5 D EMERGENT KERR TUNNELING VACUA

Abstract

Gravity is one the fundamental force of nature and understanding a quantum theory of gravity is always been a central question in physics. In this paper, we have tried to explore a gauge theory on a D-brane in 5D. The local degrees of a two form have been exploited to construct a geometric torsion dynamics in a non-perturbation theory of quantum gravity. Two different gauge choice of torsion field results into two different type of Kerr geometries that shows tunneling phenomenon.

Keywords: The local degrees of a two form have been exploited.

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I. INTRODUCTION

The bulk/boundary correspondence [1,2] is a powerful tool to establish quantum gravity through a gauge theory. It sheds light how the bulk quantum gravity theory may emerge from gauge boundary theory. In the context, emergent gravity theories have been a immediate topic of interest in theoretical physics [3,4].

As per the bulk/boundary correspondence, these non-linear U(1) gauge theories are describe on D-brane. In 5 dimensions, we may replace the one form U(1) gauge field by a two form field. A geometric torsion has been constructed on a D4-brane by exploring the local degrees of freedom of a two form. This torsion is turns out to be dynamical in the non-perturbative theory of quantum gravity. The connection is constructed by considering the iterative corrections in the theory that leads to perturbative description in gauge scenario. Hence one can visualise this non-pertubative geometric theory as the second order formalism. The gauge invariance of the U(I) theory is preserved in the curvature theory which seems to be broken in the gauge theory formalism. Henceforth the two form provides space-time curvature and becomes dynamical on a D4-brane . The constructed space time tensor, may be seen via the gauge connections which are equivalent to Christoffel symbols in General Theory of relativity (GTR). The considerations made in terms of two form ansatz leads to no-propagating torsion in the theory. It is also shown that the fourth order curvature tensor constructed form torion reduces to Rieman curvature tensor exactly for the non-dynamical case [5]..

II. MATHEMATICAL FORMULATION: GAUGE THEORETIC CURVATURE

In 5-dimensions the dynamics of a U(1) gauge field with a constant background are described by the following action

$$S=-\frac{-1}{4C_1^2}\int dx^5\sqrt{-g}\;F^2$$

Here C_1 is a dimensional constant that represents the gauge coupling, "g" is the constant background metric and F is the U(1) gauge field strength defined as $F_{\alpha\beta}=\nabla_\alpha A_\beta-\nabla_\beta A_\alpha$. ,$A_\beta$ is the one form gauge potential. In 5 dimensions, the U(1)gauge field dynamics can be re-written in terms of a two form field via Poincare duality. The modified gauge theoretical description via a two form describes a non-linear charge. The new action is

$$S=-\frac{-1}{4C_2^2}\int dx^5\sqrt{-g}\;H^2$$

Where C_2 again is a dimensional constant and provides the coupling in the two form theory, H is the field strength corresponding to a two form defined as $H_{\alpha\beta\gamma}=\nabla_\alpha B_{\beta\gamma}+\nabla_\beta B_{\gamma\alpha}+\nabla_\gamma B_{\alpha\beta}$.

In 5 dimension, we may now explore this torsion term to define a curvature scalar in space time as the dynamical degrees of freedom of the two matches exactly [6-9]. Hence an irreducible scalar $\mathcal{K}$ underlies $\mathscr{H}$ (a geometric torsion) which is basically governed by a gauge theoretic torsion $H_{\alpha\beta\gamma}=\nabla_\alpha B_{\beta\gamma}+cyclic$. Also, this modification of gauge torsion to geometric torsion requires a the redefinition of the covariant derivative in the theory, with a completely anti-symmetric connection $H^{\gamma}_{\alpha\beta}$. Hence the new covariant derivative may be defined as

$$\mathcal{D}_{\alpha}B_{\beta\gamma} = \nabla_{\alpha}B_{\beta\gamma} + \frac{1}{2} H^{\mu}_{\alpha\beta} B_{\mu\gamma} - \frac{1}{2} H^{\mu}_{\alpha\gamma} B_{\mu\beta} \tag{1}$$

Formally the geometric torsion can also be expressed as the geometric torsion with all order corrections in B_2 in a gauge theory

$$\begin{aligned} \mathscr{H}_{\alpha\beta\gamma} &= 3\mathcal{D}_{[\alpha}B_{\beta\gamma]} \\ &= 3\,\nabla_{[\alpha}B_{\beta\gamma]} + 3\,\mathscr{H}^{\mu}_{\alpha\beta} \\ &= H_{\alpha\beta\gamma} + \left(H_{\alpha\beta\delta}\, B^{\delta}_{\gamma} + cyclic\ in\ \alpha,\beta,\gamma\right) + H_{\alpha\beta\delta}B^{\delta}_{\epsilon}B^{\varepsilon}_{\gamma} + \cdots \end{aligned}$$

This geometric torsion is now utilise to construct a curvature tensor in 5 dimensions by taking the commutation relation of covariant derivative in the theory, so the effective curvature tensor obtained is defined as

$$\mathcal{K}^{\rho}_{\mu\nu\lambda} = \frac{1}{2}\, \partial_{\mu}\mathscr{H}^{\rho}_{\nu\lambda} + \frac{1}{2}\, \partial_{\nu}\mathscr{H}^{\rho}_{\mu\lambda} + \frac{1}{4}\, \mathscr{H}^{\sigma}_{\mu\lambda}\mathscr{H}^{\rho}_{\nu\sigma} + \frac{1}{4}\, \mathscr{H}^{\sigma}_{\upsilon\lambda}\mathscr{H}^{\rho}_{\mu\sigma} \tag{2}$$

One can now construct the irreducible curvature tensor $\mathcal{K}$ to write the action in 5 D. The above fourth order tensor retains a property of Riemann tensor $R_{\mu\nu\lambda}{}^{\rho}$ by showing anti-symmetricity within a pair of indices, i.e. $\mu \leftrightarrow \nu$ and $\lambda \leftrightarrow \rho$. However it is not symmetric, under an interchange of a pair of indices, like Riemann tensor. Hence, we may say that the effective gauge curvature constructed in a non-perturbative scenario can be considered as a generalized curvature tensor. It describes the geometric torsion which is propagating in a second order formalism.

This effective curvature constructed in the second order formalism preserves the gauge invariance of $\mathscr{H}$. As a result the, the space time fluctuations are given as

$$f_{\mu\upsilon} = C\, \mathscr{H}_{\mu\beta\rho}\mathscr{H}_{\upsilon}^{\beta\rho}$$

Where C is a dimensional constant. Also, the effective curvature tenor may be written though a geometric field strength $\mathscr{F}_2$ which is dual to $\mathscr{H}_3$ in 5D. Hence a geometric $\mathscr{F}_2$ may be given by

$$\begin{aligned} \mathscr{F}_{\alpha\beta} &= \mathcal{D}_{\alpha}A_{\beta} - \mathcal{D}_{\beta}A_{\alpha} \\ &= f^{z}_{\alpha\beta} + \mathscr{H}^{\lambda}_{\alpha\beta}A_{\lambda} \\ &= F_{\alpha\beta} + B_{\alpha\beta} \end{aligned}$$

Where B represents the non-dynamical modes of the two form field and already been shown towards the construction of non-linear U(1) gauge theory []. Now these fluctuations due to the dynamics and constant modes of the two form field can be added to generate the effective metric in the theory as

$$G_{\alpha\beta} = g_{\alpha\beta} - B_{\alpha\rho}\, B^{\rho}_{\beta} + C\, \mathscr{H}_{\alpha\gamma\rho}\mathscr{H}^{\gamma\rho}_{\beta} + \check{C}\mathscr{F}_{\alpha\rho}\, \mathscr{F}^{\rho}_{\beta} \tag{3}$$

Here B represents the contribution of constant(non-dynamical) modes two form, $\mathscr{H}$ shows the geometric torsion. As these non-dynamical modes are not unique they may give us a large no of vacua in the theory. The geometric torsion dynamics in 5D can be described as the effective curvature formalism [7, 8]. This new way of describing the geometric torsion in a non-perturbative formalism is thought provoking and may lead to new insights in the quantum gravity world. Importantly. The geometric action in a 5- dimensional theory can be written as

$$S = \frac{1}{3C_4^2}\int dx^4\,\sqrt{-G}\,\mathcal{K}$$

Here $C_4$is the dimensional constant in the action and G is the effective metric containing constant and non-constant modes of the two form field $B_{\mu\nu}$ as defined in equation (3). Finally $\mathcal{K}$ is the effective curvature tensor.

III. CHOICE OF BACKGROUND METRIC AND GAUGE ANSATZ

In this section we will first look into the choice background metric which is non dynamical and acts as background to the effective curvature tensor $\mathcal{K}$. A priori, these quantum modes considered provides extra term to the whole geometry but they soon decouple in the semiclassical limit in order to produce a stable Kerr blackhole solution in 5D. The background for the Kerr black hole is best described as Boyer-Lindquist coordinates which is given by

$$ds^2 = -dt^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + (r^2+a^2)sin^2\theta\, d\phi^2 + ((r^2+b^2)cos^2\theta\, d\psi^2$$

Where $\rho^2 = (r^2 + a^2cos^2\theta + b^2sin^2\theta)$ and $\Delta = (r^2+a^2)(r^2+b^2)$
The co-ordinate limits for the angular variables are: (0 <φ<2π, 0<ψ<2π, 0<θ<π). They former co-ordinates can be re-written in Cartesian form as:
$x = (r^2+a^2)\,sin\theta\,cos\phi$, $y = (r^2+a^2)\,sin\theta\,sin\phi$
$z = (r^2+b^2)\,cos\theta\,cos\psi$ and $w = (r^2+b^2)\,cos\theta\,sin\psi$

The above transformation shows ensures a ring singularity about the x-y and w-z plane.

1. **Ansatz for two form field:** The effective metric considered can now be explored to give quantum blackhole geometry by substituting different form of two form field $B_{\mu\nu}$ [9]. In the context, a gauge choice considered is

$$B_{tr} = r\sqrt{\frac{2M}{\Delta}}\ , \qquad B_{\theta\psi} = b\sqrt{2M}\,cos^2\theta\ , \qquad B_{\theta\phi} = a\sqrt{2M}\,sin^2\theta$$

$$B_{r\theta} = \rho\sqrt{2 + \frac{(2M-\rho^2)r^2}{\Delta} + \frac{2Mr^2+\Delta}{r^2\rho^2}} = \rho\sqrt{\frac{\mathscr{M}^+}{\Delta} - \frac{\mathscr{M}^+}{r^2\rho^2}}$$

$$\mathscr{M}^{\pm} = (a^2sin^2\theta + b^2cos^2\theta \pm 2M)r^2 + a^2b^2.$$

Under this choice of ansatz we obtained the Kerr Black Hole Solution in 5 dimensions. This vacua seems to tunnel to a another similar solution with the following set of B field consideration.

$$B_{tr} = r\sqrt{\frac{2M}{\Delta}} \approx \widetilde{B_{tr}}\ , \qquad B_{r\psi} = br\sqrt{\frac{2M}{\Delta}}\ , \qquad B_{r\psi} = ar\sqrt{\frac{2M}{\Delta}}$$

$$B_{r\theta} = \rho\sqrt{\frac{\mathcal{M}^+}{\Delta} - \frac{\mathcal{M}^+}{r^2\rho^2} - \frac{(2aM)r^2}{\Delta} - \frac{2Mb^2r^2}{\Delta + r^2\rho^2}}$$

Both the solution in the low energy limit seems to tunnel among themselves. We can also say that the potential considered for constructing the The U(1) gauge theory is not unique.

IV. CONCLUSION

We started with a generalized curvature theory sourced by a two form in a U(1) gauge theory. We considered two different sets of gauge ansatz for the two form field to construct quantum Kerr vacua in 5- dimensions. These Kerr black hole geometries were described via the non-perturbative formalism that underlying a geometric torsion, on a D4-brane.

Interestingly, these 5- dimensional Kerr geometries are emergent and are solely originated through the background fluctuations in a two form field on a non-BPS brane. The two form gauge ansatz, makes torsion non-propagating, which is explored via a 5 dimensional generalized curvature theory. A trivial torsion, in a non-linear U(1) gauge theory, can be seen as a trivial energy momentum tensor case. Further, these background fluctuations of two form field may be explained in presence of a U(1) electromagnetic field F2, which is dynamical as per the non-linear global description. The local F2 may be gauged away in the framework. Nevertheless, these background variations may have their significance in a two form which is dynamical field, and can be visualise as a propagating torsion, in higher dimensions.

Importantly, the two completely different sets of two form ansatz leads to same form of Kerr geometries that can be shown to tunnel within each other.

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