M-PROJECTIVE CURVATURE TENSOR OVER (κ, μ)-CONTACT RIEMANNIAN MANIFOLDS

Abstract

In 1995, the concept of (κ, μ)-contact Riemannian manifolds was introduced by Blair, Koufogiorgos, and Papantoniou [5]. Subsequently, a comprehensive investigation into the classification of contact metric (κ, μ)-spaces was conducted by Boeckx, E. [7] in 2000. Blair explored the (κ, μ)-nullity condition in the context of contact Riemannian manifolds and provided various motivations for its study. The current paper focuses on the examination of flatness conditions concerning the $\mathscr{M}$-projective curvature tensor within the framework of (κ, μ)-contact Riemannian manifolds.

Keywords: The concept of (κ, μ)-contact Riemannian manifolds,the current paper focuses on the examination

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I. INTRODUCTION

In 1958, Boothby and Wong first introduced the concept of odd-dimensional manifolds with contact and almost contact structures, primarily approaching it from a topological perspective. Subsequently, in 1961, Sasaki and Hatakeyama re-examined these structures using tensor calculus techniques.

Alternatively, in the work of Pokhariyal and Mishra, a tensor field W^* is introduced on a Riemannian manifold as (θ =Riemannian metric)

$$`W^*(\mathcal{S},\mathcal{T},\mathcal{U},\mathcal{V}) = `\mathscr{R}(\mathcal{S},\mathcal{T},\mathcal{U},\mathcal{V}) - \frac{1}{2(d-1)} \times [\rho(\mathcal{T},\mathcal{U})\theta(\mathcal{S},\mathcal{V}) - \rho(\mathcal{S},\mathcal{U})\theta(\mathcal{T},\mathcal{V}) + \theta(\mathcal{T},\mathcal{U})\rho(\mathcal{S},\mathcal{V}) - \theta(\mathcal{S},\mathcal{U})\rho(\mathcal{T},\mathcal{V})], \quad (1)$$

Where $`W^*(\mathcal{S},\mathcal{T},\mathcal{U},\mathcal{V}) = \theta(W^*(\mathcal{S},\mathcal{T})\mathcal{U},\mathcal{V})$ and $`\mathscr{R}(\mathcal{S},\mathcal{T},\mathcal{U},\mathcal{V}) = \theta(\mathscr{R}(\mathcal{S},\mathcal{T})\mathcal{U},\mathcal{V})$. The tensor field W^* is referred to as the $\mathscr{M}$-projective curvature tensor. Subsequently, Ojha conducted a comprehensive investigation of the properties of this tensor in both Sasakian and Kähler manifolds.

The category of (κ, μ)-contact Riemannian manifolds encompasses both Sasakian and non-Sasakian manifolds. Boeckx [7] provided a comprehensive categorization of (κ, μ)-contact Riemannian manifolds. These manifolds retain their properties under D-homothetic transformations.

In an earlier study [6], Blair, Kim, and Tripathi commenced an inquiry into the concircular curvature tensor of contact Riemannian manifolds. The examination of the pseudo-projective curvature tensor on a contact Riemannian manifold was recorded in [5]. More contemporarily, the investigations carried out by [14] and [15] delved into exploring the quasi-conformal curvature tensor and the E-Bochner curvature tensor on a (κ, μ)-contact Riemannian manifold, respectively. In addition to the well-known Riemannian curvature tensor, the Weyl conformal curvature tensor, and the concircular curvature tensor, the $\mathscr{M}$-projective curvature tensor emerges as a pivotal tensor within the realm of differential geometry. The curvature tensor serves as a unifying link between the conharmonic curvature tensor, the concircular curvature tensor and the conformal curvature tensor on the one hand while establishing a connection with the H-projective curvature tensor on the other.

Recently, the $\mathscr{M}$-projective curvature tensor has been a subject of study for various researchers, including Chaubey, Ojha [13], Singh [11], and others.

Expanding upon prior research, our current study investigates the symmetry and flatness characteristics of (κ, μ)-contact Riemannian manifolds in the context of the $\mathscr{M}$-projective curvature tensor. In Section 3, we review and deduce our initial findings. Subsequently, in Segment 4, we analyze $\mathscr{M}$-projectively flat (κ, μ)-contact Riemannian manifolds. Segment 5 centers on exploring ζ-$\mathscr{M}$-projectively Sasakian flat (κ, μ)-contact Riemannian manifolds, where we establish the requisite and sufficient conditions for the manifestation of ζ-$\mathcal{M}$-projective Sasakian flatness in an (κ, μ)-contact Riemannian manifold.

II. CONTACT RIEMANNIAN MANIFOLD

An almost contact structure on an $(2d + 1)$-dimensional differentiable manifold E is defined by the existence of a tensor field $\mathscr{F}$ of type (1, 1), a vector field ζ, and a 1-form η such that

$$\mathscr{F}^2 = -I + \eta \otimes \zeta, \qquad \eta(\zeta) = 1 \tag{2}$$

$$\mathscr{F}\zeta = 0, \text{ and } \eta \circ \mathscr{F} = 0 \tag{3}$$

Take into account a consistent Riemannian metric θ in conjunction with an almost contact structure $(\mathscr{F}, \zeta, \eta)$

$$\theta(\mathscr{F}\mathcal{S}, \mathscr{F}\mathcal{T}) = \theta(\mathcal{S}, \mathcal{T}) - \eta(\mathcal{S})\eta(\mathcal{T}) \tag{4}$$

Subsequently, when E^{2d+1} undergoes a transformation, it transforms into an almost contact Riemannian manifold by acquiring an almost contact metric structure represented as $(\mathscr{F}, \zeta, \eta, \theta)$. By observing equations (2) and (4), it becomes evident that

$$\theta(\mathcal{S}, \mathscr{F}\mathcal{T}) = -\theta(\mathscr{F}\mathcal{S}, \mathcal{T}), \quad \theta(\mathcal{S}, \zeta) = \eta(\mathcal{S}), \tag{5}$$

For any given vector fields $\mathcal{S}$ and $\mathcal{T}$.

The fact that the tangent sphere bundle of a Euclidean Riemannian manifold possesses a contact metric structure with the property $\mathscr{R}(\mathcal{S}, \mathcal{T})\zeta = 0$ is widely acknowledged. Conversely, in the context of a Sasakian manifold, the subsequent assertion is valid:

$$\mathscr{R}(\mathcal{S}, \mathcal{T})\,\zeta = \eta(\mathcal{T})\mathcal{S} - \eta(\mathcal{S})\mathcal{T}. \tag{6}$$

Blair et al. extended the concepts of $\mathscr{R}(\mathcal{S}, \mathcal{T})\zeta = 0$ and the Sasakian case by investigating the (κ, μ)-nullity condition on a contact Riemannian manifold. They introduced the (κ, μ)-nullity distribution $N(\kappa, \mu)$ ([3,5]) to characterize this condition on the contact Riemannian manifold.

$$N(\kappa, \mu): \mathcal{P} \to N_{\mathcal{P}}(\kappa, \mu) = \{\mathcal{U} \in T_{\mathcal{P}}E: \mathscr{R}(\mathcal{S}, \mathcal{T})\mathcal{U} = (\kappa I + \mu h)[\theta(\mathcal{T}, \mathcal{U})\mathcal{S} - \theta(\mathcal{S}, \mathcal{U})\mathcal{T}]\} \quad \dots \tag{7}$$

For any pair of vectors $\mathcal{S}$ and $\mathcal{T}$ belonging to the tangent space TE, where (κ, μ) are elements of the R^2, a Riemannian manifold E^{2d+1} possessing ζ in the set $N(\kappa, \mu)$ is referred to as a manifold with (κ, μ) characteristic. Specifically, on a manifold with (κ, μ) attributes, the following holds true

$$\mathscr{R}(\mathcal{S}, \mathcal{T})\zeta = \kappa[\eta(\mathcal{T})\mathcal{S} - \eta(\mathcal{S})\mathcal{T}] + \mu\,[\eta(\mathcal{T})h\mathcal{S} - \eta(\mathcal{S})h\mathcal{T}]\,. \tag{8}$$

On a (κ, μ)-manifold, where $\kappa \leq 1$, the structure becomes Sasakian with $h = 0$ and μ remaining indeterminate when $\kappa = 1$. When $\kappa < 1$, the (κ, μ)-nullity condition uniquely prescribes the curvature of E^{2d+1} Essentially, for a (κ, μ)-manifold, the properties of being a Sasakian manifold, a K-contact manifold, $\kappa = 1$, and $h = 0$ are all interchangeable and equivalent.

In a manifold characterized by the parameters (κ, μ), the following relationships are valid:

$$h^2 = (\kappa - 1)^2 \mathscr{F}^2 \,,\ \kappa \leq 1,$$

$$\mathscr{R}(\zeta, \mathcal{S})\mathcal{T} = \kappa[\theta(\mathcal{S}, \mathcal{T})\zeta - \eta(\mathcal{T})\mathcal{S}] + \mu[\theta(h\mathcal{S}, \mathcal{T})\zeta - \eta(\mathcal{T})h\mathcal{S}] \,,$$

$$\rho(\mathcal{S}, \zeta) = 2d\kappa\eta\,(\mathcal{S}), \tag{9}$$

$$\rho(\mathcal{S}, \mathcal{T}) = [2(d-1) - d\mu]\theta(\mathcal{S}, \mathcal{T}) + [2(d-1) + \mu]\theta(h\mathcal{S}, \mathcal{T})$$

$$+ [2(1-d) + d(2\kappa + \mu)]\,\eta(\mathcal{S})\eta(\mathcal{T}),\ n \geq 1,$$

$$\rho(\mathscr{F}\mathcal{S}, \mathscr{F}\mathcal{T}) = \rho(\mathcal{S}, \mathcal{T}) - 2d\kappa\eta(\mathcal{S})\eta(\mathcal{T}) - 2(2d - 2 + \mu)\theta(h\mathcal{S}, \mathcal{T}),$$

Where ρ is the Ricci tensor of type (0, 2), Q is the Ricci operator, that is, $\theta(Q\mathcal{S}, \mathcal{T}) = \rho(\mathcal{S}, \mathcal{T})$. Furthermore, the (κ, μ)-manifold exhibits the following property:

$$\eta(\mathscr{R}(\mathcal{S}, \mathcal{T})\mathcal{U}) = \kappa[\theta(\mathcal{T}, \mathcal{U})\eta(\mathcal{S}) - \theta(\mathcal{S}, \mathcal{U})\eta(\mathcal{T})]$$

$$+\mu[\theta(h\mathcal{T}, \mathcal{U})\eta(\mathcal{S}) - \theta(h\mathcal{S}, \mathcal{U})\eta(\mathcal{T})] \tag{10}$$

In the context of Riemannian manifold, the $\mathscr{M}$-projective curvature tensor W^* can be stated as follows [8].

$$W^*(\mathcal{S}, \mathcal{T})\mathcal{U} = \mathscr{R}(\mathcal{S}, \mathcal{T})\mathcal{U} - \frac{1}{2(d-1)} \times [\rho(\mathcal{T}, \mathcal{U})\mathcal{S} - \rho(\mathcal{S}, \mathcal{U})\mathcal{T} + \theta(\mathcal{T}, \mathcal{U})Q\mathcal{S} - \theta(\mathcal{S}, \mathcal{U})Q\mathcal{T}], \tag{11}$$

Given arbitrary vector fields $\mathcal{S}, \mathcal{T}$ and $\mathcal{U}$, where ρ represents the type of Ricci tensor (0, 2) and Q denotes the Ricci operator, θ denotes the Riemannian metric, we have the relation $\theta(Q\mathcal{S}, \mathcal{T}) = \rho(\mathcal{S}, \mathcal{T})$.

Lemma 2.1. [1] In (κ, μ)-contact Riemannian manifolds that are not Sasakian, the conditions that follow are mutually equivalent:

- η-Einstein manifold,
- $Q\mathscr{F} = \mathscr{F}Q$

Definition 2.1. An E manifold with a (k, μ)-contact metric structure is referred to as η-Einstein when the Ricci operator Q fulfills the conditions

$$Q = aI + b\eta \otimes \zeta \tag{12}$$

Smooth functions a and b are represented in the given context defined on the manifold. Notably, when b is set to zero, E qualifies as an Einstein manifold.

In the case where an $(2d+1)$-dimensional non-Sasakian (k,μ)-contact Riemannian manifold (E^{2d+1},θ) is η-Einstein, the expression for the non-zero Ricci tensor ρ takes the following form:

$$\rho(\mathcal{S},\mathcal{T}) = a\theta(\mathcal{S},\mathcal{T}) + b\eta(\mathcal{S})\eta(\mathcal{T}). \tag{13}$$

Lemma 2.2. On a non-Sasakian (k,μ)-contact Riemannian manifold (E^{2d+1},θ), $a+b=2d\kappa$
Proof. In view of (2)-(5) and (13), we have

$$Q\mathcal{S} = a\mathcal{S} + b\eta(\mathcal{S})\zeta, \tag{14}$$

such that Ricci operator Q is defined by

$$\rho(\mathcal{S},\mathcal{T}) = \theta(Q\mathcal{S},\mathcal{T}). \tag{15}$$

Again, contracting (14) with respect to $\mathcal{S}$ and using (2)-(5), we have

$$r = (2d+1)a + b. \tag{16}$$

Now, putting ζ instead of $\mathcal{S}$ and $\mathcal{T}$ in (13) and then using the equations in (2)-(5) and (9) we get

$$a + b = 2d\kappa. \tag{17}$$

Equations (16) and (17) give

$$a = \left(\frac{r}{2d} - \kappa\right) \text{ and } b = \left((2d+1)\kappa - \frac{r}{2d}\right). \tag{18}$$

Equation (18) prove the statement of the Lemma 2.2.

III. THE M-PROJECTIVE CURVATURE TENSOR W^* FOR AN (κ,μ)-CONTACT RIEMANNIAN MANIFOLDS

The curvature tensor W^* associated with $\mathscr{M}$-projective geometry on a (κ,μ)-contact Riemannian manifold is expressed as

$$W^*(\mathcal{S},\mathcal{T})\zeta = -\frac{\kappa}{(d-1)}[\eta(\mathcal{T})\mathcal{S} - \eta(\mathcal{S})\mathcal{T}] + \mu[\eta(\mathcal{T})h\mathcal{S} - \eta(\mathcal{S})h\mathcal{T}] - \frac{1}{2(d-1)}\left[\eta(\mathcal{T})Q\mathcal{S} - \eta(\mathcal{S})Q\mathcal{T}\right], \tag{19}$$

$$\eta(W^*(\mathcal{S},\mathcal{T})\,\zeta) = 0, \tag{20}$$

$$W^*(\zeta,\mathcal{T})\mathcal{U} = -W^*(\mathcal{T},\zeta)\mathcal{U} = -\frac{\kappa}{(d-1)}[\theta(\mathcal{T},\mathcal{U})\zeta - \eta(\mathcal{U})\mathcal{T}] + \mu[\theta(h\mathcal{T},\mathcal{U})\zeta - \eta(\mathcal{U})h\mathcal{T}] - \frac{1}{2(d-1)}[\rho(\mathcal{T},\mathcal{U})\zeta - \eta(\mathcal{U})Q\mathcal{T}], \quad (21)$$

$$\eta(W^*(\zeta,\mathcal{T})\mathcal{U}) = -\eta(W^*(\mathcal{T},\zeta)\mathcal{U}) = -\frac{\kappa}{(d-1)}[\theta(\mathcal{T},\mathcal{U}) - \eta(\mathcal{T})\eta(\mathcal{U})] + \mu[\theta(h\mathcal{T},\mathcal{U}) - \eta(\mathcal{U})\eta(h\mathcal{T})] - \frac{1}{2(d-1)}[\rho(\mathcal{T},\mathcal{U}) - 2d\kappa\eta(\mathcal{T})\eta(\mathcal{U})], \quad (22)$$

$$\eta(W^*(\mathcal{S},\mathcal{T})\mathcal{U}) = -\frac{\kappa}{(d-1)}[\theta(\mathcal{T},\mathcal{U})\eta(\mathcal{S}) - \theta(\mathcal{S},\mathcal{U})\eta(\mathcal{T})] + \mu[\theta(h\mathcal{T},\mathcal{U}) - \eta(\mathcal{U})\eta(h\mathcal{T})] - \frac{1}{2(d-1)}[\rho(\mathcal{T},\mathcal{U})\eta(\mathcal{S}) - \rho(\mathcal{S},\mathcal{U})\eta(\mathcal{T})]. \quad (23)$$

IV. M-PROJECTIVELY FLAT (κ, μ)-CONTACT RIEMANNIAN MANIFOLDS

The class of (κ, μ)-contact Riemannian manifolds known as $\mathscr{M}$-projectively flat manifolds is a distinctive category within contact Riemannian manifold where the geometry is such that the curvature tensor satisfies certain conditions related to the $\mathscr{M}$-projective flatness property. The parameters κ and μ are involved in the definition of the curvature conditions and can affect the geometry of the manifold.

Theorem 4.1. A (κ, μ)-contact Riemannian manifold E^{2d+1}that is $\mathscr{M}$-projectively flat exhibits the property of being an Einstein manifold.

Proof. Let $W^*(\mathcal{S},\mathcal{T},\mathcal{U},\mathcal{V}) = 0$. Subsequently, utilizing equation (11), we derive the following outcome:

$$'\mathscr{R}(\mathcal{S},\mathcal{T},\mathcal{U},\mathcal{V}) = \frac{1}{2(d-1)}[\rho(\mathcal{T},\mathcal{U})\theta(\mathcal{S},\mathcal{V}) - \rho(\mathcal{S},\mathcal{U})\theta(\mathcal{T},\mathcal{V}) + \theta(\mathcal{T},\mathcal{U})\rho(\mathcal{S},\mathcal{V}) - \theta(\mathcal{S},\mathcal{U})\theta(\mathcal{T},\mathcal{V})] \quad (24)$$

Considering e_i as an orthonormal basis of the tangent space at any point, if we set $\mathcal{T} = \mathcal{U} = e_i$ in the given equation and then sum up over i, where $1 \le i \le 2d + 1$, we arrive at the same result,

$$\rho(\mathcal{S},\mathcal{T}) = -r\theta(\mathcal{S},\mathcal{T}), \quad (25)$$

Where r-Scalar curvature of the manifold and $r = 2d(2d - 2 + \kappa - d\mu)$.

This indicates that E^{2d+1} is a manifold that satisfies the Einstein condition. This completes the proof.

V. ζ-M-PROJECTIVELY SASAKIAN FLAT (κ,μ)-CONTACT RIEMANNIAN MANIFOLDS

ζ-$\mathscr{M}$-Projectively Sasakian flat (κ, μ)-contact Riemannian manifolds likely refer to a specific class of contact Riemannian manifolds that satisfy curvature conditions related to $\mathscr{M}$-projective flatness and these manifolds also have a distinguished Reeb vector field (ζ) and Sasakian geometry. This indicates a very specialized and intricate geometric structure where various curvature conditions, contact structures, and vector fields are intertwined.

Definition 5.1. An (2d+1) (with d > 1)-dimensional (κ,μ)-contact Riemannian manifold is classified as ζ-M-projectively Sasakian flat when the condition W^* (S,T)ζ=0 holds for all S and T belonging to the tangent space TE.

Theorem 5.1. An (2d+1)-dimensional (d>1) (κ, μ)-contact Riemannian manifold exhibits ζ-$\mathscr{M}$-projective Sasakian flatness iff it possesses the characteristic of being an η-Einstein manifold.

Proof. Let W^* (S,T)ζ=0. Then, in view of (11), we have

$$\mathcal{R}(\mathcal{S},\mathcal{T})\zeta = \frac{1}{2(d-1)}[\rho(\mathcal{T},\zeta)\mathcal{S} - \rho(\mathcal{S},\zeta)\mathcal{T} + \theta(\mathcal{T},\zeta)Q\mathcal{S} - \theta(\mathcal{S},\zeta)Q\mathcal{T}] \tag{26}$$

Due to the presence of (5), (8), and (9), the equation above can be simplified to

$$\kappa[\eta(\mathcal{T})\mathcal{S} - \eta(\mathcal{S})\mathcal{T}] + \mu[\eta(\mathcal{T})h\mathcal{S} - \eta(\mathcal{S})h\mathcal{T}]$$
$$= \frac{d\kappa}{d-1}[\eta(\mathcal{T})\mathcal{S} - \eta(\mathcal{S})\mathcal{T}] + \frac{1}{2(d-1)}[\eta(\mathcal{T})Q\mathcal{S} - \eta(\mathcal{S})Q\mathcal{T}] \tag{27}$$

which by putting T=ζ, gives

$$Q\mathcal{S} = 2\kappa\,[-\mathcal{S} + (d+1)\eta(\mathcal{S})\zeta] + 2(d-1)\mu(h\mathcal{S}) \tag{28}$$

In the case of Sasakian manifolds, κ = 1, (and hence h = 0)

Now, taking the inner product of above equation with V, we get

$$\rho(\mathcal{S},\mathcal{V}) = 2[-\theta(\mathcal{S},\mathcal{V}) + (d+1)\,\eta(\mathcal{S})\eta(\mathcal{V})] \tag{29}$$

Furthermore, it can be proved that a (κ,μ)-contact Riemannian manifold represents an η-Einstein manifold. Conversely, assume that condition (29) is fulfilled. As a result of the implications of (28) and (19), we can deduce W^* (S,T)ζ=0. Thus, the proof is concluded.

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