# PENTAPARTITIONED NEUTROSOPHIC PYTHAGOREAN GENERALIZED PRE-CLOSED SETS

**Abstract**

In this paper a Pentapartitioned Neutrosophic Pythagorean generalized pre-closed sets and a Pentapartitioned Neutrosophic Pythagorean generalized pre-open sets are introduced. Some of its properties are also studied. Also we have provided some applications of Pentapartitioned Neutrosophic Pythagorean generalized pre-closed sets namely Pentapartitioned Neutrosophic Pythagorean $_{p}T_{1/2}$ space and Pentapartitioned Neutrosophic Pythagorean $_{gp}T_{1/2}$ space.

**Keywords:** Pentapartitioned Neutrosophic Pythagorean topology, Pentapartitioned Neutrosophic Pythagorean generalized pre-closed sets, Pentapartitioned Neutrosophic Pythagorean generalized pre-open sets, Pentapartitioned Neutrosophic Pythagorean $_{p}T_{1/2}$ space and Pentapartitioned Neutrosophic Pythagorean $_{gp}T_{1/2}$ space.

**Authors**

**Radha. R**
Assistant Professor
Department of Science and Humanities
Karpagam College of Engineering
Coimbatore, Tamil Nadu, India.
radharmat2020@gmail.com

**Princy. R**
Assistant Professor
Department of Science and Humanities
Karpagam College of Engineering
Coimbatore, Tamil Nadu, India.
princy.pjs@gmail.com

**Gayathri. P**
Assistant Professor
Department of Science and Humanities
Karpagam College of Engineering
Coimbatore, Tamil Nadu, India.
gayucbe@gmail.com

**Gomathi. S**
Assistant Professor
Department of Science and Humanities
Karpagam College of Engineering
Coimbatore, Tamil Nadu, India.
gomathiprakash2013@gmail.com

**Kavitha. A**
Assistant Professor
Department of Science and Humanities
Karpagam College of Engineering
Coimbatore, Tamil Nadu, India.
asrkavitha@gmail.com

**Kalamani. A**
Assistant Professor
Department of Science and Humanities
Karpagam College of Engineering
Coimbatore, Tamil Nadu, India.
20kalamani@gmail.com

## I. INTRODUCTION

Zadeh [18] introduced the idea of fuzzy sets in 1965 that permits the membership perform valued within the interval[0,1] and set theory its an extension of classical pure mathematics. Intuitionistic Fuzzy set was first introduced by K. T. Atanassov [1] in 1983. After that he introduced, the concept of Intuitionistic sets as generalization of Fuzzy sets. The concept of generalized topological structures in Fuzzy topological spaces using Intuitionistic Fuzzy sets was introduced by D. Coker [3]. D. Coker introduced the concept of Intuitionistic Fuzzy sets, Intuitionistic Fuzzy topological spaces, Intuitionistic topological spaces and Intuitionistic Fuzzy points. R. R. Yager [17] generalized Intuitionistic Fuzzy set and presented a new set called Pythagorean set.

Florentine Smarandache [15] introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. Thus neutrosophic set was framed and it includes the parts of truth membership function (T), indeterminacy membership function (I), and falsity membership function (F) severally. Neutrosophic sets deals with non normal interval of] −0 1+ [.Since neutrosophic set deals the indeterminateness effectively it plays an very important role in several applications areas embrace info technology, decision web,electronicdatabase systems, diagnosis, multicriteria higher cognitive process issues etc., Pentapartitioned neutrosophic set and its properties were introduced by Rama Malik and Surpati Pramanik [14]. In this case, indeterminacy is divided into three components: contradiction, ignorance, and an unknown membership function. Further, R. Radha and A. Stanis Arul Mary [5] outlined a brand new hybrid model of Pentapartitioned Neutrosophic Pythagorean sets (PNPS) and Quadripartioned neutrosophic pythagorean sets in 2021. The Pentapartitioned Neutrosophic Pythagorean topological spaces [13] was introduced and its properties are investigated in 2021.In this paper, we focus on the main concept of Generalized Pentapartitioned Neutrosophic Pythagorean preopen sets in Pentapartitioned Neutrosophic Pythagorean topological spaces.

## II. PRELIMINARIES

### 2.1 Definition [15]

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$

Where $T_A\ , I_A, F_A: U \rightarrow [0,1]\ and\ 0 \leq T_A(x) + I_A(x) + F_A\ (x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

$He$re, $T_A(x)$ and $F_A(x)$are dependent neutrosophic components and $I_A(x)$ is an independent component.

**2.2 Definition [5]**

Let X be a universe. A Pentapartitioned neutrosophic pythagorean [PNP] set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

$$A = \{< x, T_A, C_A, I_A, U_A, F_A >: x \in X\}$$

Where $T_A + F_A \leq 1, C_A + U_A \leq 1$ *and*

$$(T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A)^2 \leq 3$$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership and $I_A(x)$ is an unknown membership.

**2.3 Definition [14]**

Let P be a non-empty set. A Pentapartitioned neutrosophic set A over P characterizes each element p in P a truth -membership function $T_A$ , a contradiction membership function $C_A$, an ignorance membership function $G_A$, unknown membership function $U_A$ and a false membership function $F_A$ , such that for each p in P

$$T_A + C_A + G_A + U_A + F_A \leq 5$$

**2.4 Definition [5]**

The complement of a pentapartitioned neutrosophic pythagorean set A on R Denoted by $A^C$ or $A^*$ and is defined as
$A^C = \{< x, F_A(x), U_A(x), 1 - G_A(x), C_A(x), T_A(x) > : x \in X\}$

**2.5 Definition [5]**

Let A $= < x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) >$ and

B $= < x, T_B(x), C_B(x), G_B(x), U_B(x), F_B(x) >$ are pentapartitioned neutrosophic pythagorean sets. Then

$$A \cup B = < x, max(T_A(x), T_B(x)), max(C_A(x), C_B(x)), \min(G_A(x), G_B(x)), \min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)), >$$

$$A \cap B = < x, min(T_A(x), T_B(x)), min(C_A(x), C_B(x)), \max(G_A(x), G_B(x)), max(U_A(x), U_B(x)), max(F_A(x), F_B(x)) >$$

**2.6 Definition [13]**

A PNP topology on a nonempty set R is a family of a PNP sets in R satisfying the following axioms

i) 0,1$\in \tau$
ii) $R_1 \cap R_2 \in \tau$ for any $R_1, R_2 \in \tau$
iii) $\bigcup R_i \in \tau$ for any $R_i : i \in I \subseteq \tau$

The complement $R^*$ of PNP open set (PNPOS, in short) in PNP topological space [PNPTS] (R,$\boldsymbol{\tau}$), is called a PNP closed set [PNPCS].

**2.7 Definition [13]**

Let (X, $\tau$) be an PNPTS and A = $< x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) >$ be an PNPS in X. Then the interior and the closure of A are denoted by PNPInt(A) and PNPCl(A) and are defined as follows.

PNPCl(A) = ∩{K|K is an PNPCS and A ⊆ K} and
PNPInt(A) = ∪{G|G is an PNPOS and G ⊆ A}

Also, it can be established that PNPCl(A) is an PNPCS and PNPInt(A) is an PNPOS, A is an PNPCS if and only if PNPCl(A) = A and A is an PNPOS if and only if PNPInt(A) = A. We say that A is PNP- dense if PNPCl(A) = X.

**2.8 Lemma [13]**

Let (X, τ) be an PNPTS and let A and B be an Pentapartitioned Neutrosophic Pythagorean subset of X. Then the following hold.

i) PNPCl(∅) = ∅ and PNPCl(X) = X
ii) A is an PNPCS if and only if A = PNPCl(A)
iii) PNPCl(PNPCl(A)) = PNPCl(A)
iv) A ⊆ B implies that PNPCl(A) ⊆ PNPCl(B)
v) PNPCl(A ∩ B) ⊆ PNPCl(A) ∩ PNPCl(B)
vi) PNPCl(A∪B) = PNPCl(A) ∪ PNPCl(B)

**2.9 Lemma [13]**

Let (X, τ) be an PNPTS and let A and B be an Pentapartitioned Neutrosophic Pythagorean subset of X. Then the following hold.

i) PNPInt(∅) = ∅ and PNPInt(X) = X

 

ii) A is an PNPOS if and only if A = PNPInt(A)

iii) PNPInt(PNPInt(A)) = PNPInt(A)

iv) A ⊆ B implies that PNPInt(A) ⊆ PNPInt(B)

v) PNPInt(A ∩ B) = PNPInt(A) ∩ PNPInt(B)

vi) PNPInt(A∪B) ⊇ PNPInt(A) ∪ PNPInt(B)

**2.10 Lemma [13]**

Let (X, τ) be an PNPTS and let A and B be two PNPS in (X, τ). IF B is an PNPOS, then PNPCl(A) ∩ B ⊆ PNPCl(A ∩ B). So if G is PNPCS and H is any PNPS, then PNPInt(H ∪ G) ⊂ PNPInt(H) ∪ G.

**2.11 Lemma [13]**

For any Pentapartitioned Neutrosophic Pythagorean set A in (X, τ), we have X – PNPInt(A) = PNPCl(X – A) and X – PNPCl(A) = PNPInt(X – A).

**2.12 Definition [12]**

Let (X, τ) be an PNPTS and $A = \{< x, T_A, C_A, I_A, U_A, F_A >: x \in X\}$. A is an Pentapartitioned Neutrosophic Pythagorean semiopen set (PNPSOS in short) in an PNPTS (X, τ) if there is an Pentapartitioned Neutrosophic Pythagorean open set G ≠ < X, ∅, X > such that G ⊂ A ⊂ PNPCl(A). Clearly, every PNPOS is an PNPSOS and ∅ and X are PNPSOS. Also, from the definition, it follows that the closure of every PNPOS is an Pentapartitioned Neutrosophic Pythagorean semiopen set. The complement of every PNPSOS is said to be an Pentapartitioned Neutrosophic Pythagorean semiclosed set (PNPSCS in short).

The semiinterior and the semiclosure of an PNPS A is denoted by PNPSInt(A) and PNPSCl(A) and are defined as follows.

PNPSCl(A) = ∩{K|K is an PNPSCS and A ⊆ K}

PNPSInt(A) = ∪{G|G is an PNPSOS and G ⊆ A}. It can be established that PNPSCl(A) is the smallest PNPSCS contained in all PNPSCS containing A and PNPSInt(A) is the largest PNPSOS contained in A, A is an PNPSCS if and only if PNPSCl(A) = A and A is an PNPSOS if and only if PNPSInt(A) = A.

**2.13 Definition [12]**

Let (X, τ) be an PNPTS and A be an PNPS. A is said to be an Pentapartitioned Neutrosophic Pythagorean α − open set (in short PNPαOS) if A ⊆ PNPInt(PNPCl(PNPInt(A))). The family of all Pentapartitioned Neutrosophic Pythagorean α − open set is denoted by PNPαOS. The complement of an PNPαOS is called an Pentapartitioned Neutrosophic Pythagorean α − closed set (in short PNPαCS).

## III. PENTAPARTITIONED NEUTROSOPHIC PYTHAGOREAN GENERALIZED PRE-CLOSED SETS

In this section we introduce Pentapartitioned Neutrosophic Pythagorean generalized pre-closed set and studied some of its properties.

### 3.1 Definition

An PNPS A is said to be an Pentapartitioned Neutrosophic Pythagorean generalized pre-closed set (PNPGPCS in short) in (X, τ) if PNPPCl(A) ⊆ U whenever A ⊆ U and U is a PNPOS in X. The family of all PNPGPCSs of an PNPTS (X, τ) is denoted by PNPGPC(X).

### 3.2 Example

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.2, 0.1,0.6,0.7,0.8 >, < a, 0.3,0.2,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.2, 0.1,0.6,0.7,0.8 >, < a, 0.2,0.1,0.6,0.6, 0.7 >} is an PNPGPCS in X.

### 3.3 Theorem

Every PNPCS is an PNPGPCS but not conversely.

**Proof:**

Let A be an PNPCS in X and let A ⊆ U and U is an PNPOS in (X, τ). Since PNPPCl(A) ⊆ PNPCl(A) and A is an PNPCS in X, PNPPCl(A) ⊆ PNPCl(A) = A ⊆ U. Therefore A is an PNPGPCS in X.

### 3.4 Example

Let X = {a, b} and let τ = 0, T, 1} be an PNPT on X, where T = {< a, 0.2, 0.1,0.6,0.7,0.8 >, < a, 0.3,0.2,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.2,0.1,0.6,0.7, 0.8 >, < a, 0.2, 0.1,0.6,0.6,0.7 >} is an PNPGPCS in X but not an PNPCS in X.

### 3.5 Theorem

Every PNPαCS is an PNPGPCS but not conversely.

**Proof:**

Let A be an PNPαCS in X and let A ⊆ U and U is an PNPOS in (X, τ). By hypothesis, PNPCl(PNPInt(PNPCl(A))) ⊆ A. Since A ⊆ PNPCl(A), PNPCl(PNPInt(A)) ⊆ PNPCl(PNPInt(PNPCl(A))) ⊆ A. Hence PNPPCl(A) ⊆ A ⊆ U. Therefore A is an PNPGPCS in X.

 

**3.6 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.4,0.3,0.6,0.5, 0.6 >, < a, 0.2,0.1,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.3, 0.2,0.6,0.0.6,0.7 >, < a, 0.1,0,0.6,0.7, 0.8 >} is an PNPGPCS in X but not an PNP α CS in X. Since PNPCl(PNPInt(PNPCl(A))) = {< a, 0.6,0.5,0.6,0.3, 0.4 >, < a, 0.7,0.6,0.6,0.1, 0.2 >} ⊈ A.

**3.7 Theorem**

Every PNPGCS is an PNPGPCS but not conversely.

**Proof:**

Let A be an PNPGCS in X and let A ⊆ U and U is an PNPOS in (X, τ). Since PNPPCl(A) ⊆ PNPCl(A) and by hypothesis, PNPPCl(A) ⊆ U. Therefore A is an PNPGPCS in X.

**3.8 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.4, 0.3,0.6,0.5,0.6>, < a, 0.5,0.4,0.6,0.4, 0.5>}. Then the PNPS A = {< a, 0.3,0.2,0.6,0.6, 0.7 >, < a, 0.4, 0.3,0.6,0.5,0.6 >} is an PNPGPCS in X but not an PNPGCS in X since A ⊆ T but PNPCl(A) = {< a, 0.6, 0.5,0.6,0.3,0.4 >, < a, 0.5,0.4,0.6,0.4, 0.5 >} ⊈ T.

**3.9 Theorem**

Every PNPRCS is an PNPGPCS but not conversely.

**Proof:**

Let A be an PNPRCS in X. By Definition , A = PNPCl(PNPInt(A)). This implies PNPCl(A) = PNPCl(PNPInt(A)). Therefore PNPCl(A) = A. That is A is an PNPCS in X. By Theorem 3.3, A is an PNPGPCS in X.

**3.10 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.6, 0.5,0.6,0.3,0.4 >, < a, 0.7,0.6,0.6,0.1,0.2 >}. Then the PNPS A = {< a, 0.3,0.2,0.6,0.6, 0.7 >, < a, 0.2, 0.1,0.6,0.7,0.8 >} is an PNPGPCS but not an PNPRCS in X since PNPCl(PNPInt(A)) = 0 ≠ A.

**3.11 Theorem**

Every PNPPCS is an PNPGPCS but not conversely.

**Proof:**

Let A be an PNPPCS in X and let A ⊆ U and U is an PNPOS in (X, τ). By Definition, PNPCl(PNPInt(A)) ⊆ A. This implies that PNPPCl(A) = A ∪ PNPCl(PNPInt(A)) ⊆ A. Therefore PNPPCl(A) ⊆ U. Hence A is an PNPGPCS in X.

**3.12 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.6,0.5,0.6,0.3, 0.4 >, < a, 0.3,0.2,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.8,0.7,0.6,0.1, 0.2 >, < a, 0.3,0.2,0.6,0.6, 0.7 >} is an PNPGPCS but not an PNPPCS in X since PNPCl(PNPInt(A)) = 1 ⊈ A.

**3.13 Theorem**

Every PNPαGCS is an PNPGPCS but not conversely.

**Proof:**

Let A be an PNPαGCS in X and let A ⊆ U and U is an PNPOS in (X, τ). By Definition, A ∪ PNPCl(PNPInt(PNPCl(A))) ⊆ U. This implies PNPCl(PNPInt(PNPCl(A))) ⊆ U and PNPCl(PNPInt(A)) ⊆ U. Therefore PNPPCl(A) = A ∪ PNPCl(PNPInt(A)) ⊆ U. Hence A is an PNPGPCS in X.

**3.14 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.5, 0.4,0.6,0.4,0.5 >, < a, 0.6,0.5,0.6,0.3, 0.4 >}. Then the PNPS A = {< a, 0.4,0.3,0.6,0.5, 0.6 >, < a, 0.5,0.4,0.6,0.4, 0.5 >} is an PNPGPCS but not an PNPαGCS in X since PNPαcl(A) = 1 ⊈ T.

**3.15 Proposition**

PNPSCS and PNPGPCS are independent to each other.

**3.16 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.5, 0.4,0.6,0.4,0.5 >, < a, 0.2,0.1,0.6,0.5,0.6 >}. Then the PNPS A = T is an PNPSCS but not an PNPGPCS in X since A ⊆ T but PNPPCl(A) = {< a, 0.5, 0.4,0.6,0.4,0.5 >, < a, 0.6, 0.5,0.6,0.1,0.2 >} ⊈ T.

**3.17 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.8, 0.7,0.6,0.1,0.2 >, < a, 0.8,0.7,0.6,0.1, 0.2 >}. Then the PNPS A = {< a, 0.8,0.7,0.6,0.1, 0.2 >, < a, 0.7,0.6,0.6,0.1,0.2 >} is an PNPGPCS but not an PNPSCS in X since PNPInt(PNPCl(A)) ⊈ A.

**3.18 Proposition**

PNPGSCS and PNPGPCS are independent to each other.

**3.19 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.5, 0.4,0.6,0.4,0.5 >, < a, 0.2,0.1,0.6,0.5,0.6 >}. Then the PNPS A = T is a PNPSCS but not a PNPGPCS in X since A ⊆ T but PNPPCl(A) = {< a, 0.5, 0.4,0.6,0.4,0.5 >, < a, 0.6, 0.5,0.6,0.1,0.2 >} ⊈ T.

**3.20 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.7, 0.6,0.6,0.2,0.3 >, < a, 0.9, 0.8,0.6,0,0.1 >}. Then the PNPS A = {< a, 0.6,0.5,0.6,0.3, 0.4 >, < a, 0.7,0.6,0.6,0.2, 0.3 >} is an PNPGPCS but not an PNPGSCS in X since A ⊆ T but PNPscl(A) = 1 ⊈ T.

**3.21 Remark**

The union of any two PNPGPCSs is not an PNPGPCS in general as seen in the following example.

**3.22 Example**

Let X = {a, b} be an PNPTS and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.6, 0.5,0.6,0.3,0.4 >, < a, 0.8,0.7,0.6,0.1, 0.2 >}. Then the PNPSs A = {< a, 0.1, 0,0.6,0.8,0.9 >, < a, 0.8,0.7,0.6,0.1,0.2 >}, B = {< a, 0.6,0.5,0.6,0.3, 0.4 >, < a, 0.7, 0.6,0.6.0.2,0.3 >} are PNPGPCSs but A ∪ B is not an PNPGPCS in X.

## IV. PENTAPARTITIONED NEUTROSOPHIC PYTHAGOREAN GENERALIZED PRE-OPEN SETS

In this section we introduce Pentapartitioned Neutrosophic Pythagorean generalized pre-open sets and studied some of its properties.

**4.1 Definition**

A PNPS A is said to be an Pentapartitioned Neutrosophic Pythagorean generalized pre-open set (PNPGPOS in short) in (X, τ) if the complement $A^c$ is an PNPGPCS in X.

The family of all PNPGPOSs of an PNPTS (X,τ) is denoted by PNPGPO(X).

**4.2 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.7, 0.3,0.6,0.1,0.2 >, < a, 0.6, 0.5,0.6,0.2,0.3 >}. Then the PNPS A = {< a, 0.8, 0.7,0.6,0.1,0.2 >, < a, 0.7, 0.6,0.6,0.1,0.2 >} is an PNPGPOS in X.

 

**4.3 Theorem**

For any PNPTS (X,τ), we have the following:

- Every PNPOS is an PNPGPOS
- Every PNPSOS is an PNPGPOS
- Every PNP$\alpha$OS is an PNPGPOS
- Every PNPPOS is an PNPGPOS.

But the converses is not true in general

**Proof:** Straight forward.

The converse of the above statements need not be true which can be seen from the following examples.

**4.4 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.2, 0.1,0.6,0.7,0.8 >, < a, 0.3,0.2,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.8, 0.7,0.6,0.1,0.2 >, < a, 0.7, 0.6,0.6,0.1,0.2 >} is an PNPGPOS in (X, τ) but not an PNPOS in X.

**4.5 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.6, 0.5,0.6,0.3,0.4 >, < a, 0.4, 0.3,0.6,0.5,0.6 >}. Then the PNPS A = {< a, 0.2,0.1,0.6,0.7, 0.8 >, < a, 0.7, 0.6,0.6,0.2,0.3 >} is an PNPGPOS but not an PNPSOS in X.

**4.6 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.4, 0.3,0.6,0.2,0.6 >, < a, 0.2,0.1,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.7,0.6,0.6,0.2, 0.3 >, < a, 0.8,0.7,0.6,0, 0.1 >} is an PNPGPOS but not an PNP$\alpha$OS in X.

**4.7 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.6, 0.5,0.6,0.3,0.4 >, < a, 0.5,0.4,0.6,0.4, 0.5 >}. Then the PNPS A = {< a, 0.7, 0.6,0.6,0.2,0.3 >, < a, 0.6, 0.3,0.6,0.2,0.4 >} is an PNPGPOS but not an PNPPOS in X.

**4.8 Theorem**

Let (X, τ) be an PNPTS. If A ∈ PNPGPO(X) then V ⊆ PNPInt(PNPCl(A)) whenever V ⊆ A and V is PNPCS in X.

**Proof:**

Let A ∈ PNPGPO(X). Then $A^c$ is an PNPGPCS in X. Therefore PNPPCl($A^c$) ⊆ U whenever $A^c$ ⊆ U and U is an PNPOS in X. That is PNPCl(PNPInt($A^c$)) ⊆ U. This implies $U^c$ ⊆ PNPInt(PNPCl(A)) whenever $U^c$ ⊆ A and $U^c$ is PNPCS in X. Replacing $U^c$ by V, we get V ⊆ PNPInt(PNPCl(A)) whenever V ⊆ A and V is PNPCS in X.

**4.9 Theorem**

Let (X, τ) be an PNPTS. Then for every A ∈ PNPGPO(X) and for every B ∈ PNPS(X), PNPPInt(A) ⊆ B ⊆ A implies B ∈ PNPGPO(X).

**Proof:**

By hypothesis $A^c$ ⊆ $B^c$ ⊆ (PNPPInt(A))$^c$ . Let $B^c$ ⊆ U and U be an PNPOS. Since $A^c$ ⊆ $B^c$ , $A^c$ ⊆ U. But $A^c$ is an PNPGPCS, PNPPCl( $A^c$ ) ⊆ U. Also $B^c$ ⊆ (PNPPInt(A))$^c$ = PNPPCl($A^c$). Therefore PNPPCl($B^c$) ⊆ PNPPCl($A^c$) ⊆ U. Hence $B^c$ is an PNPGPCS. Which implies B is an PNPGPOS of X.

**4.10 Remark**

The intersection of any two PNPGPOSs is not an PNPGPOS in general.

**4.11 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPTS be an PNPT on X, where T = {< a, 0.6,0.5,0.6,0.3, 0.4 >, < a, 0.8,0.7,0.6,0.5, 0.2 >}. Then the PNPSs A = {< a, 0.9, 0.8,0.6,0,0.1 >, < a, 0.2,0.1,0.6,0.7, 0.8 >} and B = {< a, 0.4, 0.3,0.6,0.5,0.6 >, < a, 0.3, 0.2,0.6,0.6,0.7 >} are PNPGPOSs but A ∩ B is not an PNPGPOS in X.

**4.12 Theorem**

An PNPS A of an PNPTS (X, τ) is an PNPGPOS if and only if F ⊆ PNPPInt(A) whenever F is an PNPCS and F ⊆ A.

**Proof:**

**Necessity:** Suppose A is an PNPGPOS in X. Let F be an PNPCS and F ⊆ A. Then $F^c$ is an PNPOS in X such that $A^c$ ⊆ $F^c$. Since $A^c$ is an PNPGPCS, we have PNPPCl($A^c$) ⊆ $F^c$. Hence (PNPPInt(A))$^c$⊆ $F^c$. Therefore F ⊆ PNPPInt(A).

**Sufficiency:** Let A be an PNPS of X and let F ⊆ PNPPInt(A) whenever F is an PNPCS and F ⊆ A. Then $A^c$ ⊆ $F^c$ and $F^c$ is an PNPOS. By hypothesis, (PNPPInt(A))$^c$ ⊆ $F^c$. Which implies PNPPCl($A^c$) ⊆ $F^c$. Therefore $A^c$ is an PNPGPCS of X. Hence A is an PNPGPOS of X.

**4.13 Corollary**

An PNPS A of an PNPTS $(X, \tau)$ is an PNPGPOS if and only if F ⊆ PNPInt(PNPCl(A)) whenever F is an PNPCS and F ⊆ A.

**Proof:**

**Necessity:** Suppose A is an PNPGPOS in X. Let F be an PNPCS and F ⊆ A.
Then $F^c$ is an PNPOS in X such that $A^c \subseteq F^c$. Since $A^c$ is an PNPGPCS,

We have PNPPCl( $A^c$) ⊆ $F^c$ . Therefore PNPCl(PNPInt( $A^c$ ) ⊆ $F^c$ .Hence $(\mathrm{PNPInt}(\mathrm{PNPCl}(A)))^c \subseteq F^c$.

Therefore F⊆ PNPInt(PNPCl(A)).

**Sufficiency:** Let A be an PNPS of X and let F ⊆ PNPInt(PNPCl(A)) whenever F is an

PNPCS and F ⊆ A. Then $A^c \subseteq F^c$ and $F^c$ is an PNPOS. By hypothesis,
$(\mathrm{PNPInt}(\mathrm{PNPCl}(A)))^c \subseteq F^c$.Hence PNPCl(PNPInt($A^c$)) ⊆ $F^c$ , which implies pcl($A^c$) ⊆

$F^c$. Hence A is an PNPGPOS of X.

**4.14 Theorem**

For an PNPS A, A is an PNPOS and an PNPGPCS in X if and only if A is an PNPROS in X.

**Proof:**

**Necessity:** Let A be an PNPOS and an PNPGPCS in X. Then pcl(A) ⊆ A. This implies PNPCl(PNPInt(A)) ⊆ A. Since A is an PNPOS, it is an PNPPOS. Hence A ⊆ PNPInt(PNPCl(A)). Therefore A = PNPInt(PNPCl(A)). Hence A is an PNPROS in X.

**Sufficiency:** Let A be an PNPROS in X. Therefore A = PNPInt(PNPCl(A)). Let A ⊆ U and U is an PNPOS in X. This implies pcl(A) ⊆ A. Hence A is an PNPGPCS in X.

## V. APPLICATIONS OF PENTAPARTITIONED NEUTROSOPHIC PYTHAGOREAN GENERALIZED PRE-CLOSED SETS

In this section we provide some applications of Pentapartitioned Neutrosophic Pythagorean generalized pre-closed sets.

**5.1 Definition**

An PNPTS $(X, \tau)$ is said to be an Pentapartitioned Neutrosophic Pythagorean ${}_pT_{1/2}$ ($PNP_pT_{1/2}$ in short) space if every PNPGPCS in X is an PNPCS in X.

**5.2 Definition**

An PNPTS (X, τ) is said to be an Pentapartitioned Neutrosophic Pythagorean $_{gp}T_{1/2}$ ($PNP_{gp}T_{1/2}$ in short) space if every PNPGPCS in X is an PNPPCS in X.

**5.3 Theorem**

Every $PNP_pT_{1/2}$ space is an $PNP_{gp}T_{1/2}$ space. But the converse is not true in general.

**Proof:**

Let X be an $PNP_pT_{1/2}$ space and let A be an PNPGPCS in X. By hypothesis A is an PNPCS in X. Since every PNPCS is an PNPPCS, A is an PNPPCS in X. Hence X is an $PNP_{gp}T_{1/2}$ space.

**5.4 Example**

Let X = {a, b} and let τ = {0, T, 1} be an PNPT on X, where T = {< a, 0.9, 0.8,0.6,0,0.1 >, < a, 0.9, 0.8,0.6,0,0.1 >}.Then (X, τ) is an $PNP_{gp}T_{1/2}$ space. But it is not an $PNP_pT_{1/2}$ space since the PNPS A = {< a, 0.2,0.1,0.6,0.7, 0.8 >, < a, 0.3, 0.2,0.6,0.6,0.7 >} is PNPGPCS but not an PNPCS in X.

**5.5 Theorem**

Let (X, τ) be an PNPTS and X is an $PNP_pT_{1/2}$ space then

i) Any union of PNPGPCSs is an PNPGPCS.
ii) Any intersection of PNPGPOSs is an PNPGPOS.

**Proof:**

i) Let $\{A_i\}_{i \in J}$ is a collection of PNPGPCSs in an $PNP_pT_{1/2}$ space (X, τ). Therefore every PNPGPCS is an PNPCS. But the union of PNPCS is an PNPCS. Hence the union of PNPGPCS is an PNPGPCS in X.

ii) It can be proved by taking complement in (i).

**5.6 Theorem**

An PNPTS X is an $PNP_{gp}T_{1/2}$ space if and only if PNPGPO(X) = PNPPO(X).

**Proof:**

**Necessity:** Let A be an PNPGPOS in X, then $A^c$ is an PNPGPCS in X. By hypothesis $A^c$ is an PNPPCS in X. Therefore A is an PNPPOS in X. Hence PNPGPO(X) = PNPPO(X).

**Sufficiency:** Let A be an PNPGPCS in X. Then $A^c$ is an PNPGPOS in X. By hypothesis $A^c$ is an PNPPOS in X. Therefore A is an PNPPCS in X. Hence X is an $PNP_{gp}T_{1/2}$ space.

## VI. CONCLUSION

In this paper, we have defined a Pentapartitioned Neutrosophic Pythagorean generalized pre-closed sets and a Pentapartitioned Neutrosophic Pythagorean generalized pre-open sets and its characterizations were discussed. Also we have studied applications of Pentapartitioned Neutrosophic Pythagorean generalized pre-closed sets namely Pentapartitioned Neutrosophic Pythagorean $_{p}T_{1/2}$ space and Pentapartitioned Neutrosophic Pythagorean $_{gp}T_{1/2}$ space.In future study, we will develop this set into continuous mappings and homomorphisms of Pentapartitioned Neutrosophic Pythagorean Topological Spaces.

## REFERENCES

[1] K. Atanassov, Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*.1986,volume 20 87-96.
[2] Broumi S, Smarandache F (2014) Rough Neutrosophic sets, *Ital J Pure Appl Math ,2014, volume* 32:493-502.
[3] Coker D., An introduction to Intuitionistic Fuzzy points, submitted to notes on IFS.
[4] R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic pythagorean Soft set, *IRJMETS*, 2021 , *Volume 3(2)*,905-914.
[5] R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic Pythagorean Set, *IRJASH*, 2021, *volume 3,* 62-82.
[6] R. Radha, A. Stanis Arul Mary. Heptapartitioned neutrosophic sets, *IRJCT, 2021 ,volume 2*,222-230.
[7] R. Radha, A. Stanis Arul Mary, F. Smarandache. Quadripartitioned Neutrosophic Pythagorean soft set, *International journal of Neutrosophic Science, 2021, volume14(1)*,9-23.
[8] R. Radha, A. Stanis Arul Mary. Neutrosophic Pythagorean soft set, *Neutrosophic sets and systems*, 2021, *vol 42*,65-78.
[9] R. Radha, A. Stanis Arul Mary, Pentapartitioned Neutrosophic Pythagorean Lie algebra, (Accepted)
[10] R, Radha, A. Stanis Arul Mary, Improved Correlation Coefficients of Quadripartitioned Neutrosophic Pythagorean sets using MADM, *Journal of Computational Mathematica*, volume 5(1),2021, 142-153
[11] R. Radha, A.Stanis Arul Mary, Pentapartitioned Neutrosophic Generalized semi-closed sets, *Journal of Computational Mathematica*,volume5(1),2021,123-131
[12] R. Radha, A. Stanis Arul Mary, Pentapartitioned Neutrosophic Pythagorean Semi-closed Sets (Communicated)
[13] R. Radha, A.Stanis Arul Mary, Pentapartitioned Neutrosophic Pythagorean Topological Spaces, Journal of XI' AN University of Architecture and Technology,
[14] Rama Malik, Surapati Pramanik. Pentapartitioned Neutrosophic set and its properties, *Neutrosophic Sets and Systems*, 2020, *Vol 36*,184-192.
[15] F.Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, *American Research Press, Rehoboth*.
[16] Wang H, Smarandache F, Zhang YQ, Sunderraman R Single valued neutrosophic sets, *Multispace Multistruct ,2010 ,volume* 4:410-413.
[17] R.R.Yager, Pythagorean Fuzzy subsets, In, Proc Joint IFSA World Congress and NAFIPS Journal, L.Zadeh , Fuzzy sets, *Information and Control* 1965, volume 8, 87-96.