QUADRIPARTITIONED SINGLE VALUED NEUTROSOPHIC REFINED CONTRA GENERALIZED PRE CONTINUOUS MAPPINGS

Abstract

The focus of this paper is to introduce and study the notions of quadripartitioned single valued neutrosophic refined contra generalized pre-continuous mappings in quadripartitioned single valued neutrosophic refined topological spaces.We examine some of its basic characteristics and properties.

Keywords: Quadripartitioned single valued neutrosophic refined topology, QNRCGP-conti.mapping, QNRC-conti.mapping, QNRCG-closed set

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I. INTRODUCTION

L.A. Zadeh [9]was the first to explain fuzzy sets and fuzzy set operations. Fuzzy topological spaces were first introduced and developed by Chang [4]. The earliest publication of the "Intuitionistic fuzzy set" notion was made by Atanassov [1].Fuzzy sets and neurothosophic sets, an expansion of intuitionistic fuzzy sets, were first described by Smarandache[6]. Neutosophic set theory addresses the problem of uncertainty. As an extension of intuitionistic fuzzy sets, fuzzy sets, and the classical set, Wang [7] proposed single-valued neutrosphic sets.Four membership functions make up Chatterjee's quadripartitioned single valued neutrosophic sets: truth, contradiction, unknown, and falsity. Deli et al.'s [5] development of intuitionistic fuzzy multisets and fuzzy multisets was the introduction of neurosophic refined sets.This paper is arranged in the following manner:Section 2 consists of basic concepts.Section 3 consists of quadripartitioned single valued neutrosophic refined contra generalized pre continuous mapping and its characterizations.

II. PRELIMINARIES

Definition 2.1 *[2] A QSVNRS ϱ on Λ can be defined by $\varrho = \{\langle \kappa, T_{\varrho}^{J}(\kappa), D_{\varrho}^{J}(\kappa), Y_{\varrho}^{J}(\kappa), F_{\varrho}^{J}(\kappa)\rangle : \kappa \in \Lambda\}$*

where$T_{\varrho}^{J}(\kappa), D_{\varrho}^{J}(\kappa), Y_{\varrho}^{J}(\kappa), F_{\varrho}^{J}(\kappa):\Lambda \to[0,1]$ such that $0\leq T_{\varrho}^{J}+D_{\varrho}^{J}+Y_{\varrho}^{J}+F_{\varrho}^{J} \leq 4$ (J=1,2,...P) and for every $\kappa \in \Lambda$. $T_{\varrho}^{J}(\kappa), D_{\varrho}^{J}(\kappa), Y_{\varrho}^{J}(\kappa)andF_{\varrho}^{J}(\kappa)$ are the truth membership sequence,a contradiction membership sequence,an unknown membership sequence and falsity membership sequence of the element x respectively. P is also referred to as the QSVNRS(ϱ) dimension.

Definition 2.2 *[2] Let $\varrho, \zeta \in$ QSVNRS(Λ) havimg the form*

$\varrho = \{\langle \kappa, T_{\varrho}^{J}(\kappa), D_{\varrho}^{J}(\kappa), Y_{\varrho}^{J}(\kappa), F_{\varrho}^{J}(\kappa)\rangle : \kappa \in \Lambda\}$ (J=1,2,...P)

$\zeta = \{\langle \kappa, T_{\zeta}^{J}(\kappa), D_{\zeta}^{J}(\kappa), Y_{\zeta}^{J}(\kappa), F_{\zeta}^{J}(\kappa)\rangle : \kappa \in \Lambda\}$ (J=1,2,...P).Then

1. $\varrho \widetilde{\subseteq} \zeta$ if $T_{\varrho}^{J}(\kappa) \leq T_{\zeta}^{J}(\kappa)$, $D_{\varrho}^{J}(\kappa) \leq D_{\zeta}^{J}(\kappa)$, $Y_{\varrho}^{J}(\kappa) \leq Y_{\zeta}^{J}(\kappa)$ and $F_{\varrho}^{J}(\kappa) \leq F_{\zeta}^{J}(\kappa)$ J=1,2,...P)

2. $\varrho^{\tilde{c}} = \{\langle \kappa, F_{\varrho}^{J}(\kappa), Y_{\varrho}^{J}(\kappa), D_{\varrho}^{J}(\kappa), T_{\varrho}^{J}(\kappa)\rangle : \kappa \in \Lambda\}$ (J=1,2,...P)

3.$\varrho \widetilde{\cup} \zeta = \omega_1$ and is defined by

$T_{\omega_1}^{J}(\kappa)=\max\{T_{\varrho}^{J}(\kappa), T_{\zeta}^{J}(\kappa)\}$, $D_{\omega_1}^{J}(\kappa)=\max\{D_{\varrho}^{J}(\kappa), D_{\zeta}^{J}(\kappa)\}$, $Y_{\omega_1}^{J}(\kappa)=\min\{Y_{\varrho}^{J}(\kappa), Y_{\zeta}^{J}(\kappa)\}$, $F_{\omega_1}^{J}(\kappa)=\min\{F_{\varrho}^{J}(\kappa), F_{\zeta}^{J}(\kappa)\}$ for all $\kappa \in \Lambda$ and J=1,2...P.

4. $\varrho \widetilde{\cap} \zeta = \omega_1$ and is defined by

$T_{\omega_1}^{J}(\kappa)=\min\{T_{\varrho}^{J}(\kappa), T_{\zeta}^{J}(\kappa)\}$, $D_{\omega_1}^{J}(\kappa)=\min\{D_{\varrho}^{J}(\kappa), D_{\zeta}^{J}(\kappa)\}$, $Y_{\omega_1}^{J}(\kappa)=\max\{Y_{\varrho}^{J}(\kappa), Y_{\zeta}^{J}(\kappa)\}$, $F_{\omega_1}^{J}(\kappa)=\max\{F_{\varrho}^{J}(\kappa), F_{\zeta}^{J}(\kappa)\}$ for all $\kappa \in \Lambda$ and J=1,2...P.

Definition 2.3 *[2] A QSVNRTS on Λ^* in a family $\mathfrak{T}$ of QSVNRS in Λ^* which satisfy the following axioms.*

1. $\widetilde{\Phi}_{QNR}$, $\widetilde{\mathbb{X}}_{QNR} \in \mathfrak{T}$.
2. $H_1 \tilde{\cap} H_2 \in \mathfrak{T}$ for any $H_1, H_2 \in \mathfrak{T}$.
3. $\tilde{\cup}\, H_i \in \mathfrak{T}$ for every$\{ H_i$: i ∈ I$\} \tilde{\subseteq} \mathfrak{T}$.

Here the pair $(\Lambda^*, \mathfrak{T})$ is called a QSVNRTS and any QSVNRS in $\mathfrak{T}$ is said to be quadripartitioned single valued neutrosophic refined open set (QNROS) in Λ^*. The complement of $\varrho^{\tilde{c}}$ of a QNROS ϱ in a QSVNRTS $(\Lambda^*,\mathfrak{T})$ is known as quadripartitioned single valued neutrosophic refined closed set (QNRCS) in Λ^*.

Definition 2.4 *[2] Let $(\Lambda^*,\mathfrak{T})$ be a QSVNRTS and $\varrho = \{\langle \kappa, T_\varrho^J(\kappa), D_\varrho^J(\kappa), Y_\varrho^J(\kappa), F_\varrho^J(\kappa)\rangle : \kappa \in \Lambda^*\}$ for J=1,2,...P be QSVNRS in X.Then quadripartitioned single valued neutrosophic refined closure (QNR($\mathfrak{cl}(\varrho)$) and quadripartitioned single valued neutrosophic refined interior (QNR$\mathfrak{int}(\varrho)$) are defined by*

$$\text{QNR}\mathfrak{cl}(\varrho) = \tilde{\cap}\{K:K \text{ is a QNRCS in } \mathbb{X} \text{ and } \varrho \tilde{\subseteq} K\}$$

$$\text{QNR}\mathfrak{int}(\varrho) = \tilde{\cup}\{L:L \text{ is a QNROS in } \mathbb{X} \text{ and } L \tilde{\subseteq} \varrho\}$$

Definition 2.5 *[2] Let $(\Lambda^*,\mathfrak{T})$ be a QSVNRTS is known as*

1.Quadripartitioned single valued neutrosophic refined semi closed set(QNRSCS) if QNRint(QNRcl(ϱ)) $\tilde{\subseteq} \varrho$.

2.Quadripartitioned single valued neutrosophic refined pre-closed set(QNRPCS) if QNRcl(QNRint(ϱ)) $\tilde{\subseteq} \varrho$.

3.Quadripartitioned single valued neutrosophic refined α -closed set(QNR α CS) if QNRcl(QNRint(QNRcl(ϱ))) $\tilde{\subseteq} \varrho$.

4.Quadripartitioned single valued neutrosophic refined regular closed (QNRRCS) if ϱ = QNRcl(QNRint(ϱ)).

5.Quadripartitioned single valued neutrosophic refined semi-pre closed set(QNRSPCS) if QNRint(QNRcl(QNRint(ϱ))) $\tilde{\subseteq} \varrho$.

Definition 2.6 *[2] Let $(\Lambda^*,\mathfrak{T})$ be a QSVNRTS is known as*

1.generalized closed set (QNRGCS) if QNRcl(ϱ) $\tilde{\subseteq}$ L whenever $\varrho \tilde{\subseteq}$ L and L is a QNROS in Λ^*.

2.generalized pre closed set (QNRGPCS) if QNRPcl(ϱ) $\tilde{\subseteq}$ L whenever $\varrho \tilde{\subseteq}$ L and L is a QNROS in Λ^*.

3. generalized semi closed set (QNRGSCS) if QNRScl(ϱ) $\tilde{\subseteq}$ L whenever ϱ $\tilde{\subseteq}$ L and L is a QNROS in Λ^*.
4. generalized α closed set (QNRGαCS) if QNRαcl(ϱ) $\tilde{\subseteq}$ L whenever ϱ $\tilde{\subseteq}$ L and L is a QNROS in Λ^*.
5. generalized semi-pre closed set (QNRGSPCS) if QNRSPcl(ϱ) $\tilde{\subseteq}$ L whenever ϱ $\tilde{\subseteq}$ L and L is a QNROS in Λ^*.

Definition 2.7 *[3] Let (ω^*,Λ) and (κ^*,Γ) be any two QSVNRTS. A map δ:(ω^*,Λ) → (κ^*,Γ) is known as,*

- Quadripartitioned single valued neutrosophic refined continuous (QNR conti) if $\delta^{-1}(\xi_{Q1})$ ∈ QNRCS(ω^*) for all QNRCS ξ_{Q1} of (κ^*,Γ).
- Quadripartitioned single valued neutrosophic refined semi-continuous (QNRS conti) if $\delta^{-1}(\xi_{Q1})$ ∈ QNRSCS(ω^*) for all QNRCS ξ_{Q1} of (κ^*,Γ).
- Quadripartitioned single valued neutrosophic refined pre-continuous (QNRP conti) if $\delta^{-1}(\xi_{Q1})$ ∈ QNRPCS(ω^*) for all QNRCS ξ_{Q1} of (κ^*,Γ).
- Quadripartitioned single valued neutrosophic refined semi pre-continuous (QNRSP conti) if $\delta^{-1}(\xi_{Q1})$ ∈ QNRSPCS(ω^*) for all QNRCS ξ_{Q1} of (κ^*,Γ).
- Quadripartitioned single valued neutrosophic refined α-continuous (QNRα-conti) if $\delta^{-1}(\xi_{Q1})$ ∈ QNRαCS(ω^*) for all QNRCS ξ_{Q1} of (κ^*,Γ).
- Quadripartitioned single valued neutrosophic refined regular continuous (QNRR conti) if $\delta^{-1}(\xi_{Q1})$ ∈ QNRRCS(ω^*) for all QNRCS ξ_{Q1} of (κ^*,Γ).
- Quadripartitioned single valued neutrosophic refined generalized continuous (QNRG conti) if $\delta^{-1}(\xi_{Q1})$ ∈ QNRGCS(ω^*) for all QNRCS ξ_{Q1} of (κ^*,Γ).
- Quadripartitioned single valued neutrosophic refined generalized semi continuous (QNRGS conti) if $\delta^{-1}(\xi_{Q1})$ ∈ QNRGSCS(ω^*) for all QNRCS ξ_{Q1} of (κ^*,Γ).
- Quadripartitioned single valued neutrosophic refined generalized semi pre-continuous (QNRGSP conti) if $\delta^{-1}(\xi_{Q1})$ ∈ QNRGSPCS(ω^*) for all QNRCS ξ_{Q1} of (κ^*,Γ).
- Quadripartitioned single valued neutrosophic refined α generalized-continuous (QNRαG conti) if $\delta^{-1}(\xi_{Q1})$ ∈ QNRαGCS(ω^*) for all QNRCS ξ_{Q1} of (κ^*,Γ).

III. QUADRIPARTITIONED SINGLE VALUED NEUTROSOPHIC REFINED CONTRA GENERALIZED PRE CONTINUOUS MAPPINGS

Definition 3.1 A map δ : $(A, \tau) \rightarrow (B, \kappa)$ is known as Quadripartitioned single valued neutrosophic refined contra generalized continuous(QNRCGP-conti)mapping if $\delta^{-1}(\zeta)$ is a QNRGPCS in (A,τ) for every QNROS ζ in (B,κ).

Example 3.2 *Let A = {e,f} and B = {w,x}*
U_1={⟨e,{0.4,0.6,0.7,0.8},{0.5,0.6,0.8,0.3},{0.5,0.7,0.3,0.4}⟩,

⟨f,{0.6,0.7,0.4,0.5},{0.7,0.8,0.5,0.6}, {0.8,0.6,0.5,0.7}⟩}

U_2={⟨w,{0.6,0.8,0.2,0.3},{0.7,0.8,0.3,0.5},{0.6,0.5,0.4,0.3}⟩,

⟨x,{0.8,0.7,0.4,0.6},{0.6,0.5,0.4,0.3}, {0.7,0.6,0.3,0.5}⟩}

Then τ = {0_{QNR},1_{QNR}, U_1} and κ = {0_{QNR},1_{QNR}, U_2} are QSVNRTS on A and B.Define a mapping δ : (A,τ) → (B,κ) by δ(e) = w and δ(f) = x.Then δ is QNRCGP-conti.mapping.

Theorem 3.3 *Every QNRC-conti.mapping is a QNRCGP-conti.mapping but not conversely.*

Proof. Let δ : (A,τ) → (B,κ) be a QNRC-conti.mapping.Let ζ be a QNROS in B.Then $\delta^{-1}(\zeta)$ is a QNRCS in A.Since every QNRCS is a QNRGPCS,$\delta^{-1}(\zeta)$ is a QNRGPCS in A.Hence δ is a QNRCGP-conti.mapping.

Example 3.4 *Let A = {e,f} and B = {w,x}*

U_1={⟨e,{0.4,0.3,0.6,0.5},{0.6,0.5,0.8,0.7},{0.5,0.3,0.6,0.8}⟩,

⟨f,{0.5,0.4,0.7,0.6},{0.3,0.4,0.7,0.5}, {0.6,0.4,0.8,0.7}⟩}

U_2={⟨w,{0.5,0.6,0.3,0.4},{0.4,0.3,0.5,0.6},{0.6,0.5,0.5,0.7}⟩,

⟨x,{0.4,0.3,0.6,0.5},{0.8,0.7,0.6,0.4}, {0.5,0.6,0.7,0.6}⟩}

Then τ = {0_{QNR},1_{QNR}, U_1} and κ = {0_{QNR},1_{QNR}, U_2} are QSVNRTS on A and B.Define a mapping δ : (A,τ) → (B,κ) by δ(e) = w and δ(f) = x.Then δ is QNRCGP-conti.mapping but not QNRC-conti.mapping.

Theorem 3.5 *Every QNRCα-conti.mapping is a QNRCGP-conti.mapping but not conversely.*

Proof. Let δ : (A,τ) → (B,κ) be a QNRCα-conti.mapping.Let ζ be a QNROS in B.Then $\delta^{-1}(\zeta)$ is a QNRαCS in A.Since every QNRαCS is a QNRGPCS,$\delta^{-1}(\zeta)$ is a QNRGPCS in A.Hence δ is a QNRCGP-conti.mapping.

Example 3.6 *Let A = {e,f} and B = {w,x}*

U_1={⟨e,{0.3,0.4,0.7,0.6},{0.4,0.5,0.7,0.8},{0.4,0.5,0.6,0.7}⟩,

⟨f,{0.4,0.5,0.8,0.7},{0.3,0.4,0.6,0.8}, {0.6,0.5,0.7,0.8}⟩}

U_2={⟨w,{0.5,0.6,0.6,0.5},{0.5,0.7,0.6,0.7},{0.5,0.6,0.6,0.5}⟩,

⟨x,{0.6,0.7,0.7,0.6},{0.4,0.5,0.6,0.7}, {0.7,0.5,0.6,0.7}⟩}

Then τ = {0_{QNR},1_{QNR}, U_1} and κ = {0_{QNR},1_{QNR}, U_2} are QSVNRTS on A and B.Define a mapping δ : (A,τ) → (B,κ) by δ(e) = w and δ(f) = x.Then δ is QNRCGP-conti.mapping but not QNRCα-conti.mapping.

Theorem 3.7 *Every QNRCP-conti.mapping is a QNRCGP-conti.mapping but not conversely.*

Proof. Let δ : (A,τ) → (B,κ) be a QNRCP-conti.mapping.Let ζ be a QNROS in B.Then $\delta^{-1}(\zeta)$ is a QNRPCS in A.Since every QNRPCS is a QNRGPCS,$\delta^{-1}(\zeta)$ is a QNRGPCS in A.Hence δ is a QNRCGP-conti.mapping.

Example 3.8 *Let A = {e,f} and B = {w,x}*

U_1={⟨e,{0.6,0.5,0.8,0.7},{0.4,0.3,0.5,0.6},{0.5,0.4,0.5,0.7}⟩,

⟨f,{0.4,0.3,0.5,0.8},{0.5,0.6,0.8,0.9}, {0.4,0.5,0.8,0.7}⟩}

U_2={⟨w,{0.7,0.8,0.7,0.6},{0.6,0.5,0.3,0.4},{0.6,0.4,0.4,0.6}⟩,

⟨x,{0.7,0.5,0.3,0.6},{0.6,0.7,0.7,0.8}, {0.7,0.6,0.5,0.6}⟩}

Then τ = {0_{QNR},1_{QNR}, U_1} and κ = {0_{QNR},1_{QNR}, U_2} are QSVNRTS on A and B.Define a mapping δ : (A,τ) → (B,κ) by δ(e) = w and δ(f) = x.Then δ is QNRCGP-conti.mapping but not QNRCP-conti.mapping.

Theorem 3.9 *Let δ : (A,τ) → (B,κ) be a mapping.Then the following conditions are equivalent.*

1. δ is a QNRCGP-conti.mapping.
2. $\delta^{-1}(\zeta)$ is a QNRGPOS in A for every QNRCS in B.

Proof:

i) ⇒ 2):Let ζ be a QNRCS in B.Then $\zeta^{\tilde{c}}$ is a QNROS in B.By statement,$\delta^{-1}(\zeta^{\tilde{c}})$ is a QNRGPCS in A.Hence $\delta^{-1}(\zeta)$ is a QNRGPOS in A.

ii) ⇒ 1):Let ζ be a QNROS in B.Then $\zeta^{\tilde{c}}$ is a QNRCS in B.By statement,$\delta^{-1}(\zeta^{\tilde{c}})$ is a QNRGPOS in A.Hence $\delta^{-1}(\zeta)$ is a QNRGPCS in A.Thus δ is a QNRCGP-conti.mapping.

Theorem 3.10 *Let δ : (A, τ) → (B, κ) is a QNRCGP-conti.mapping if δ^{-1}(QNRPcl(ζ))$\widetilde{\subseteq}$QNRint($\delta^{-1}(\zeta)$) for every ζ in B.*

Proof: Let ζ be a QNRCS in B.Then QNRcl(ζ) = ζ.Since every QNRCS is a QNRPCS,this implies QNRPcl(ζ) = ζ.By hypothesis,$\delta^{-1}(\zeta) = \delta^{-1}$(QNRPcl($\zeta$))$\widetilde{\subseteq}$ QNRint($\delta^{-1}(\zeta)$)$\widetilde{\subseteq}$ $\delta^{-1}(\zeta)$.This implies $\delta^{-1}(\zeta)$ is a QNROS in A.Therefore δ is a QNRC-conti.mapping,since every QNRC-conti.mapping is a QNRCGP-conti.mapping,δ is a QNRCGP-conti.mapping.

Theorem 3.11 *A QNR-conti.mapping δ : (A,τ) → (B,κ) is a QNRCGP-conti.mapping if QNRGPO(A) = QNRGPC(A).*

Proof: Let ζ be a QNROS in Y.By hypothesis $\delta^{-1}(\zeta)$ is a QNROS in A and hence is a QNRGPOS in A.since QNRGPO(A) = QNRGPC(A),$\delta^{-1}(\zeta)$ is a QNRGPCS in A.Therefore δ is a QNRCGP-conti.mapping.

IV. CONCLUSION

In this paper,we introduced uadripartitioned single valued neutrosophic refined contra generalized pre-continuous mappings and some of this characterizations.

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