CONFORMAL CURVATURE TENSOR COMPILED WITH A METRIC SEMI-SYMMETRIC CONNECTION OF ALMOST HYPERBOLIC TACHIBANA MANIFOLDS

Abstract

Negi, et. al. (2019) has established an analytic HP-transformation in almost Kaehlerian spaces. Also, study on Projective recurrent and Symmetric tensor in Almost Kaehlerian Spaces. After that, Negi and Preeti Chauhan (2021), have accomplished Kaehlerian Manifolds with H-Projective and Bochner Recurrent Curvature Tensor of first order. In this chapter, we have calculated Conformal curvature tensor compiled with a metric semi-symmetric connection of Almost Hyperbolic Tachibana Manifolds and some theorems established.

Keywords: Conformal curvature tensor, Recurrent curvature tensor, Riemannian manifolds, Kaehlerian Manifolds and Almost Hyperbolic Tachibana Manifolds.

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I. INTRODUCTION

Let ***n***-dimensional differential manifold (M^n, g) (n >2), with the structure $F_i^{\ h}$ is a tensor field of type (1, 1) and $F_{i,j}^{h}$ is a covariant derivative with respect to Riemannian curvature tensor. A metric semi-symmetric connection ∇ and Riemannian curvature tensor with coefficients $\Gamma_{i,j}^{h}$ and $\begin{Bmatrix} h \\ i\ j \end{Bmatrix}$ [Yano and Imai (1982)] and if the torsion tensor Γ of the connection ∇ on (M^n, g) (n >2) satisfies. Then the manifold is called hyperbolic Tachibana manifold which satisfies equations (1), (2) and (3), (4), (5) and (6) respectively:

$$F_j^{\ i} F_i^{\ h} = \delta_j^{\ h}, \tag{1}$$

$$F_{ij} = -F_{ji},\ (F_{ij} = g_{jk} F_i^{\ k}), \tag{2}$$

$$F_{i,j}^{\ \ h} = 0, \tag{3}$$

$$\Gamma_{jk}^{\ \ i} = p_j A_k^{\ i} - p_k A_j^{\ i}, \tag{4}$$

$$\Gamma_{jk}^{\ \ i} = \begin{Bmatrix} i \\ jk \end{Bmatrix} - p_k U_j^{\ i} + p_j V_k^{\ i} - p^i V_{jk}, \tag{5}$$

$$U_{ij} = \frac{1}{2}(A_{ij} - A_{ji}),\ \ V_{ij} = \frac{1}{2}(A_{ij} + A_{ji}), \tag{6}$$

Also, $A_j^{\ i}$ denotes the components of the tensor of the type (1, 1) as well as ∇g = 0 and p_i are the components of a 1- form. Then equation (1.6) written as:

$$A_{ij} = U_{ij} + V_{ij}. \tag{7}$$

Taking $V_{ij} = g_{ij}$ and $U_{ij} = F_{ij}$ in (5) [Nevena Pusic (2003)], then we obtain:

$$\Gamma_{jk}^{\ \ i} = \begin{Bmatrix} i \\ jk \end{Bmatrix} - p_k F_j^{\ i} + p_j \delta_k^{\ i} - p^i g_{jk}. \tag{8}$$

The relation between Riemannian curvature tensor (r-4) relating to a metric semi-symmetric connection is given by [Nevena Pusic (2003)]:

$$\bar{R}_{ijkh} = R_{ijkh} - g_{ih}p_{kj} + g_{ik}p_{hj} - g_{jk}p_{hi} + g_{hj}p_{ki} + p_j p_h F_{ik} + p_i p_k F_{jh} - p_j p_k F_{ih} - p_i p_h F_{jk}, \tag{9}$$

Where

$$p_{jk} = \nabla_j p_k - p_j p_k + p_k q_j + \frac{1}{2} p_s p^s g_{jk}. \tag{10}$$

Again, the Ricci tensor and the scalar curvature are given by [Nevena Pusic (2003)]:

$$\bar{R}_{jk} = R_{jk} - (n-2)p_{kj} - g_j p_m^{\ m} - p_j q_k + p_k g_j - p^s p_s F_{kj}, \tag{11}$$

$$\bar{R} = R - 2(n-1)p_m^{\ m}. \tag{12}$$

$$p_j = p^h g_{jh}\ ;\ q_i = F_{ti}p^t\ ;\ p^r = g^{ir}p_i. \tag{13}$$

II. CONFORMAL CURVATURE TENSOR COMPILED WITH A METRIC SEMI-SYMMETRIC CONNECTION OF ALMOST HYPERBOLIC TACHIBANA MANIFOLDS

We have Conformal curvature tensor (rank-4) in a Riemannian manifolds is defined as following:

$$C_{ijkh} = R_{ijkh} - \frac{1}{n-2}(R_{jk}g_{ih} - R_{ik}g_{jh} + R_{ih}g_{jk} - R_{jh}g_{ki}) + \frac{R}{(n-1)(n-2)}(g_{ih}g_{jk} - g_{jh}g_{ki}) \tag{14}$$

The Conformal curvature tensor (rank-4) relating to a metric semi-symmetric connection is given by:

$$\bar{C}_{ijkh} = \bar{R}_{ijkh} - \frac{1}{n-2}(\bar{R}_{jk}g_{ih} - \bar{R}_{ik}g_{jh} + \bar{R}_{ih}g_{jk} - \bar{R}_{jh}g_{ki}) + \frac{\bar{R}}{(n-1)(n-2)}(g_{ih}g_{jk} - g_{jh}g_{ki}) \tag{15}$$

Theorem 1: The Conformal curvature tensor (r-4) relating to a metric semi-symmetric connection will be equal to the Conformal curvature tensor (r-4) with respect to a Riemannian curvature tensor (r-4) in almost hyperbolic Tachibana manifold, if the following conditions satisfies:

$$p_hF_{ik} = p_kF_{ih}. \tag{16}$$

Proof. Wed have from (9), (11) and (12) in (15), we obtain:

$$\begin{aligned}\bar{C}_{ijkh} = {} & R_{ijkh} - g_{ih}p_{kj} + g_{ik}p_{hj} - g_{jk}p_{hi} + g_{hj}p_{ki} + p_jp_hF_{ik} + p_ip_kF_{jh} - p_jp_kF_{ih} \\ & - p_ip_hF_{jk} - \frac{1}{n-2}[g_{ih}(R_{jk} - (n-2)p_{kj} - g_{jk}{p_m}^m - p_jq_k + p_kq_j - p^sp_sF_{kj}) \\ & - g_{jh}(R_{ik} - (n-2)p_{ki} - g_{ik}{p_m}^m - p_iq_k + p_kq_i - p^sp_sF_{ki}) \\ & + g_{kj}(R_{ih} - (n-2)p_{hi} - g_{ih}{p_m}^m - p_iq_h + p_hq_i - p^sp_sF_{hi}) \\ & - g_{ik}(R_{jh} - (n-2)p_{hj} - g_{jh}{p_m}^m - p_jq_h + p_hq_j - p^sp_sF_{hj})] \\ & + \frac{R - 2(n-1){p_m}^m}{(n-1)(n-2)}(g_{ih}g_{jk} - g_{jh}g_{ik})\end{aligned} \tag{17}$$

Again, from (13) and (17), we get:

$$\begin{aligned}\bar{C}_{ijkh} = {} & R_{ijkh} - \frac{1}{(n-2)}(R_{jk}g_{ih} - R_{ik}g_{jh} + R_{ih}g_{jk} - R_{jh}g_{ki}) \\ & + \frac{R}{(n-1)(n-2)}(g_{ih}g_{jk} - g_{jh}g_{ki}) + \frac{n+1}{n-2}(p_jp_hF_{ik} - p_jp_kF_{ih}) \\ & + \frac{n+1}{n-2}(p_ip_kF_{jh} - p_ip_hF_{jk})\end{aligned} \tag{18}$$

Also, from (14) and (18), we get:

$$\bar{C}_{ijkh} = C_{ijkh} + \frac{n+1}{n-2}(p_j p_h F_{ik} - p_j p_k F_{ih}) + \frac{n+1}{n-2}(p_i p_k F_{jh} - p_i p_h F_{jk}) \tag{19}$$

If we take $p_h F_{ik} = p_k F_{ih}$ then (19) reduces to the form:

$$\bar{C}_{ijkh} = C_{ijkh}. \tag{20}$$

Hence completes the proof.

Theorem 2: The Conformal curvature tensor (r-4) relating to a metric semi-symmetric connection in almost hyperbolic Tachibana manifold satisfies Bianchi identity if:

$$p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0. \tag{21}$$

Proof. We have Interchanging i, j *and* k in a cyclic order in (3.5), we get:

$$\bar{C}_{ijkh} = C_{ijkh} + \frac{n+1}{n-2}(p_j p_h F_{ik} - p_j p_k F_{ih}) + \frac{n+1}{n-2}(p_i p_k F_{jh} - p_i p_h F_{jk}), \tag{22}$$

$$\bar{C}_{jkih} = C_{jkih} + \frac{n+1}{n-2}(p_k p_h F_{ji} - p_k p_i F_{jh}) + \frac{n+1}{n-2}(p_j p_i F_{kh} - p_j p_h F_{ki}) \tag{23}$$

And

$$\bar{C}_{kijh} = C_{kijh} + \frac{n+1}{n-2}(p_i p_h F_{kj} - p_i p_j F_{kh}) + \frac{n+1}{n-2}(p_k p_j F_{ih} - p_k p_h F_{ij}) \tag{24}$$

Adding (3.9), (3.10) and (3.11), we obtain:

$$\bar{C}_{ijkh} + \bar{C}_{jkih} + \bar{C}_{kijh} = C_{ijkh} + C_{jkih} + C_{kijh} + 2\left(\frac{n+1}{n-2}\right) p_h (p_j F_{ik} + p_i F_{kj} + p_k F_{ji}). \tag{25}$$

Since, the Conformal curvature tensor (r-4) in a Riemannian manifold satisfies the condition:

$$C_{ijkh} + C_{jkih} + C_{kijh} = 0, \tag{26}$$

By using (3.13) in (3.12), we find

$$\bar{C}_{ijkh} + \bar{C}_{jkih} + \bar{C}_{kijh} = 2\left(\frac{n+1}{n-2}\right) p_h (p_j F_{ik} + p_i F_{kj} + p_k F_{ji}). \tag{27}$$

If we take $p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0$ then from (3.14), we get

$$\bar{C}_{ijkh} + \bar{C}_{jkih} + \bar{C}_{kijh} = 0. \tag{28}$$

Hence completes the proof.

Theorem 3: The Conformal curvature tensor (r-4) relating to a metric semi-symmetric connection in almost hyperbolic Tachibana manifold satisfies the following properties:

$$\bar{C}_{ijkh} = -\bar{C}_{jikh}, \quad \bar{C}_{ijkh} = -\bar{C}_{ijhk}.$$

Proof. We have Interchanging i *and* j in (3.5), we get:

$$\bar{C}_{jikh} = C_{jikh} + (\frac{n+1}{n-2})(p_i p_h F_{jk} - p_i p_k F_{jh}) + (\frac{n+1}{n-2})(p_j p_k F_{ih} - p_j p_h F_{ik}). \quad (29)$$

Adding (5) and (16), we obtain

$$\bar{C}_{ijkh} + \bar{C}_{jikh} = C_{ijkh} + C_{jikh}. \quad (30)$$

Since the conformal curvature tensor (r-4) in a Riemannian manifold satisfies:

$$C_{ijkh} + C_{jikh} = 0, \quad (31)$$

Now using (18) and (17), we get the result.
Again, interchanging k and h in (3.5), we have

$$\bar{C}_{ijhk} = C_{ijhk} + \frac{n+1}{n-2}(p_j p_k F_{ih} - p_j p_h F_{ik}) + \frac{n+1}{n-2}(p_i p_h F_{jk} - p_i p_k F_{jh}). \quad (33)$$

Adding (3.5) and (3.19), we have

$$\bar{C}_{ijkh} + \bar{C}_{ijhk} = C_{ijkh} + C_{ijhk}. \quad (34)$$

Since the conformal curvature tensor (r-4) in a Riemannian manifold satisfies:

$$C_{ijkh} + C_{ijhk} = 0, \quad (35)$$

Now by using (21) and (20), we get the result.

Theorem 4: The Conformal curvature tensor (r-4) relating to a metric semi-symmetric connection relating to Riemannian recurrent curvature tensor (r-4) in almost hyperbolic Tachibana manifold, if following condition satisfies:

$$C_{rjkh} p_i + C_{irkh} p_j + C_{ijrh} p_k + C_{ijkr} p_h = 0. \quad (36)$$

Proof. We have by taking covariant differentiation of the Conformal curvature tensor (r-4) relating to the Riemannian manifolds and metric semi-symmetric connection respectively, we get:

$$D_m C_{ijkh} = \partial_m C_{ijkh} - C_{rjkh}\begin{Bmatrix} r \\ mi \end{Bmatrix} - C_{irkh}\begin{Bmatrix} r \\ mj \end{Bmatrix} - C_{ijrh}\begin{Bmatrix} r \\ mk \end{Bmatrix} - C_{ijkr}\begin{Bmatrix} r \\ mh \end{Bmatrix} \quad (37)$$

And

$$\nabla_m C_{ijkh} = \partial_m C_{ijkh} - C_{rjkh}\Gamma_{mi}{}^r - C_{irkh}\Gamma_{mj}{}^r - C_{ijrh}\Gamma_{mk}{}^r - C_{ijkr}\Gamma_{mh}{}^r \quad (38)$$

Subtracting (3.23) from (3.24), we get:

$$\nabla_m C_{ijkh} - D_m C_{ijkh} = C_{rjkh}(\begin{Bmatrix} r \\ mi \end{Bmatrix} - \Gamma_{mi}{}^r) + C_{irkh}(\begin{Bmatrix} r \\ mj \end{Bmatrix} - \Gamma_{mj}{}^r) + C_{ijrh}(\begin{Bmatrix} r \\ mk \end{Bmatrix} - \Gamma_{mk}{}^r) + C_{ijkr}(\begin{Bmatrix} r \\ mh \end{Bmatrix} - \Gamma_{mh}{}^r) \quad (39)$$

Now using (1.8) in (3.25), we find:

$$\nabla_m C_{ijkh} - D_m C_{ijkh} = C_{rjkh}(p_i F_m{}^r - p_m \delta_i{}^r + p^r g_{mi}) + C_{irkh}(p_j F_m{}^r - p_m \delta_j{}^r + p^r g_{mj})$$

$$+ C_{ijrh}(p_k F_m{}^r - p_m \delta_k{}^r + p^r g_{mk}) + C_{ijkr}(p_h F_m{}^r - p_m \delta_h{}^r + p^r g_{mh}) \quad (40)$$

Again using (1.13) in (3.26), we obtain:

$$\nabla_m C_{ijkh} - D_m C_{ijkh} = (C_{rjkh} p_i + C_{irkh} p_j + C_{ijrh} p_k + C_{ijkr} p_h) F_m{}^r \quad (41)$$

If (3.22) is satisfied, then we find:

$$\nabla_m C_{ijkh} = D_m C_{ijkh}. \quad (42)$$

Hence we get the result.

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