STRESS DISTRIBUTION OF PVC/ POLYSTYRENE MATERIAL TUBE HAVING INTERNAL PRESSURE AND THERMO- MECHANICAL LOAD

Abstract

This article deals with the study of thermal stress distribution in a tube made of polyvinyl chloride/ polystyrene material and subjected to internal pressure and mechanical load. Through the acquired outcomes, it is remarked that value of pressure increases with increasing temperature Θ_1 =0.0175 and decreases with increasing mechanical loads (i.e. L_0 = 0.1 and 0.2) at the inner surface of polyvinyl chloride material tube and also in a polystyrene material tube for the initial /fully –plastic stage . The values of the circumferential / radial stress also increase with increasing temperature /mechanical loads in the compression/tension region of the tube. The polyvinyl chloride material tube is more convenient than that of polystyrene material tube.

Keywords: Tube ; load ; stresses ; pressure ; temperature

Authors

Kanav Gupta
Department of Physics
ICFAI University
Himachal Pradesh
India.
kanav17aug@gmail.com

Neeru Gupta
Department of Mathematics
ICFAI University
Himachal Pradesh
India.
neeruguptagupta087@gmail.com

Sukhvinder
Department of Mathematics
Swami Premanand Mahavidyalaya
Hoshiarpur,Punjab
India.
Sukhvindermaths@gmail.com

Anish Kumar
Department of Mathematics
ICFAI University
Himachal Pradesh
India.
anishnegi483@gmail.com

Shreya Gupta
Department of Mathematics
Jammu and Kashmir (UT), India.
shreyagupta409@gmail.com

I. INTRODUCTION

The analysis of Elasto-plastic thick-walled tubes have enticed a lot of concern due to their essential applications in chemical industry engineering, petrochemical industry, agricultural irrigation, urban construction, and electric power industry. For structural use in bridges, piling pipe, piers, roads, building structures, etc. and also body transport in gas, steam, liquefied petroleum gas, etc. The analytical solutions of stress distribution are given for idealized elasto-plastic by Timoshenko [2] and work hardening by Chadwick [3] for homogeneous materials. Bland [1] has analyzed the problem of thick- walled tubes with the addition of heated gradient and consistent pressure.Gamer et al. [4] achieved the analytical solution of stresses in thick walled tube through employing Tresca's yield condition. Bree [5] has discussed the deformation of elasto - plastic stresses in a closed tube scheduled for thermal stresses. Mufit et al. [6] proposed heat generating tube with yield stress showing thermal stress distribution through employing Tresca's yield condition and its corresponding flow rule. Xin et al. [7] studied the elasto-plastic behavior of functionally graded thick-walled tube with the addition to internal pressure by applying the supposition of a invariable strain field within the volume element and the Tresca yield criterion. Matvienko et al. [10] analysed the aluminum tube elastoplastic deformation of under inner/ outer pressure. Futher Matvienko et al. [11] examined mathematical modeling in dispersion-hardened aluminum alloy tube with non- homogeneous thermal field.Qian et al. [9] developed mechanical properties of extremely well organized heat interchange tubes. Gupta et al. [8] has investigated the deformation of stress in a thick walled tube made of isotropic material with the inclusion of inner pressure and thermo mechanical load through employing transition theory.It has been seen that, with the introduction of mechanical loads the value of pressure decreases. The objective of this article is to be investigating stress distribution in a thermo mechanical loaded tube made of PVC/ polystyrene material and subjected to uniform pressure.

II. MATERIALS USED

1. **Polyvinyl Chloride (PVC):** It is a prudent and adaptable thermoplastic polymer. To construct the building of industries , schools , universities , govt buildings ,hospitals , factories , gas storage tanks ,marines etc. (i.e to produce wire and cable insulation , door and window profiles, pipes (drinking and wastewater), medical devices, etc.) . It is a good dimensional stability at room temperature and low cost , flexible and high impact strength , good electrical insulation and vapor barrier properties etc.

2. **Polystyrene:** It is a monomer styrene polymer (i.e liquid hydrocarbon obtained from petroleum). It is solid plastic products that require clarity (i.e. laboratory ware). After the combination of different colorants, additives /various plastics, it is useful to instruct gardening pots, research labs material / appliances/ equipments, automobile parts, toys, electronics etc. It provides superior insulation, water resistant, resistant to bacterial growth, excellent shock absorbers.

III. MATHEMATICAL MODEL

1. Abbreviations and Acronyms

λ, μ - lame's constants,

e_{kk} - first strain invariant, C- compressibility factor,

$C\xi = \alpha E(2-C)$, $C = 2\mu/(\lambda+2\mu) = (1-2\nu)/(1-\nu)$

r_i, r_0 - inner and outer radii,

u,v,w- displacement components

ν - poisson's ratio,

$\tau_{ij}, \varepsilon_{ij}$ - components of stress and strain

Y, Y^* - yielding stress,

$Y = \mu(1+\nu) = (3-2C)\mu/(2-C)$

δ_{ij} - kronecker's delta $p_{i.}$ - Inner surface of pressure

P_i - initial yielding stage pressure Pf - fully-plastic stage pressure

l_0 - load at the outer surface,

η - function of r ,

r - function of x and y

T - function of η ,

Θ - Temperature,

A - Constant of integration

2. Non-Dimensional Quantities

$R = r/r_0$ $R_0 = r_i/r_0$ -Radii ratio,
$\sigma_r = \tau_r/Y$ -Radial stress component,
$\sigma_\theta = \tau_\theta/Y$ -Circumferential stress component,

$L_0 = l_0 / Y$ -Mechanical load**,**
$\Theta_1 = \alpha E \Theta_0 / Y$ -Temperature**,**
and $P_i = p_i / Y$. - Pressure.

3. **Equations:** The tube is taken in the form of cylinder made of Polystyrene and Polyvinyl chloride material, with an inner radius r_i and external radius r_0 ($r_i < r_0$)/ (a<b) with the inclusion of invariable pressure p_i respectively. Let us consider uniform temperature Θ_0 be applied at the inner surface of the tube. Further, if we suppose that there are body couples, no body forces and couple stresses on the tube, and if only a steady deformation problem is considered as shown in Figure 1.

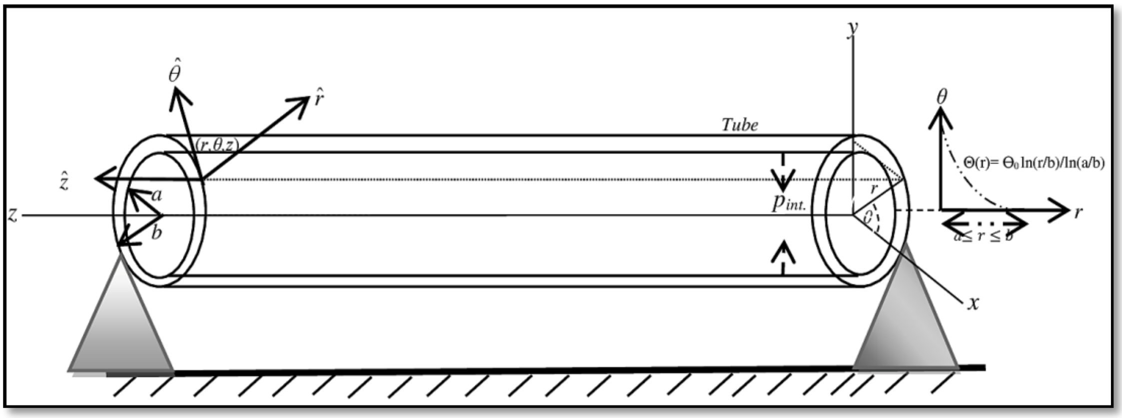


Figure 1 : Polystyrene/ Poly Vinyl Chloride Materials Tube

Basic Governing Equation: The cylindrical polar coordinates (r, θ, z) are shown [12-16, 8]:

$$u = r(1-\eta),\ v = 0,\ w = dz \tag{1}$$

The strain components of Almansi are shown [16,17]:

$$\varepsilon_r = \frac{1}{2}\left[1-(r\eta' + \eta)^2\right],\ \varepsilon_\theta = \frac{1}{2}\left[1-\eta^2\right]$$

$$\varepsilon_z = \frac{1}{2}\left[1-(1-d)^2\right]$$

$$\varepsilon_{r\theta} = \varepsilon_{\theta z} = \varepsilon_{zr} = 0 \tag{2}$$

Isotropic material relations are shown [12,14]:

$$T_{ij} = \lambda \delta_{ij} I_1 + 2\mu \varepsilon_{ij} - \xi \Theta \delta_{ij},\ (i, j = 1,2,3) \tag{3}$$

Using (2) into (3),we get

$$\tau_r = \lambda\left\{1-\frac{1}{2}\left[(r\eta'+\eta)^2+\eta^2\right]\right\}+\mu\left[1-(r\eta'+\eta)^2\right]-\xi\Theta$$

$$\tau_\theta = \lambda\left\{1-\frac{1}{2}\left[(r\eta'+\eta)^2+\eta^2\right]\right\}+\mu\left(1-\eta^2\right)-\xi\Theta,$$

$$\tau_z = \lambda\left\{1-\frac{1}{2}\left[(r\eta'+\eta)^2+\eta^2\right]\right\}-\xi\Theta=0,\ \tau_{r\theta}=\tau_{\theta z}=\tau_{zr}=0 \qquad (4)$$

The equilibrium equation is shown as:

$$\frac{\partial\tau_r}{\partial r}+\frac{(\tau_r-\tau_\theta)}{r}=0 \qquad (5)$$

Using the equation $\nabla^2\Theta=\frac{1}{r}\frac{d}{dr}\left(r\frac{d\Theta}{dr}\right)=0$ and $\Theta=\Theta_o$ at $r=r_i$, $\Theta=0$ at $r=r_0$. After applying

These conditions we obtained the new relation :

$$\Theta=\overline{\Theta}_o\ln(r/r_0)$$

Where $\overline{\Theta}_o=\frac{\Theta_o}{\ln(r_i/r_0)}$.

Boundary condition: The boundary conditions of the tube (say compression /tension region) are:

$$\tau_r=-p_i \text{ at r = } r_i \text{ and } \tau_r=l_0 \text{ at r = } r_0 \qquad (7)$$

Using (4) into (5) then we obtained the differential equation:

$$(\lambda+2\mu)\left[\eta^2+(r\eta'+\eta^2)\right]+2\mu\int r\eta'^2dr-(2\lambda+2\mu)+2\xi\Theta=A_0 \qquad (8)$$

where A_0 is the constant of integration. Differentiating (8) w .r. t to r, given as under:

$$\left[T^2+\left(2+\frac{1}{2}C\right)T+2\right]\frac{d\eta}{dT}+\eta(1+T)-\frac{C\xi\Theta_0}{\mu}=0 \qquad (9)$$

Problem solutions:

(a) Case of compression : The transition function Π as:

$$\Pi \cong (2-C) - \frac{C}{\mu}\left[\tau_r + C\xi\Theta\right] \tag{10}$$

By applying differentiation, integrating in (10) and using (9) as well, we obtained the function :

$$\Pi = Ar^{-C} \tag{11}$$

Comparing, (10) and (11), we obtain the radial stress:

$$\tau_r = \frac{\mu}{C}\left[(2-C) - Ar^{-C}\right] - c\xi\Theta_0 \frac{\ln(r_0/r)}{\ln(r_0/r_i)} \tag{12}$$

The yielding stress in compression is given [8-17]. Now substituting the value of yielding stress condition in (12), we get

$$\tau_r = \frac{(2-C)Y}{C(3-2C)}\left[(2-C) - Ar^{-C}\right] - c\xi\Theta_0 \frac{\ln(r_0/r)}{\ln(r_0/r_i)} \tag{13}$$

By applying (7) into (13), necessary relation obtained

$A = (2-C)r_0^{\,C} - \dfrac{l_0 C(3-2C)r_0^{\,C}}{Y(2-C)}$. Further, (13) become:

$$\tau_r = \frac{(2-C)^2 Y}{C(3-2C)}\left[1 - \left(\frac{r_0}{r}\right)^C\right] + l_0\left(\frac{r_0}{r}\right)^C - \alpha E(2-C)\Theta_0 \frac{\ln(r_0/r)}{\ln(r_0/r_i)} \tag{14}$$

Now by applying (7) into (14), necessary relation obtained

$$\Rightarrow \frac{(2-C)^2 Y}{C(3-2C)} = \frac{p_i + l_0(r_i/r_0)^{-C}}{\left\{(r_i/r_0)^{-C} - 1\right\}} - \frac{\alpha E(2-C)\Theta_0}{\left\{(r_i/r_0)^{-C} - 1\right\}} \tag{15}$$

Substituting (15) into (14) and using (7) :

$$\tau_r = \left\{\begin{array}{l} \dfrac{p_i + l_0(r_i/r_0)^{-C}}{\left[\left(\dfrac{r_i}{r_0}\right)^{-C} - 1\right]}\left[1 - \left(\dfrac{r_0}{r}\right)^C\right] + l_0\left(\dfrac{r_0}{r}\right)^C \\ -\alpha E(2-C)\Theta_0\left[\dfrac{1-(r_0/r)^C}{(r_i/r_0)^{-C} - 1} + \dfrac{\ln(r_0/r)}{\ln(r_0/r_i)}\right] \end{array}\right\} \tag{16}$$

$$\tau_\theta = \left\{ \begin{array}{l} \dfrac{p_i + l_0 (r_i / r_0)^{-C}}{\left[\left(\dfrac{r_i}{r_0}\right)^{-C} - 1\right]} \left[1 - (1-C)\left(\dfrac{r_0}{r}\right)^C\right] + l_0 (1-C)\left(\dfrac{r_0}{r}\right)^C \\ + \alpha E (2-C)\Theta_0 \left[\dfrac{1-(r_0/r)^C (1+C)}{(r_i/r_0)^{-C} - 1} + \dfrac{1 - \ln(r_0/r)}{\ln(r_0/r_i)}\right] \end{array} \right\} \quad (17)$$

From (16) and (17), we get:

$$\tau_\theta - \tau_r = \frac{(p_i + l_0) C (r_0/r)^C}{(r_i/r_0)^{-C} - 1} + \alpha E (2-C)\Theta_0 \left[\frac{2-(r_0/r)^C (2+C)}{(r_i/r_0)^{-C} - 1} + \frac{1}{\ln(r_0/r_i)}\right] \quad (18)$$

Initial yielding stage: Taking (18), and using r = r_i, therefore yielding becomes:

$$\left|\tau_\theta - \tau_r\right|_{r=r_i} = \left| \frac{(p_i + l_0) C (r_0/r_i)^C}{\{(r_i/r_0)^{-C} - 1\}} + \alpha E (2-C)\Theta_0 \left[\frac{2-(r_0/r_i)^C (2+C)}{(r_i/r_0)^{-C} - 1} + \frac{1}{\ln(r_0/r_i)}\right] \right| = Y \text{ (say)};$$

$$P_i = \frac{p_i}{Y} = \left|\frac{(r_i/r_0)^{-C} - 1}{C (r_0/r_i)^C}\right| \left(1 - \left|\alpha E (2-C)\Theta_0 \left[\frac{2-(r_0/r_i)^C (2+C)}{(r_i/r_0)^{-C} - 1} + \frac{1}{\ln(r_0/r_i)}\right]\right|\right) - \left|\frac{l_0}{Y}\right| \quad (19)$$

(16), (17) and (19), in non-dimensional form becomes:

$$\sigma_r = \frac{(P_i + L_0 R_0^{-C})(1 - R^{-C})}{(R_0^{-C} - 1)} + L_0 R^{-C} - \Theta_1 (2-C)\left[\frac{1 - R^{-C}}{R_0^{-C} - 1} + \frac{\ln R}{\ln R_0}\right],$$

$$\sigma_\theta = \frac{(P_i + L_0 R_0^{-C})}{(R_0^{-C} - 1)}\left[1 - (1-C) R^{-C}\right] + L_0 (1-C) R^{-C} + \Theta_1 (2-C)\left[\frac{1 - R^{-C}(1+C)}{R_0^{-C} - 1} - \frac{1 + \ln R}{\ln R_0}\right],$$

$$P_i = \left|\frac{R_0^{-C} - 1}{R_0^{-C} C}\right| \left|1 - \Theta_1 (2-C)\left[\frac{2 - R_0^{-C}(2+C)}{R_0^{-C} - 1} - \frac{1}{\ln R_0}\right]\right| - \left|L_0\right| \quad (20)$$

Fully-plastic stage: (20) obtained :

$$\sigma_r = -(P_f + L_0)\frac{\ln R}{\ln R_0} - \frac{4\Theta_1 \ln R}{\ln R_0} + L_0,$$

$$\sigma_\theta = -\left(P_f + L_0\right)\left(\frac{\ln R + 1}{\ln R_0}\right) - \frac{4\Theta_1\left(1 + \ln R\right)}{\ln R_0} + L_0 \,,$$

$$P_f = \left|\ln R_0\right| - \left|4\Theta_1 \ln R_0\right| - \left|L_0\right| \tag{21}$$

(b) Case of tension :The transition function Π as:

$$\Pi \cong \left(2 - C\right) - \frac{C}{\mu}\left[\tau_r + C\xi\Theta\right] \tag{22}$$

By applying differentiation, integrating in (22) and using (9) as well, we obtained the function:

$$\Pi = Br^{C/(1-C)} \tag{23}$$

Comparing, (22) and (23), we obtain the radial stress

$$\tau_r = \frac{\mu}{C}\left[\left(2 - C\right) - Br^{C/(1-C)}\right] - c\xi\Theta_0 \frac{\ln(r_0 / r)}{\ln(r_0 / r_i)} \tag{24}$$

The yielding stress in tension is given [8-17]: Now substituting the value of yielding stress condition in (24), we get

$$\tau_r = \frac{\left(2 - C\right)Y}{C\left(3 - 2C\right)}\left[\left(2 - C\right) - Ar^{C/(1-C)}\right] - c\xi\Theta_0 \frac{\ln(r_0 / r)}{\ln(r_0 / r_i)} \tag{25}$$

Now By applying (7) into (25), necessary relation obtained

$$B = \left(2 - C\right)r_0^{-C/(1-C)} - \frac{l_0 C\left(3 - 2C\right)r_0^{-C/(1-C)}}{Y\left(2 - C\right)}.$$ Further, (25) become:

$$\tau_r = \frac{\left(2 - C\right)^2 Y}{C\left(3 - 2C\right)}\left[1 - \left(\frac{r}{r_0}\right)^{C/(1-C)}\right] + l_0\left(\frac{r}{r_0}\right)^{C/(1-C)} - \alpha E\left(2 - C\right)\Theta_0 \frac{\ln(r_0 / r)}{\ln(r_0 / r_i)} \tag{26}$$

Now By applying (7) into (26), necessary relation obtained

$$\Rightarrow \frac{\left(2 - C\right)^2 Y}{C\left(3 - 2C\right)} = \frac{p_i + l_0\left(r_i / r_0\right)^{C/(1-C)}}{\left\{\left(r_i / r_0\right)^{C/(1-C)} - 1\right\}} - \frac{\alpha E\left(2 - C\right)\Theta_0}{\left\{\left(r_i / r_0\right)^{C/(1-C)} - 1\right\}} \tag{27}$$

Substituting (27) into (26) and using (7) :

$$\tau_r = \left\{ \begin{array}{l} \dfrac{p_i + l_0 (r_i / r_0)^{C/(1-C)}}{\left[\left(\dfrac{r_i}{r_0}\right)^{C/(1-C)} - 1\right]} \left[1 - \left(\dfrac{r}{r_0}\right)^{C/(1-C)}\right] + l_0 \left(\dfrac{r}{r_0}\right)^{C/(1-C)} \\ -\alpha E(2-C)\Theta_0 \left[\dfrac{1-(r/r_0)^{C/(1-C)}}{(r_i/r_0)^{C/(1-C)} - 1} + \dfrac{\ln(r_0/r)}{\ln(r_0/r_i)}\right] \end{array} \right\} \tag{28}$$

$$\tau_\theta = \left\{ \begin{array}{l} \dfrac{p_i + l_0 (r_i / r_0)^{C/(1-C)}}{\left[\left(\dfrac{r_i}{r_0}\right)^{C/(1-C)} - 1\right]} \left[1 - \dfrac{1}{(1-C)}\left(\dfrac{r}{r_0}\right)^{C/(1-C)}\right] + \dfrac{l_0}{(1-C)} \left(\dfrac{r}{r_0}\right)^{C/(1-C)} \\ -\alpha E(2-C)\Theta_0 \left[\dfrac{1-(r/r_0)^{C/(1-C)}\left(\dfrac{1}{1-C}\right)}{(r_i/r_0)^{C/(1-C)} - 1} - \dfrac{1-\ln(r_0/r)}{\ln(r_0/r_i)}\right] \end{array} \right\} \tag{29}$$

From (28) and (29), we get:

$$\tau_\theta - \tau_r = \left\{ \begin{array}{l} \dfrac{(p_i + l_0) C (r/r_0)^{C/(1-C)}}{(1-C)\left\{1-(r_i/r_0)^{C/(1-C)}\right\}} \\ +\alpha E(2-C)\Theta_0 \left[\dfrac{(r/r_0)^{C/(1-C)}(2-C)-(2-2C)}{(1-C)\left\{(r_i/r_0)^{C/(1-C)} - 1\right\}} + \dfrac{1}{\ln(r_0/r_i)}\right] \end{array} \right\} \tag{30}$$

Initial yielding stage: Taking (30), and using r = r_0 , therefore yielding becomes:

$$\left|\tau_\theta - \tau_r\right|_{r=r_0} = \left| \dfrac{(p_i + l_0) C}{(1-C)\left\{1-(r_i/r_0)^{C/(1-C)}\right\}} + \alpha E(2-C)\Theta_0 \left[\dfrac{C}{(1-C)\left\{(r_i/r_0)^{C/(1-C)} - 1\right\}} + \dfrac{1}{\ln(r_0/r_i)}\right]\right| = Y^*$$

$$P_i = \frac{p_i}{Y^*} = \left| \begin{array}{c} \dfrac{\left\{1-(r_i/r_0)^{C/(1-C)}\right\}(1-C)}{C} \\ \left(1 - \alpha E(2-C)\Theta_0 \left[\dfrac{C}{(1-C)\left\{(r_i/r_0)^{C/(1-C)} - 1\right\}} + \dfrac{1}{\ln(r_0/r_i)}\right]\right) \end{array} \right| - \left|\frac{l_0}{Y^*}\right| \tag{31}$$

(28)-(29) and (31), in non-dimensional form becomes:

$$\sigma_r^* = \frac{P_i + L_0 R_0^{C/(1-C)}}{\left\{R_0^{C/(1-C)} - 1\right\}}\left[1 - R^{\frac{C}{1-C}}\right] + \Theta_1(2-C)\left[\frac{1 - R^{C/(1-C)}}{R_0^{C/(1-C)} - 1} - \frac{\ln R}{\ln R_0}\right] + L_0 R^{\frac{C}{1-C}},$$

$$\sigma_\theta^* = \begin{pmatrix} \dfrac{P_i + L_0 R_0^{C/(1-C)}}{\left\{R_0^{C/(1-C)} - 1\right\}}\left[1 - \left(\dfrac{1}{1-C}\right)R^{\frac{C}{1-C}}\right] + \dfrac{L_0 R^{\frac{C}{1-C}}}{(1-C)} \\ -\Theta_1(2-C)\left[\dfrac{1 - R^{C/(1-C)}\left(\dfrac{1}{1-C}\right)}{R_0^{C/(1-C)} - 1} + \dfrac{1 + \ln R}{\ln R_0}\right] \end{pmatrix},$$

and

$$P_i^* = \left|\frac{\left\{1 - R_0^{C/(1-C)}\right\}(1-C)}{C}\right|\left(1 - \Theta_1(2-C)\left[\frac{C}{(1-C)\left\{R_0^{C/(1-C)} - 1\right\}} - \frac{1}{\ln R_0}\right]\right) - |L_0| \qquad (32)$$

Fully-plastic stage: (32) obtained :

$$\sigma_r^* = -(P_f + L_0)\left(\frac{\ln R}{\ln R_0}\right) - \frac{4\Theta_1 \ln R}{\ln R_0} + L_0,$$

$$\sigma_\theta^* = -(P_f + L_0)\left(\frac{\ln R + 1}{\ln R_0}\right) - \frac{4\Theta_1(1 + \ln R)}{\ln R_0} + L_0,$$

$$P_f^* = |\ln R_0| - |4\Theta_1 \ln R_0| - |L_0| \qquad (33)$$

IV. VALIDATION OF RESULTS

By applying $\Theta_1 \rightarrow 0$ into (20), (32), and (21) , (33) we get the Initial Yielding/ Fully plastic stage:

$$\sigma_r = \frac{\left(P_i + L_0 R_0^{-C}\right)\left(1 - R^{-C}\right)}{\left(R_0^{-C} - 1\right)} + L_0 R^{-C},$$

$$\sigma_\theta = \frac{\left(P_i + L_0 R_0^{-C}\right)}{\left(R_0^{-C} - 1\right)}\left[1 - (1-C)R^{-C}\right] + L_0(1-C)R^{-C},$$

$$P_i = \left| \frac{R_0^{-C} - 1}{R_0^{-C} C} \right| - |L_0| \tag{34}$$

in the case of compression region .

$$\sigma_r^* = \frac{P_i + L_0 R_0^{C/(1-C)}}{\{R_0^{C/(1-C)} - 1\}} \left[1 - R^{\frac{C}{1-C}} \right] + L_0 R^{\frac{C}{1-C}},$$

$$\sigma_\theta^* = \frac{P_i + L_0 R_0^{C/(1-C)}}{\{R_0^{C/(1-C)} - 1\}} \left[1 - \left(\frac{1}{1-C} \right) R^{\frac{C}{1-C}} \right] + \frac{L_0 R^{\frac{C}{1-C}}}{(1-C)}, \tag{35}$$

$$P_i^* = \left| \frac{\{1 - R_0^{C/(1-C)}\}(1-C)}{C} \right| - |L_0|$$

in the case of tension region .

$$\sigma_r = \sigma_r^* = -(P_f + L_0) \frac{\ln R}{\ln R_0} + L_0,$$

$$\sigma_\theta = \sigma_\theta^* = -(P_f + L_0) \left(\frac{\ln R + 1}{\ln R_0} \right) + L_0,$$

$$P_f = P_f^* = |\ln R_0| - |L_0| \tag{36}$$

in the compression /tension region. The present results obtained from (34) - (36) are same as given by Gupta et al. [8] in the tension/compression region.

1. **Figures:** To see the combined effect of stress distribution and pressure in a cylindrical tube made of Polyvinyl chloride , PVC (say C = 0.3333 or v = 0.4) and Polystyrene, PS (say C = 0.46154 or v = 0.35) [17]: L0 = 0, 0.1, 0.2; Θ1 = 0, 0.0175, 0.07; ri = 1 and r0 = 2 respectively.

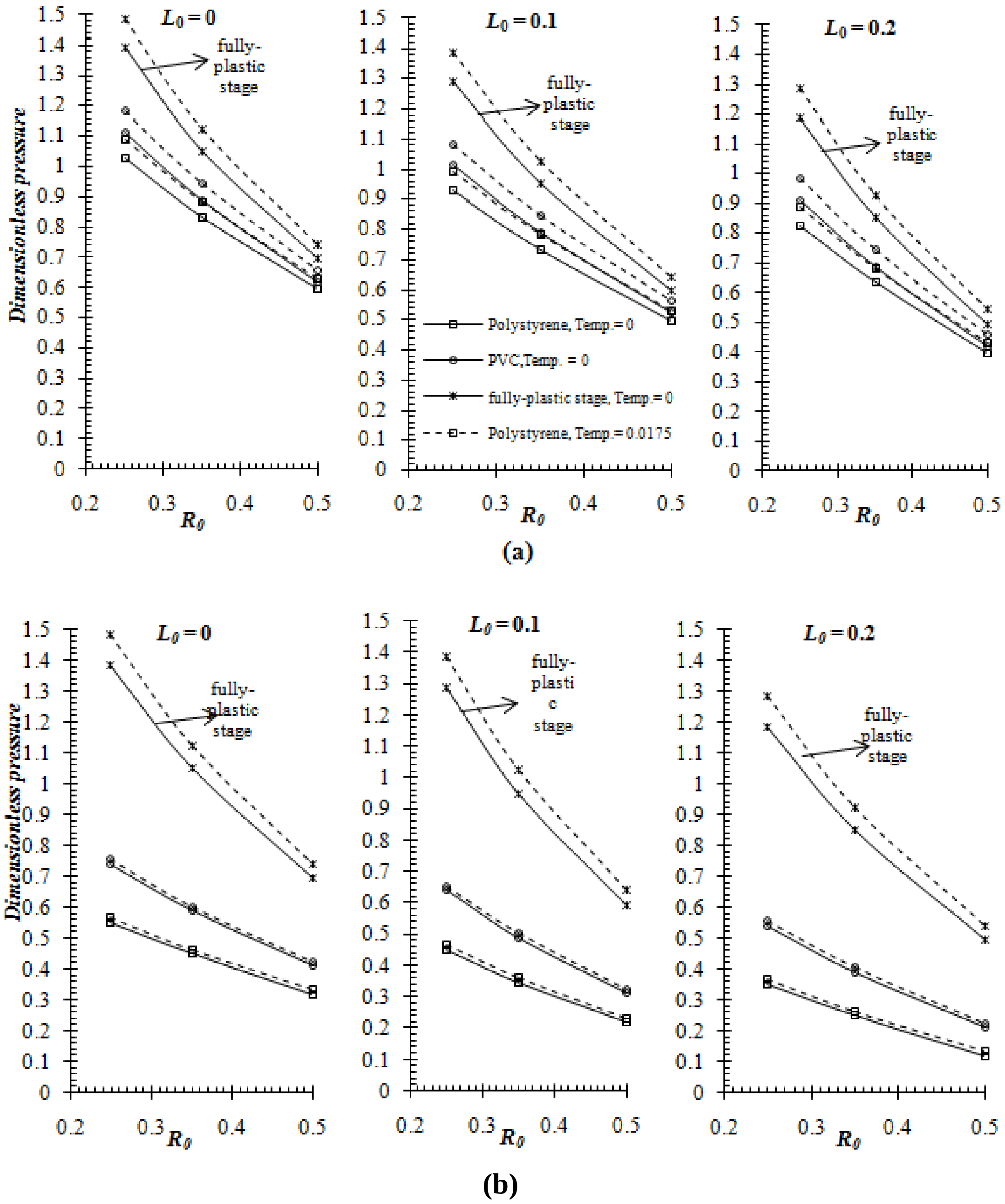
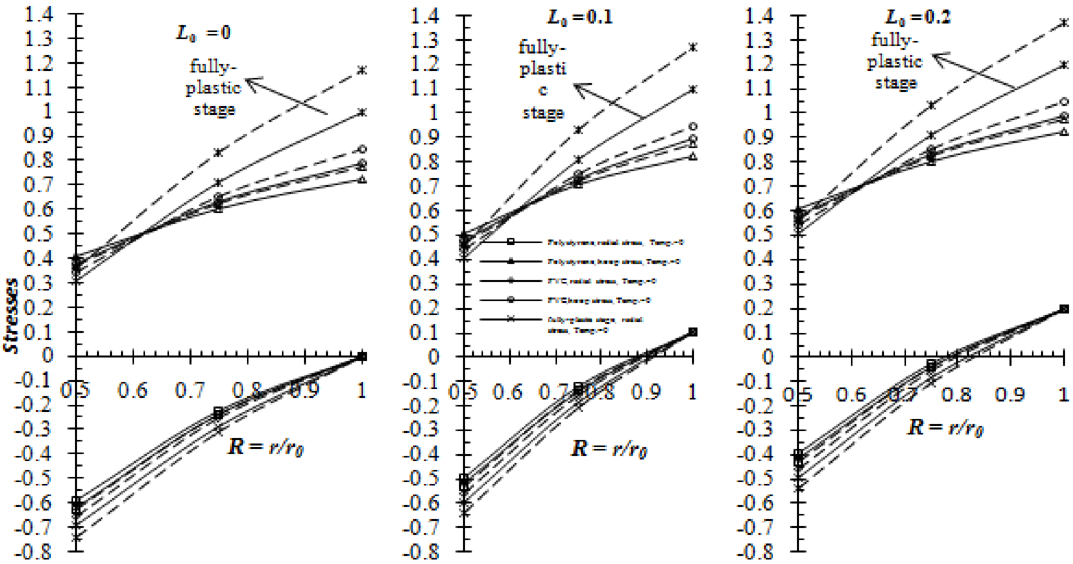


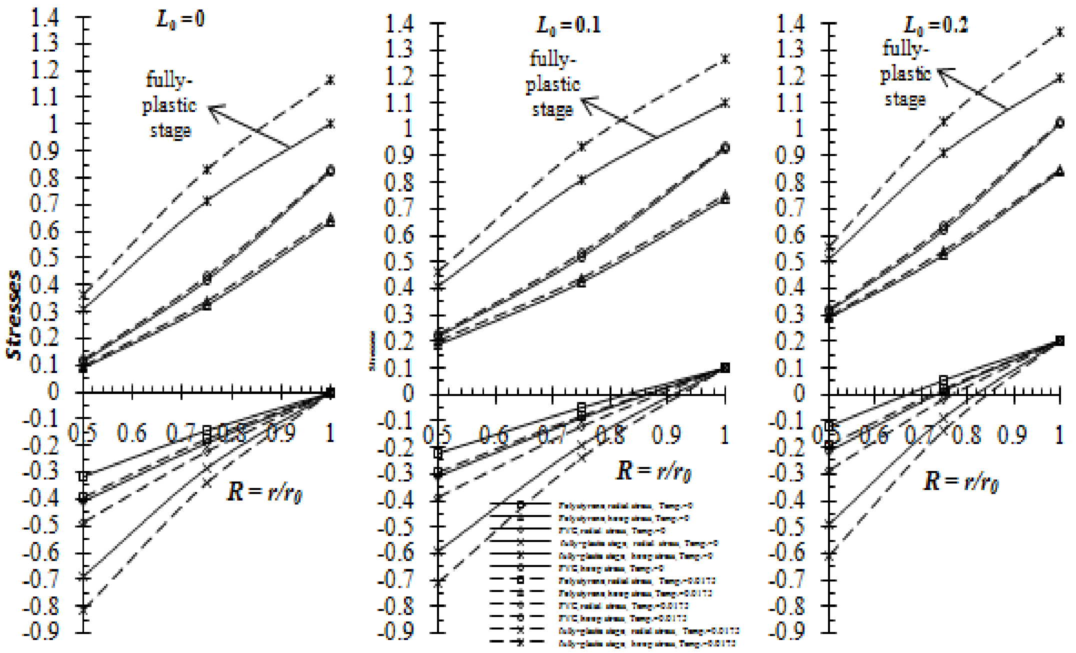
Figure 2: Dimensionless Pressure Versus R_0 in the (a) Compression (b) Tension Region

In Figure.2, the graph is plotted between radii ratio R_0 versus dimensionless pressure for the compression / tension region with . $L_0 = 0, 0.1, 0.2$ and $\Theta_1 = 0, 0.0175$ resp. It has been seen that in the initial yielding stage polyvinyl chloride material tube requires maximum pressure to yield at the inner surface as compared to the polystyrene material tube . With increasing $\Theta_1 = 0.0175$ / $L_0 = 0.1$ and 0.2 the value of pressure increases / decreases respectively .

Figure 3 graph has been drawn between stress distribution versus radii ratio R with L_0 = 0, 0.1, 0.2 and Θ_1 = 0, 0.0175 respectively. Firstly, noticed that at the outer surface PVC tube desires greatest circumferential stress in collation to polystyrene material tube. Wth increasing Θ_1 = 0.175/ L_0 = 0.1, 0.2 the hoop / radial stress values also increases .



(a)



(b)

Figure 3: Stress Distribution Versus R in the (a) Compression (b) Tension Region

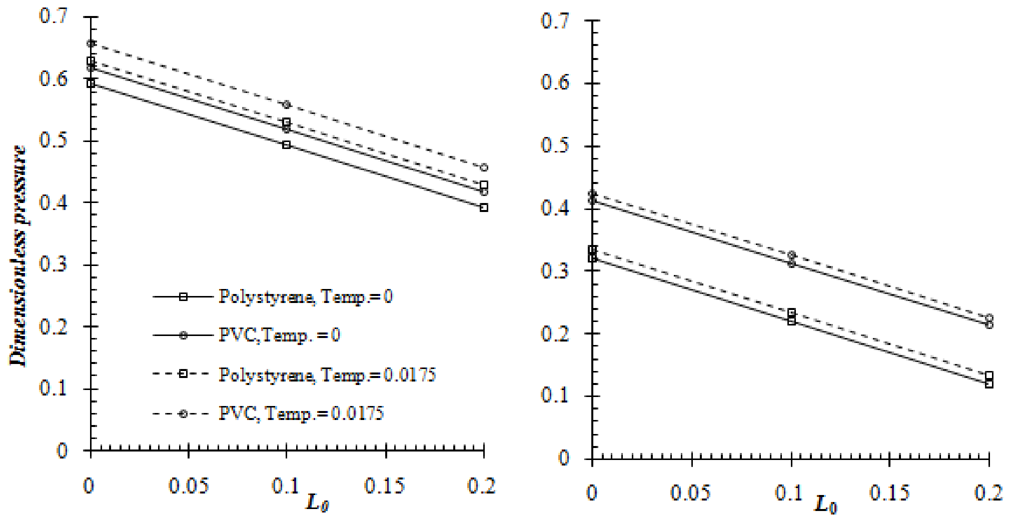


Figure 4: Pressure versus Mechanical Load at R_0 = 0.5

Figure. 4 graph has been drawn between pressure versus mechanical load and having temperature Θ_1 = 0, 0.0175, at R_0 = 0.5. With increasing L_0 = 0.1, 0.2 and Θ_1 = 0.175 the value of pressure decrease and increases respectively. Further with increasing L_0 , the PVC material tube wishes greatest pressure in collation to polystyrene material tube.

Figure. 5 is plotted between pressure versus temperature with L_0 = 0, 0.1, 0.2 at R_0 = 0.5. In the compression region, the value of pressure increases with increasing temperature (i.e. Θ_1 = 0, 0.0175, 0.07), but in the case of tension region, the value of pressure neither increases nor decreasing. Furthermore, the values of pressure decreases with the mechanical load L_0 = 0.1, 0.2 the tube made of polyvinyl chloride/ polystyrene material .

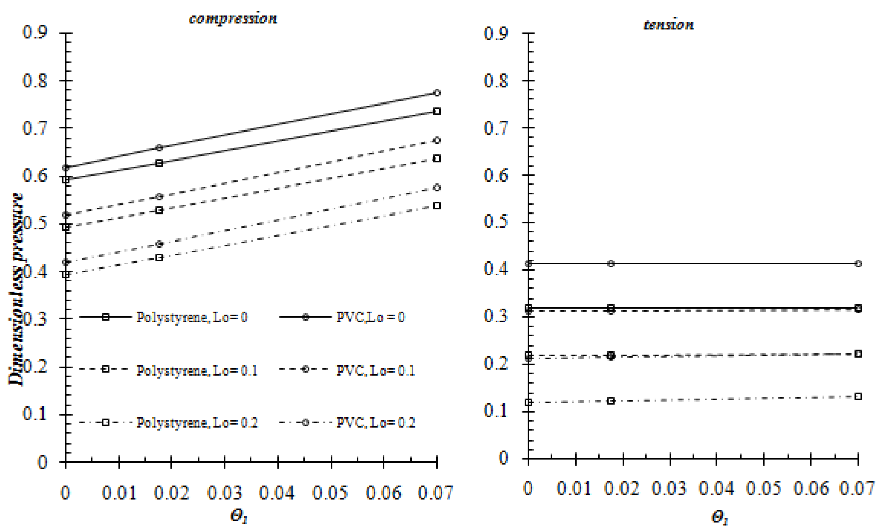


Figure 5: Pressure versus Temperature at R_0 = 0.5

V. CONCLUSIONS

It has been seen that the PVC material tube requires maximum dimensionless pressure to yield at the inner surface in comparison to the Polystyrene material tube for the initial yielding stage. The tube made of Polyvinyl chloride material requires maximum pressure in contrast to Polystyrene material tube with increasing mechanical loads. The value of pressure decreases with the mechanical load L_0 = 0.1, 0.2 in compression / tension region of PVC/ Polystyrene material tube. The PVC tube requires higher circumferential stress at the outer surface as compared to polystyrene material tube. With increasing Θ_1 = 0.175 and L_0 = 0.1, 0.2, the circumferential / radial stress also increases in the compression /tension region of the tube. In tension region, tube requisite higher circumferential stress situated at the outer surface of the initial yielding stage as compared to compression region and inverse results are obtained with the involvement of temperature conditions. By taking $\Theta_1 \rightarrow 0$ in the resulting equations, Gupta et al. [8] results can be obtained.

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