 2\}$ defined as

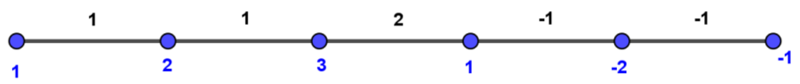
$$f(u_iu_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1 \\ 2 & ; i = \left\lceil \frac{n}{2} \right\rceil \\ -1 & ; Otherwise \end{cases}$$

Table 11

$n \geq 3$	Edge Conditions	Vertex Conditions
n is even	$e_f(1) = \frac{n-2}{2} = e_f(-1)$ $e_f(2) = 1, e_f(-2) = 0$	$v_{f^*}(1) = 2, v_{f^*}(-1) = 1$ $v_{f^*}(2) = \frac{n-4}{2} = v_{f^*}(-2)$ $v_{f^*}(3) = 1, v_{f^*}(-3) = 0$
n is odd	$e_f(1) = \frac{n-1}{2}, e_f(-1) = \frac{n-3}{2}$ $e_f(2) = 1, e_f(-2) = 0$	$v_{f^*}(1) = 2, v_{f^*}(-1) = 1$ $v_{f^*}(2) = \frac{n-4}{2}, v_{f^*}(-2) = \frac{n-6}{2}$ $v_{f^*}(3) = 1, v_{f^*}(-3) = 0$

Hence, P_n is H_3 cordial.

Illustration2.14 H_3 cordial labeling of P_6 is as shown in below Figure.



Remarks 2.15
Consider path P_n. As per barycentric subdivision of $P_n (n \geq 2)$ is again a path P_{2n-1} which is also is H_3 cordial as per Theorem 2.13. Hence we have the following.

Theorem 2.16 Thebarycentricsubdivision of path $S(P_n)(n \geq 2)$ is H_3 cordial.

Theorem 2.17 The super subdivision of path $SS(P_n)(n \geq 3)$ is H_3 cordial.

Proof: Let $V(SS(P_n)) = \{u_i, u_{ij}, u_n; 1 \leq i \leq n-1, 1 \leq j \leq m\}$ and $E(SS(P_n)) = \{u_i u_{ij}, u_{ij} u_{i+1}; 1 \leq i \leq n-1, 1 \leq j \leq m\}$.

Consider a function $f: E \to \{-2, -1, 1, 2\}$ defined as

Type 1: m is even and n is odd.

$$f(u_i u_{i1}) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n-3}{2} \\ 2 & ; i = \frac{n-1}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_{i1}u_{i+1}) = \begin{cases} 1 & ; 1 \le i \le \frac{n-3}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_i u_{i2}) = f(u_{i2}u_{i+1}) = (-1)^{i+1} \; ; 1 \le i \le n-1$$

$$f(u_i u_{ij}) = f(u_{ij}u_{i+1}) = (-1)^j \; ; 3 \le j \le m.$$

Table 12

$n \ge 3$	Edge Conditions	Vertex Conditions
m is even n is odd	$e_f(1) = m(n-1),$ $e_f(-1) = m(n-1) - 1$ $e_f(2) = 1, e_f(-2) = 0$	$v_{f^*}(1) = 1, v_{f^*}(-1) = 0$ $v_{f^*}(2) = \frac{(m+1)(n-1)}{2}$ $v_{f^*}(-2) = \frac{(m+1)(n-1)-2}{2}$ $v_{f^*}(3) = 1, v_{f^*}(-3) = 0$

Type 2: m and n bothareeven

$$f(u_i u_{i1}) = f(u_{i1}u_{i+1}) = \begin{cases} 1 & ; 2 \le i \le \frac{n}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_i u_{ij}) = f(u_{ij}u_{i+1}) = \begin{cases} 1 & ; j = 1,2 \; ; i = 1 \\ -1 & ; j = 1,2 \; ; i = n-1 \end{cases}$$

$$f(u_i u_{i2}) = \begin{cases} -2 & ; 2 \le i \le \frac{n}{2} \\ 2 & ; Otherwise \end{cases}$$

$$f(u_{i2}u_{i+1}) = \begin{cases} -1 & ; 2 \le i \le \frac{n}{2} \\ 1 & ; Otherwise \end{cases}$$

$$f(u_i u_{ij}) = f(u_{ij}u_{i+1}) = (-1)^j \; ; 3 \le j \le m.$$

Table 13

$n \ge 3$	Edge Conditions	Vertex Conditions
m is even n is even	$e_f(1) = \frac{(2m-1)(n-1)+3}{2}$ $e_f(-1) = \frac{(2m-1)(n-1)+1}{2}$ $e_f(2) = \frac{n-4}{2}, e_f(-2) = \frac{n-2}{2}$	$v_{f^*}(1) = \frac{n-2}{2}, v_{f^*}(-1) = \frac{n-4}{2}$ $v_{f^*}(2) = \frac{(m-1)(n-1)+5}{2} = v_{f^*}(-2)$ $v_{f^*}(3) = \frac{n-4}{2}, v_{f^*}(-3) = \frac{n-2}{2}$

Type 3: m is odd and $n \geq 3$

$$f(u_i u_{i1}) = \begin{cases} 1 & ; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1 \\ 2 & ; i = \left\lceil \frac{n}{2} \right\rceil \\ -1 & ; Otherwise \end{cases}$$

$$f(u_{i1} u_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1 \\ -1 & ; Otherwise \end{cases}$$

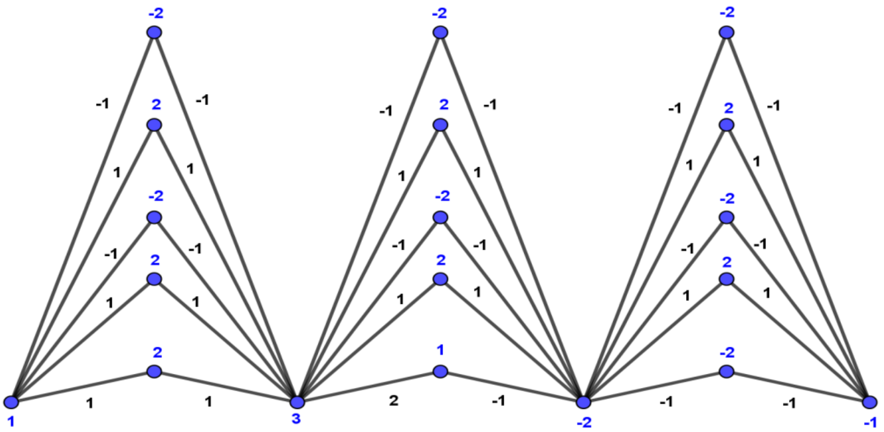
$f(u_i u_{ij}) = f(u_{ij} u_{i+1}) = (-1)^j$; $2 \leq j \leq m$.

Table 14

$n \geq 3, m$ odd	Edge Conditions	Vertex Conditions
n is even	$e_f(1) = m(n-1) - 1,$ $e_f(-1) = m(n-1)$ $e_f(2) = 1, e_f(-2) = 0$	$v_{f^*}(1) = 2, v_{f^*}(-1) = 1$ $v_{f^*}(2) = \frac{(m+1)(n-1)-4}{2}$ $v_{f^*}(-2) = \frac{(m+1)(n-1)-2}{2}$ $v_{f^*}(3) = 1, v_{f^*}(-3) = 0$
n is odd	$e_f(1) = m(n-1),$ $e_f(-1) = m(n-1) - 1$ $e_f(2) = 1, e_f(-2) = 0$	$v_{f^*}(1) = 2, v_{f^*}(-1) = 1$ $v_{f^*}(2) = \frac{(m+1)(n-1)-2}{2}$ $v_{f^*}(-2) = \frac{(m+1)(n-1)-4}{2}$ $v_{f^*}(3) = 1, v_{f^*}(-3) = 0$

Hence, $SS(P_n)$ is H_3 cordial.

Illustration 2.18 $SS(P_4)$ with$m = 5$ is H_3cordial shown in Figure.



Theorem 2.19 The H -super subdivision of path $HSS(P_n)(n \geq 2)$ is H_3cordial.

Proof: Let$V(HSS(P_n)) = \{u_i, u_{ij}\,; 1 \leq i \leq n\,, 1 \leq j \leq 4\}$
and$E\big(HSS(P_n)\big) = \{u_i u_{i1}, u_{i3}u_{i+1}, u_{i1}u_{i3}, u_{i1}u_{i2}, u_{i3}u_{i4}; 1 \leq i \leq n-1\}$.
Consider a function $f: E(HSS(P_n)) \to \{-2, -1, 1, 2\}$ defined as

Type 1: n is odd.

$$f(u_i u_{i1}) = f(u_{i1}u_{i3}) = \begin{cases} 1 & ; 1 \leq i \leq \dfrac{n-1}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_{i+1}u_{i3}) = \begin{cases} 1 & ; 1 \leq i \leq \dfrac{n-3}{2} \\ 2 & ; i = \dfrac{n-1}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_{i1}u_{i2}) = f(u_{i3}u_{i4}) = \begin{cases} -1 & ; 1 \leq i \leq \dfrac{n-1}{2} \\ 1 & ; Otherwise \end{cases}$$

Table 15

	Edge Conditions	Vertex Conditions
n is odd	$e_f(1) = \frac{5n-7}{2}, e_f(-1) = \frac{5n-5}{2}$ $e_f(2) = 1, e_f(-2) = 0$	$v_{f^*}(1) = 2n - 1 = v_{f^*}(-1)$ $v_{f^*}(2) = \frac{n-1}{2}, v_{f^*}(-2) = \frac{n-3}{2}$

Hence f satisfies the conditions H_2cordiallabeling in this Type.

Type 2: n is even.

$f(u_i u_{i1}) = f(u_{i+1} u_{i3}) = (-1)^i; 2 \leq i \leq n-1$

$f(u_1 u_{11}) = 1$

$f\left(u_n u_{(n-1)3}\right) = -1$

$f(u_{i1} u_{i3}) = f(u_{i3} u_{i4}) = (-1)^i; 1 \leq i \leq n-1$

$f(u_{i1} u_{i2}) = (-1)^{i+1}; 1 \leq i \leq n-1.$

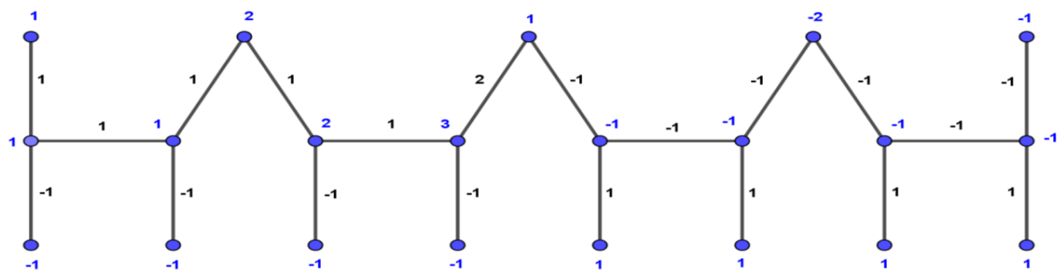
Table 16

	Edge Conditions	Vertex Conditions
n is even	$e_f(1) = \frac{5n-6}{2}, e_f(-1) = \frac{5n-4}{2}$	$v_{f^*}(1) = 2n-1, v_{f^*}(-1) = 2n-2$ $v_{f^*}(2) = \frac{n-2}{2} = v_{f^*}(-2)$ $v_{f^*}(3) = 0, v_{f^*}(-3) = 1$

Hence f satisfies the conditions H_3 cordiallabeling in this Type.

Hence, $HSS(P_n)$ is H_3 cordialgraph.

Illustration 2.20 H_3 cordial labeling of $HSS(P_5)$ as shown in below Figure.



Theorem 2.21 Cycle $C_n (n \geq 4)$ is H_3 cordial.

Proof: Let $V(C_n) = \{u_i; 1 \leq i \leq n\}$ and $E(C_n) = \{u_i u_{i+1}, u_1 u_n; 1 \leq i \leq n-1\}$.

Consider a function $f: E(C_n) \rightarrow \{-2, -1, 1, 2\}$ defined as

$$f(u_i u_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1 \\ 2 & ; i = \left\lceil \frac{n}{2} \right\rceil \\ -1 & ; \left\lceil \frac{n}{2} \right\rceil + 1 \leq i \leq n-1 \end{cases}$$

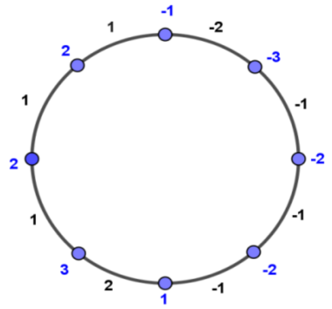
$f(u_n u_1) = -2.$

Table 17

$n \geq 4$	Edge Conditions	Vertex Conditions
n is even	$e_f(1) = \frac{n-2}{2} = e_f(-1)$ $e_f(2) = 1 = e_f(-2)$	$v_{f^*}(1) = 1 = v_{f^*}(-1)$ $v_{f^*}(2) = \frac{n-4}{2} = v_{f^*}(-2)$ $v_{f^*}(3) = 1 = v_{f^*}(-3)$
n is odd	$e_f(1) = \frac{n-1}{2}, e_f(-1) = \frac{n-3}{2}$ $e_f(2) = 1 = e_f(-2)$	$v_{f^*}(1) = 1 = v_{f^*}(-1)$ $v_{f^*}(2) = \frac{n-3}{2}, v_{f^*}(-2) = \frac{n-5}{2}$ $v_{f^*}(3) = 1 = v_{f^*}(-3)$

Hence, C_n is H_3 cordial.

Illustration 2.22 H_3–cordial labeling of cycle C_8 is shown in below Figure.



Remarks 2.23 Consider cycle C_n. As per barycentric subdivision of $C_n (n \geq 3)$ is again a path C_{2n} which is also is H_3 cordial as per Theorem 2.19. Hence we have the following.

Theorem 2.24 Thebarycentricsubdivision of cycle $S(C_n)(n \geq 2)$ is H_3 cordial.

Theorem 2.25 Thesuper subdivision of cycle $SS(C_n)(n \geq 4)$ is H_3 cordial.

Proof: Let $V(SS(C_n)) = \{u_i, u_{ij}\,; 1 \le i \le n\,, 1 \le j \le m\}$and
$E(SS(C_n)) = \{u_i u_{ij}\,, u_{ij}\, u_{i+1},, u_1 u_{nj}\,; 1 \le i \le n, 1 \le j \le m\}$.

Consider a function $f: E(SS(C_n)) \to \{-2, -1, 1, 2\}$ defined as

Type 1: m is evenand n is odd.

$$f(u_i u_{i1}) = \begin{cases} 1 & ; 1 \le i \le \frac{n-1}{2} \\ 2 & ; i = \frac{n+1}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_{i1} u_{i+1}) = \begin{cases} 1 & ; 1 \le i \le \frac{n-1}{2} \\ -1 & ; \frac{n+1}{2} \le \; i \le n-1 \end{cases}$$

$f(u_1 u_{n1}) = -2$

$f(u_1 u_{n2}) = 2$

$f(u_n u_{n2}) = 1$

$f(u_i u_{i2}) = f(u_{i2} u_{i+1}) = (-1)^{i+1}\; ; 1 \le i \le n-1$

$f(u_i u_{ij}) = f(u_{ij}\, u_{i+1}) = (-1)^j\; ; 3 \le j \le m.$

Table 18

	Edge Conditions	Vertex Conditions
m is even and n is odd	$e_f(1) = mn-1, e_f(-1) = mn-2$ $e_f(2) = 2, e_f(-2) = 1$	$v_{f^*}(1) = 1, v_{f^*}(-1) = 0$ $v_{f^*}(2) = \frac{n(m+1)-3}{2}$ $v_{f^*}(-2) = \frac{n(m+1)-5}{2}$ $v_{f^*}(3) = 2, v_{f^*}(-3) = 1$

Type 2: m and n both are even.

$f(u_1 u_{n1}) = -2$

$$f(u_i u_{i1}) = \begin{cases} 1 & ; 1 \le i \le \frac{n}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_{i1}u_{i+1}) = \begin{cases} 1 & ; 1 \le i \le \frac{n-2}{2} \\ 2 & ; i = \frac{n}{2} \\ -1 & ; \frac{n+2}{2} \le \ i \le n-1 \end{cases}$$

$$f(u_i u_{i2}) = f(u_{i2}u_{i+1}) = (-1)^{i+1}\ ; 1 \le i \le n$$

$$f\left(u_i u_{ij}\right) = f\left(u_{ij}\, u_{i+1}\right) = (-1)^j\ ; 3 \le j \le m, 1 \le i \le n.$$

Table 19

	Edge Conditions	Vertex Conditions
m and n both are even	$e_f(1) = nm - 1 = e_f(-1)$ $e_f(2) = 1 = e_f(-2)$	$v_{f^*}(1) = 1 = v_{f^*}(-1)$ $v_{f^*}(2) = \frac{n(m+1)-4}{2} = v_{f^*}(-2)$ $v_{f^*}(3) = 1 = v_{f^*}(-3)$

Type 3: m and n both are odd.

$$f(u_1 u_{n1}) = -2$$

$$f(u_i u_{i1}) = \begin{cases} 1 & ; 1 \le i \le \frac{n-1}{2} \\ 2 & ; i = \frac{n+1}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_{i1}u_{i+1}) = \begin{cases} 1 & ; 1 \le i \le \frac{n-1}{2} \\ -1 & ; \frac{n+1}{2} \le \ i \le n-1 \end{cases}$$

$$f\left(u_i u_{ij}\right) = f\left(u_{ij}\, u_{i+1}\right) = (-1)^j\ ; 2 \le j \le m\ , 1 \le i \le n.$$

Type 4: m is odd and$n$ is even.

$$f(u_i u_{i1}) = \begin{cases} 1 & ; 1 \le i \le \frac{n}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_{i1}u_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n-2}{2} \\ 2 & ; i = \frac{n}{2} \\ -1 & ; \frac{n+2}{2} \leq i \leq n-1 \end{cases}$$

$f(u_1 u_{n1}) = -2$

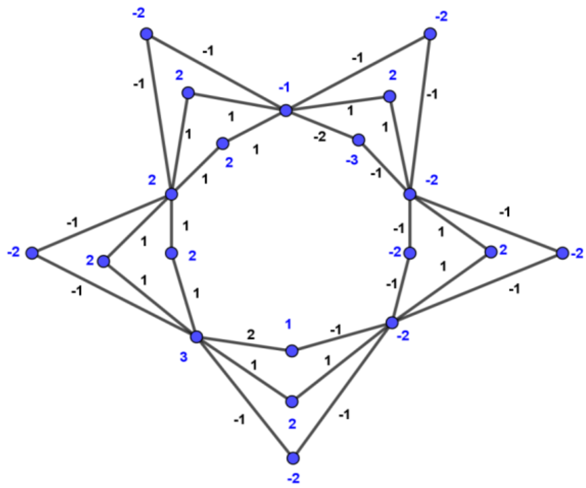
$f(u_i u_{ij}) = f(u_{ij} u_{i+1}) = (-1)^j$; $2 \leq j \leq m, 1 \leq i \leq n.$

Table 20

$n \geq 4$	Edge Conditions	Vertex Conditions
m is odd	$e_f(1) = nm - 1 = e_f(-1)$ $e_f(2) = 1 = e_f(-2)$	$v_{f^*}(1) = 1 = v_{f^*}(-1)$ $v_{f^*}(2) = \frac{n(m+1)-4}{2} = v_{f^*}(-2)$ $v_{f^*}(3) = 1 = v_{f^*}(-3)$

Hence, $SS(C_n)$ is H_3cordial.

Illustration 2.26 $SS(C_5)$ with$m = 3$ is H_3cordial shown in Figure.



Theorem 2.27 The H-super subdivision of cycle $HSS(C_n)(n \geq 3)$ is H_2cordial.

Proof: Let $V(HSS(C_n)) = \{u_i, u_{ij}; 1 \leq i \leq n, 1 \leq j \leq 4\}$and
$E(HSS(C_n)) = \{u_i u_{i1}, u_{i3} u_{i+1}, u_{i1} u_{i3}, u_{i1} u_{i2}, u_{i3} u_{i4}, u_1 u_{n3}; 1 \leq i \leq n-1\}.$

Consider a function $f: E(HSS(C_n)) \to \{-1,1\}$ definedas

Type 1: n is odd.

$$f(u_iu_{i1}) = \begin{cases} 1 & ; 1 \le i \le \frac{n+1}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_{i+1}u_{i3}) = f(u_{i1}u_{i3}) = \begin{cases} 1 & ; 1 \le i \le \frac{n-1}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_1u_{n3}) = 1$$

$$f(u_{i1}u_{i2}) = \begin{cases} -1 & ; 1 \le i \le \frac{n+1}{2} \\ 1 & ; Otherwise \end{cases}$$

$$f(u_{i3}u_{i4}) = \begin{cases} -1 & ; 1 \le i \le \frac{n-1}{2} \\ 1 & ; Otherwise \end{cases}$$

Table 21

	Edge Conditions	Vertex Conditions
n is odd	$e_f(1) = \frac{5n+1}{2}, e_f(-1) = \frac{5n-1}{2}$	$v_{f^*}(1) = 2n = v_{f^*}(-1)$ $v_{f^*}(2) = \frac{n+1}{2}, v_{f^*}(-2) = \frac{n-1}{2}$

Type 2: n is even.

$$f(u_iu_{i1}) = f(u_{i1}u_{i3}) = f(u_{i+1}u_{i3}) = \begin{cases} 1 & ; 1 \le i \le \frac{n}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(u_1u_{n3}) = 1$$

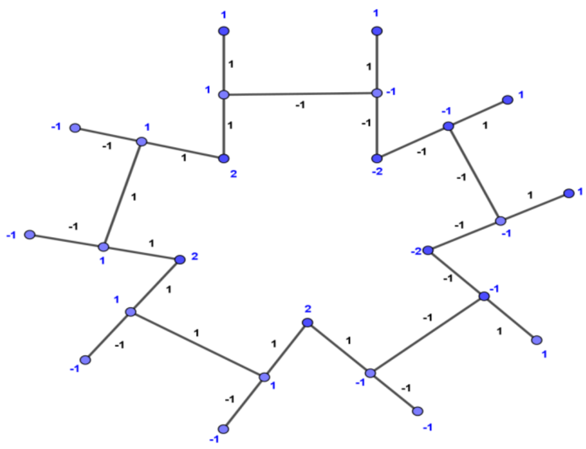
$$f(u_{i1}u_{i2}) = f(u_{i3}u_{i4}) = \begin{cases} -1 & ; 1 \le i \le \frac{n}{2} \\ 1 & ; Otherwise \end{cases}$$

Table 22

	Edge Conditions	Vertex Conditions
n is even	$e_f(1) = \frac{5n}{2} = e_f(-1)$	$v_{f^*}(1) = 2n = v_{f^*}(-1)$ $v_{f^*}(2) = \frac{n}{2} = v_{f^*}(-2)$

Hence, $HSS(C_n)$ is H_2cordial.

Illustration 2.28 H_2cordial labeling of $HSS(C_5)$ asshown in Figure.



III. CONCLUSION

Path, star and cycle graph are basic graphs which we have proved to be H_k –cordial graphs. We have derived the results on these graphs by considering operations such as barycentric subdividion, super subdivision and H- super subdivision.

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