NEUTROSOPHIC CHAOTIC B-CLOSED SET IN NEUTROSOPHIC CHAOTIC TOPOLOGICAL SPACE

Abstract

This article focuses on devising the novel idea of framing a b-open set in neutrosophic chaotic topological space. We further devote this article to the study of the properties posed by this newly developed set suitable examples are provided as and when required.

Keywords: neutrosophic chaotic set**,** neutrosophic chaotic topological space, neutrosophic chaotic open sets, neutrosophic chaotic b-closed set.

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I. INTRODUCTION

Zadeh [12] introduced fuzzy sets in 1965, allowing elements to have varying degrees of membership in the set. The real unit interval [0, 1] is where the membership degrees are found. Developed by Atnassov [1] in 1983, the intuitionistic fuzzy set (IFS) permits both membership and non-membership to the elements. In 1998, Smarandache [9] introduced the neutrosophic set by adding a single extra component to the IFS set. The truth, indeterminacy, and falsity membership functions are the three components of the neutrosophic set, in that order. This neutrosophic set aids in the efficient handling of the ambiguous and inconsistent data. P.ishwarya and K. Bagherathi introduced the concept of neutrosophic semi-open sets in neutrosophic topological space in 2016[4]. The concept of neutrosophic pre-closed in neutrosophic topological spaces and neutrosophic pre-open sets was introduced in 2017 by V. Venkateswara Rao and Y. Srinivasa Rao [11]. In 2018 P. Evanzalin Ebenanjar et al[3] introduced the concept of neutrosophic b-open sets in neutrosophic topological space The concept of chaotic function in general metric space was introduced by R.L.Devaney[2]. It has many applications in trafficforecasting, animation, computer graphics, medical field, image processing, etc. T.Thrivikraman and P.B. Vinod Kumar[10] defined chaos and fractals in general topological spaces. The idea of the fuzzy chaotic set was introduced by R.Malathi and M.K. Uma[5] in 2018. In [6] we introduced the concept of neutrosophic chaotic continuous functions. In this we extend the neutrosophic b-closed set in neutosophic chaotic topological space.

II. PRELIMINARIES

2.1 Definition [9] Let X be a universe. A Neutrosophic set ($\mathcal{NS}$) A on X can be defined as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

Where T_A , $I_A, F_A: U \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A\,(x) \leq 3$

Here,$T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancyand $F_A(x)$ is the degree of non-membership.

2.2 Definition [9] Let X be a non empty set, M = $<x,M^T,M^I,M^F>$ and V = $<x,V^T,V^I,V^F>$ be neutrosophic sets on X, and let $\{A_i : i\in J\}$ be an arbitrary family of neutrosophic sets in X, where $M_i = <x,M^T, M^I, M^F>$

(i) $M \subseteq V$ if and only if $M^T \leq V^T$, $M^I \geq V^I$ and $M^F \geq V^F$

(ii) M = V if and only if $M \leq V$ and $V \leq M$.

(iii) $\bar{M} = <x,M^F,1-M^I,M^T>$

(iv) $M \cap V = <x,M^T \wedge V^T,M^I \vee V^I,M^F \vee V^F>$

(v) $M \cup V = \langle x, M^T \vee V^T, M^I \wedge V^I, M^F \wedge V^F \rangle$

(vi) $\cup M_i = \langle x, \vee M_i^T, \wedge M_i^I, \wedge M_i^F \rangle$

(vii) $\cap M_i = \langle x, \wedge M_i^T, \vee M_i^I, \vee M_i^F \rangle$

(viii) $M - V = M \wedge \bar{V}$.

(ix) $0_N = \langle x,0,1,1 \rangle$; $1_N = \langle x,1,0,0 \rangle$.

2.3 Definition [8] A neutrosophic topology ($\mathcal{NT}$ for short) on a nonempty set X is a family τ of neutrosophic set in X satisfying the following axioms:

(i) $0_N, 1_N \in \tau$.

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.

(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a neutrosophic topological space (NTS for short) and any neutrosophic set in τ is called a neutrosophic open set(NOS for short) in X. The complement A of a neutrosophic open set A is called a neutrosophic closed set (NCS for short) in X.

2.4 Definition [8] Let (X, τ) be a neutrosophic topological space and $A = \langle X, A^T, A^I, A^F \rangle$ be a set in X. Then the closure and interior of A are defined by
$Ncl(A) = \cap\{K : K$ is a neutrosophic closed set in X and $A \subseteq K\}$,
$Nint(A) = \cup\{G : G$ is a neutrosophic open set in X and $G \subseteq A\}$.

2.5 Definition[7] Let X be a nonempty set and f : X → X be any mapping. Let α be any neutrosophic set in X. The neutrosophic orbit $O_f(\alpha)$ of α under the mapping f is defined as $O_{fT}(\alpha) = \{\alpha, f^1(\alpha), f^2(\alpha), ... f^n(\alpha)\}$, $O_{fI}(\alpha) = \{\alpha, f^1(\alpha), f^2(\alpha), ... f^n(\alpha)\}$, $O_{fF}(\alpha) = \{\alpha, f^1(\alpha), f^2(\alpha), ... f^n(\alpha)\}$ for $\alpha \in X$ and $n \in Z^+$.

2.6 Definition[7] Let X be a nonempty set and let f : X→ X be any mapping.

The neutrosophic orbit set of α under the mapping f is defined as $NO_f(\alpha) = \langle \alpha, O_{fT}(\alpha), O_{fI}(\alpha), O_{fF}(\alpha) \rangle$ for $\alpha \in X$, where $O_{fT}(\alpha) = \{\alpha \wedge f^1(\alpha) \wedge f^2(\alpha) \wedge ... \wedge f^n(\alpha)\}$, $O_{fI}(\alpha) = \{\alpha \vee f^1(\alpha) \vee f^2(\alpha) \vee ... \vee f^n(\alpha)\}$, $O_{fF}(\alpha) = \{\alpha \vee f^1(\alpha) \vee f^2(\alpha) \vee ... \vee f^n(\alpha)\}$.

2.7 Definition[6] Let X be a nonempty set and let f : X → X be any mapping. Then a neutrosophic set of X is called neutrosophic periodic set with respect to f if $f_n(\Upsilon) = \Upsilon$, for some $n \in Z^+$. smallest of these n is called neutrosophic periodic of X.

2.8 Definition [6] Let (X, T) be a neutrosophic topological space. Let f : X → X be any mapping. The neutrosophic periodic set with respect to f which is in neutrosophic topology τ is called neutrosophic periodic open set with respect to f. Its complement is called a neutrosophic periodic closed set with respect to f.

2.9 Notation P = ∩{neutrosophic periodic open sets with respect to f }.

2.10 Definition [6] Let (X, τ) be a neutrosophic topological space and $\lambda \in$ NF(X) (Where NF(X) is a collection of all nonempty neutrosophic compact subsets of X). Let f : X → X be any mapping. Then f is neutrosophic chaotic with respect to λ if

(i) cl NO_f (λ) = 1,

(ii) P is neutrosophic dense.

2.11 Example Let X = {a, b, c}. Define τ = {0, 1, μ_1, μ_2, μ_3, μ_4} where μ_1, μ_2, μ_3, μ_4 : X → [0,1] are defined as μ_1 (a) =<a,0.4,0.3,0.6>, μ_1 (b) =<b,0.8,0.7,0.2>, μ_1 (c) = <c,0.4,0.3,0.6>, μ_2 (a) = <a,0.4,0.3,0.6>, μ_2(b) =<b, 0.8,0.2,0.2>, μ_2 (c) = <c,0.5,0.2,0.5> μ_3 (a)=<a,0.8,0.2,0.2>, μ_3 (b)=<b,0.8,0.7,0.2>, μ_3 (c)=<c,0.6,0.2,0.4>, μ_4 (a)=<a,0.9,0.2,0.1>, μ_4 (b)=<a,0.8,0.7,0.2>, μ_4 (c)=<a,0.9,0.2,0.1>,

Let λ: X → I be defined as λ (a) =<a,0.3,0.2,0.7> (b) = <b,0.6,0.5,0.4> λ(c) =<c, 0.3,0.2,0.7>. Define f : X → X as f(a) = b, f(b) = c, f(c) = a. The neutrosophic orbit set of λ under the mapping f is defined as NO_f (λ) = λ ∩f(λ) ∩ f2(λ) ∩…⇒ NO_f (λ)(a) = <a,0.3,0.2,0.7>, NO_f (λ)(b) = <b,0.6,0.5,0.4>, NO_f (λ)(c) =<c, 0.3,0.2,0.7>. Therefore cl(NO_f (λ)) = 1. Here P(a) = <a,0.4,0.3,0.6>, P(b) =<b, 0.8,0.7,0.2> ,P(c) = <c,0.4,0.3,0.6> and cl (P) is neutrosophic dense. Hence f is neutrosophic chaotic with respect to λ.

2.12 Notation

(i) NC (λ) = {f: X → X / f is neutrosophic chaotic with respect to λ}.

(ii) NCH(λ) = {$\lambda \in$ NF(X) / NC(λ) ≠ ϕ}.

2.13 Definition A neutrosophic topological space (X, τ) is called a neutrosophic chaos space if NCH (λ) ≠ ϕ. If (X, τ) is neutrosophic chaos space then the element of the NCH(X) are called chaotic sets in X.

III. NEUTROSOPHIC CHAOTIC B-CLOSED SET IN NEUTROSOPHIC CHAOTIC TOPOLOGICAL SPACE

3.1 Definition: Let (X, τ) be a neutrosophic (neu-) chaos space. Let $\mathfrak{C}$ be the collection of neu- chaotic sets in X satisfying the following conditions:

(i) $0_{NC}, 1_{NC} \in \mathfrak{C}$,

(ii) If $A_1, A_2 \in \mathfrak{C}$,then $A_1 \cap A_2 \in \mathfrak{C}$

(iii) If $\{A_j : j \in J\} \subset \mathfrak{C}$, then $\bigcup_{j\in J} A_j \in \mathfrak{C}$.

Then $\mathfrak{C}$ is called the neu- chaotic topological space in X. The triple (X, τ, $\mathfrak{C}$) is called a neu- chaotic topological space. The element of $\mathfrak{C}$ are called neu- chaotic open sets. The complement of neu- chaotic open set is called neu- chaotic closed set.

3.2 Example: Let X = {a, b, c}. Define τ = {0, 1, μ_1, μ_2, μ_3, μ_4} where μ_1, μ_2, μ_3, μ_4 : X → [0,1] are such that μ_1 (a) =<a,0.4,0.3,0.6>, μ_1 (b) =<b,0.8,0.7,0.2>, μ_1 (c) = <c,0.4,0.3,0.6>, μ_2 (a) = <a,0.4,0.3,0.6>, μ_2(b) =<b, 0.8,0.2,0.2>, μ_2 (c) = <c,0.5,0.2,0.5> μ_3 (a)=<a,0.8,0.2,0.2>, μ_3 (b)=<b,0.8,0.7,0.2>, μ_3 (c)=<c,0.6,0.2,0.4>, μ_4 (a)=<a,0.9,0.2,0.1>, μ_4 (b)=<a,0.8,0.7,0.2>, μ_4 (c)=<a,0.9,0.2,0.1>.Let $\mathfrak{C} = \{0, 1, \mu_1, \mu_2, \mu_3\}$.Clearly (X,T, $\mathfrak{C}$) is called neu- chaotic topological space.

3.3 Definition: Let (X,T, $\mathfrak{C}$) be neu- chaotic topological space and A=<x, T_A,I_A,F_A> be neu- chaotic set in X. then the neu- chaotic interior and neu- chaotic closure are defined by

i) $int_{NC}(A) = \bigcup\{M/M \text{ is a } NCOS \text{ in } X \text{ and } M \subseteq A\}$,

ii) $cl_{NC}(A) = \bigcap\{N/N \text{ is a } NCCS \text{ in } X \text{ and } A \subseteq N\}$.

Note that for any neu- chaotic set A in (X,T, $\mathfrak{C}$) , we have $cl_{NC}(A^c)=(int_{NC}(A))^c$ and $int_{NC}(A^c)=(cl_{NC}(A))^c$.

It can be also shown that $cl_{NC}(A)$ is NCCS and $int_{NC}(A)$ is NVOS in X.

a) A is NCCS in X if and only if $cl_{NC}(A)=A$.
b) A is NCOS in X if and only if $int_{NC}(A)=A$.

3.4 Proposition: Let A be any neu- chaotic set in X. Then

i) $int_{NC}(1_{NC}-A) = 1_{NC}-(cl_{NC}(A))$ and
ii) $cl_{NC}(1_{NC}-A) = 1_{NC}-(int_{NC}(A))$

Proof:

i) By definition $cl_{NC}(A) = \bigcap\{N/N \text{ is a } NCCS \text{ in } X \text{ and } A \subseteq N\}$.

$$\begin{aligned} 1_{NC}-(cl_{NC}(A)) &= 1_{NC}-\bigcap\{N/N \text{ is a } NCCS \text{ in } X \text{ and } A \subseteq N\} \\ &= \bigcup\{1_{NC}-N/N \text{ is a } NCCS \text{ in } X \text{ and } A \subseteq N\} \\ &= \bigcup\{M/M \text{ is a } NCOS \text{ in } X \text{ and } M \subseteq 1_{NC}-A\} \\ &= int_{NC}(1_{NC}-A) \end{aligned}$$

ii) The proof is similar to (i).

3.5 Proposition: Let (X, τ, $\mathfrak{C}$) be neu- chaotic topological space and A, B be neu-chaotic sets in X. Then the following properties hold:

a) $int_{NC}(A) \subseteq A$
b) $A \subseteq cl_{NC}(A)$
c) $A \subseteq B \Rightarrow int_{NC}(A) \subseteq int_{NC}(B)$
d) $A \subseteq B \Rightarrow cl_{NC}(A) \subseteq cl_{NC}(B)$
e) $int_{NC}(int_{NC}(A))=int_{NC}(A)$
f) $cl_{NC}(cl_{NC}(A))=cl_{NC}(A)$
g) $int_{NC}(A \cap B) = int_{NC}(A) \cap int_{NC}(B)$
h) $cl_{NC}(A \cup B) = cl_{NC}(A) \cup cl_{NC}(B)$
i) $int_{NC}(1_{NC}) = 1_{NC}$
j) $cl_{NC}(0_{NC}) = 0_{NC}$

Proof:

(a),(c) and (i) are obvious, (e) follows from (a) g) From $int_{NC}(A \cap B) \subseteq (A)$ and $int_{NC}(A \cap B) \subseteq (B)$ we obtain $int_{NC}(A \cap B) \subseteq int_{NC}(A) \cap int_{NC}(B)$. On the other hand, from the facts $int_{NC}(A) \subseteq A$ and $int_{NC}(B) \subseteq B \Rightarrow int_{NC}(A) \cap int_{NC}(B) \subseteq A \cap B$ and $int_{NC}(A) \cap int_{NC}(B) \in \mathfrak{C}$ we see that $int_{NC}(A) \cap int_{NC}(B) \subseteq int_{NC}(A \cap B)$, for which we obtain the required result.

(a)-(j) They can be easily deduced from (a)-(i).

3.6 Definition. Let A be a neu- chaotic set of a neu- chaotic topological space. Then A is said to be neu- chaotic pre open [NCPO]set of X if there exists a neu- chaotic open set NCO such that $NCO \subseteq A \subseteq NCO(cl_{NC}(A))$.

3.7 Definition: A neu- chaotic set M=<x, T_M,I_M,F_M> in neutrosophic chaotic topological space (X, τ, $\mathfrak{C}$) is said to be

i) A neu- chaotic pre- open set if $M \subseteq int_{NC}(cl_{NC}(M))$ and neu- chaotic pre-closed set if $cl_{NC}(int_{NC}(M)) \subseteq M$.
ii) A neu- chaotic α-open set if $M \subseteq int_{NC}(cl_{NC}(Nint(M)))$ and neu- chaotic α-closed set if $cl_{NC}(int_{NC}(cl_{NC}(M))) \subseteq M$.
iii) neu- chaotic semi-open set $M \subseteq cl_{NC}(int_{NC}(M))$ and neu- chaotic semi-closed set if $int_{NC}(cl_{NC}(M)) \subseteq M$.
iv) neu- chaotic b-open set if $M \subseteq int_{NC}(cl_{NC}(M)) \cup cl_{NC}(int_{NC}(M))$ and neu- chaotic b-closed set $int_{NC}(cl_{NC}(M)) \cap cl_{NC}(int_{NC}(M)) \subseteq M$.

v) a neu- chaotic β-open set, if $M \subseteq cl_{NC}(int_{NC}(cl_{NC}(M)))$ and neu- chaotic β-closed set if $int_{NC}(cl_{NC}(int_{NC}(M))) \subseteq M$.

vi) neu- chaotic regular open set if $M = int_{NC}(cl_{NC}(AM))$ and neu- chaotic regular closed set, if $M = cl_{NC}(int_{NC}(M))$.

3.8 Definition Let (X, τ, ℭ) be a neu- chaotic topological space and M=<x, T_M,I_M,F_M> be a NCS in X. The neu- chaotic b interior of A and denoted by $bint_{NC}(M)$ is defined to be the union of all neu- chaotic b-open sets of X which are contained in M. The intersection of all neu- chaotic b-closed sets containing M is called the neu- b-closure of M and is denoted by $bcl_{NC}(M)$.

i) $bint_{NC}(M) = \cup\{U/U \text{ is a } NCbOS \text{ in } X \text{ and } U \subseteq M\}$,

ii) $bcl_{NC}(M) = \cap\{N/N \text{ is a } NCbCS \text{ in } X \text{ and } M \subseteq N\}$.

3.9 Theorem In a neu- chaotic topological space X

i) An arbitrary union of neu- chaotic b-open sets is a neu- chaotic b-open set.
ii) An arbitrary intersection of neu- chaotic b-cosed sets is a neu- chaotic b-closed set.

Proof:

i) Let $\{M_\alpha\}$ be a collection of neu- chaotic b-cosed sets. Then for each α, $M_\alpha \subseteq cl_{NC}(int_{NC}(M_\alpha)) \cup int_{NC}\big(cl_{NC}(M_\alpha)\big)$.Now

$\cup M_\alpha \subseteq \cup\, (cl_{NC}(int_{NC}(M_\alpha)) \cup int_{NC}\big(cl_{NC}(M_\alpha)\big)) \subseteq cl_{NC}(int_{NC}(\cup M_\alpha)) \cup int_{NC}\big(cl_{NC}(\cup M_\alpha)\big)$. Thus $\cup M_\alpha$ is a neu- chaotic b-open set.

ii) Similarly by taking complements.

3.10 Theorem.

i) Every neu- chaotic open set in the neu- chaotic topological space in X is neu- chaotic pre-open set in X.

ii) Every neu- chaotic pre-open set in the neu- chaotic topological spaces (X, τ, ℭ) is neu- chaotic b-open set in (X, τ, ℭ).

iii) Every neu- chaotic semi-open set in the neu- chaotic topological spaces (X, τ, ℭ) is neu- chaotic b-open set in (X, τ, ℭ).

iv) Every neu- chaotic α -open set in the neu- chaotic topological spaces (X, τ, ℭ) is neu- chaotic b-open set in (X, τ, ℭ).

v) Every neu- chaotic regular-open set in the neu- chaotic topological spaces (X, τ, $\mathfrak{C}$) is neu- chaotic b-open set in (X, τ, $\mathfrak{C}$).

vi) Every neu- chaotic β-open set in the neu- chaotic topological spaces (X, τ, $\mathfrak{C}$) is neu- chaotic b-open set in (X, τ, $\mathfrak{C}$).

Proof:

i) Consider M be neu- chaotic open set in neu- chaotic topological space. Then M=int_{NC}(M).Clearly M $\subseteq$ cl_{NC}(M) taking interior on both sides we get int_{NC}(M) $\subseteq$ int_{NC}(cl_{NC}(M)). Since M=int_{NC}(M), M$\subseteq$$int_{NC}$($cl_{NC}$(M)). A is a neu- chaotic pre-open set in X.

ii) Assune M be neu- chaotic pre-open set in a neu- chaotic topological space. Then M$\subseteq$ int_{NC}(cl_{NC}(M)) which implies M$\subseteq$$int_{NC}$($cl_{NC}$(M))$\cup$$int_{NC}(M)\subseteq$ int_{NC} (cl_{NC} (M))$\cup$$cl_{NC}$ (int_{NC} M). Hence M is a neu- chaotic b-closed sets.

iii) Consider M be neu- chaotic semi-open set in a neu- chaotic topological space. Then M$\subseteq$ cl_{NC}(int_{NC}(M)) which implies M $\subseteq$ cl_{NC}(int_{NC}(M)) $\cup$ int_{NC}(M) $\subseteq$ cl_{NC}(int_{NC}(M)) $\cup$ int_{NC}(cl_{NC}(M)). Hence M is a neu- chaotic b-closed sets.

iv) v) and vi) Proof is obvious from above Definition.

3.11 Remark. The converse of above theorem need not be true as shown by the following examples

3.12 Example Let X={x_1,x_2}. Define $f: X \to X$ as f(x_1)=x_2, f(x_2)=x_1. Let $\mathfrak{C} = \{0_{NC}, 1_{\mathrm{NC}}, \mu_1, \mu_2\}$ be neu- chaotic topology on X. Here μ_1 (x) =<(x_1,0.5,0.6,0.4)(x_2,0.3,0.2,0.5)>, μ_2 (x) = <(x_1,0.5,0.6,0.4)(x_2,0.3,0.2,0.5)> . Define A=<(x_1,0.5,0.4,0.3)(x_2,0.2,0.1,0.5)> Then the set A is neu- chaotic b-open set but not neu- chaotic regular open set. Since A = cl_{NC}(int_{NC}(A))=1_{NC}≠A.

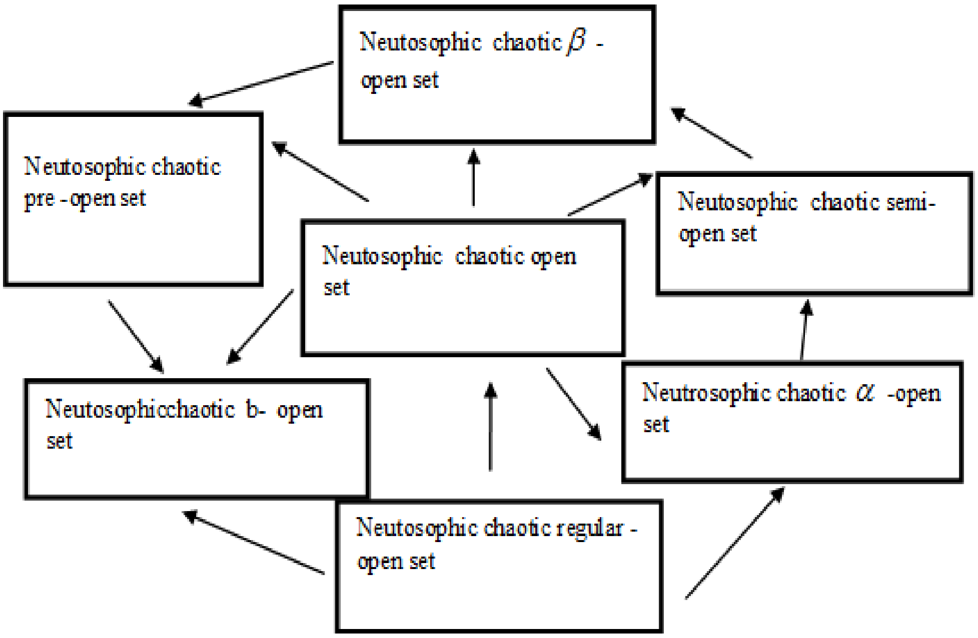
3.13 Example. Let X={x}. Define $f: X \to X$ as f(x)=x. Let $\mathfrak{C} = \{0_{NC}, 1_{\mathrm{NC}}, \mu_1, \mu_2\}$ be neu- chaotic topology on X. Here μ_1 (x) =<x,0.3,0.5,0.8>, μ_2 (x) = <x,0.4,0.6,0.7>. Let $\mathfrak{C} = \{0_{NC}, 1_{\mathrm{NC}}, \mu_1, \mu_2\}$ be neu- chaotic topology on X. A={<x,0.1, 0.3, 0.5>}. Then the set A is neu- chaotic b- open set A$\subseteq$$cl_{NC}$($int_{NC}$(A))$\cup$$int_{NC}$($cl_{NC}$(A)) $\subseteq$ $1_{Nc.}$ but not neu- chaotic semi-open set. Since A$\nsubseteq$ cl_{NC} (int_{NC}(A))$\nsubseteq$$0_{NC}$.

3.14 Example. Let X={x}. Define $f: X \to X$ as f(x)=x. Let $\mathfrak{C} = \{0_{NC}, 1_{NC}, \mu_1, \mu_2\}$ be neu- chaotic topology on X. Here μ_1 (x) =<x,0.5,0.6,0.5>, μ_2 (x) = <x,0.4,0.7,0.8>. Let $\mathfrak{C} = \{0_{NC}, 1_{NC}, \mu_1, \mu_2\}$ be neu- chaotic topology on X. A={<x,0.4, 0.4, 0.5>}.Then the set A is neu- chaotic b- open set but not neu- pre- open set. Since A$\not\subseteq$$int_{NC}$(N cl_{NC} (A))$\not\subseteq$<0.5, 0.6, 0.5>

3.15 Example. Let X={x}. Define $f: X \to X$ as f(x)=x. Let $\mathfrak{C} = \{0_{NC}, 1_{NC}, \mu_1, \mu_2\}$ be neu- chaotic topology on X. Here μ_1 (x) =<x,0.5,0.6,0.5>, μ_2 (x) = <x,0.4,0.7,0.8>. Let $\mathfrak{C} = \{0_{NC}, 1_{NC}, \mu_1, \mu_2\}$ be neu- chaotic topology on X. A={<x,0.4, 0.4, 0.5>}.Then the set A is neu- chaotic b- open set but not neu- α- open set. Since A$\not\subseteq$$int_{NC}$(N cl_{NC} (int_{NC} (A)))$\not\subseteq$<0.5, 0.6, 0.5>

3.12 Example Let X={x_1,x_2}. Define $f: X \to X$ as f(x_1)=x_2, f(x_2)=x_1. Let $\mathfrak{C} = \{0_{NC}, 1_{NC}, \mu_1, \mu_2\}$ be neu- chaotic topology on X. Here μ_1 (x) =<(x_1,0.5,0.6,0.5)(x_2,0.3,0.2,0.5)>, μ_2 (x) = <(x_1,0.4,0.8,0.5)(x_2,0.2,0.3,0.6)> . Define A=<(x_1,0.5,0.4,0.6)(x_2,0.5,0.8,0.9)> Then the set A is neu- chaotic β-open set but not neu- chaotic b-open set. Since A $\not\subseteq$ $cl_{NC}(int_{NC}(A)) \cup int_{NC}(cl_{NC}(A))$ $\not\subseteq$ <(x_1,0.4,0.7,0.8)(x_2,0.4,0.5,0.6)>.

3.11 Remark. The diagrammatic representation of above theorem.



3.15 Theorem: Let A be a neu- chaotic set in neu- chaotic topological space. then

i) $scl_{NC}(A) = A \cup int_{NC}(cl_{NC}(A))$ and

$sint_{NC}(A) = A \cap cl_{NC}(int_{NC}(A))$

ii) $pcl_{NC}(A) = A \cup cl_{NC}(int_{NC}(A))$ and

$pint_{NC}(A) = A \cap int_{NC}(cl_{NC}(A))$.

Proof:

i) $scl_{NC}(A) \supseteq int_{NC}(cl_{NC}(scl_{NC}(A)) \supseteq int_{NC}(cl_{NC}(A))$.

$A \cup scl_{NC}(A) = scl_{NC}(A) \supseteq A \cup int_{NC}(cl_{NC}(A))$.

So $A \cup int_{NC}(cl_{NC}(A)) \subseteq scl_{NC}(A)$ ----------------(1)

Also $A \subseteq scl_{NC}(A)$

$int_{NC}(cl_{NC}(A)) \subseteq int_{NC}(cl_{NC}(scl_{NC}(A)) \subseteq scl_{NC}(A)$.

$A \cup int_{NC}(cl_{NC}(A)) \subseteq scl_{NC}(A) \cup A \subseteq scl_{NC}(A)$ ------------------(2)

From (1) and (2), $scl_{NC}(A) = A \cup int_{NC}(cl_{NC}(A))$.

$sint_{NC}(A) = A \cap cl_{NC}(int_{NC}(A))$ can be proved by taking the complement of $scl_{NC}(A) = A \cup int_{NC}(cl_{NC}(A))$. This proves (i).

The proof for (ii) is analogous.

3.16 Theorem: Let A be a neu- chaotic set in neu-chaotic topological space. then

i) $bcl_{NC}(A) = scl_{NC}(A) \cap pcl_{NC}(A)$

ii) $bint_{NC}(A) = sint_{NC}(A) \cap pint_{NC}(A)$

Proof: Since $bcl_{NC}(A)$ is a neu- chaotic b-closed set.

We have $bcl_{NC}(A) \supseteq int_{NC}(cl_{NC}(bcl_{NC}(A)) \cap cl_{NC}(int_{NC}(bcl_{NC}(A)) \supseteq int_{NC}(cl_{NC}(A)) \cap cl_{NC}(int_{NC}(A))$ and also $bcl_{NC}(A) \supseteq A \cup int_{NC}(cl_{NC}(A)) \cap cl_{NC}(int_{NC}(A)) = scl_{NC}(A) \cap pcl_{NC}(A)$. The reverse inclusion is clear. Therefore $bint_{NC}(A) = sint_{NC}(A) \cap pint_{NC}(A)$.

Analogously (ii) can be proved.

3.17 Theorem: Let A be a neu- chaotic set in neu- chaotic topological space. Then

(i) $scl_{NC}(sint_{NC}(A)) = sint_{NC}(A) \cup int_{NC}(cl_{NC}(int_{NC}(A)))$

(ii) $sint_{NC}(scl_{NC}(A)) = scl_{NC}(A) \cap cl_{NC}(int_{NC}(cl_{NC}(A)))$

Proof: We have $scl_{NC}(sint_{NC}(A)) = sint_{NC}(A) \cup int_{NC}(cl_{NC}(sint_{NC}(A))) = sint_{NC}(A) \cup int_{NC}(cl_{NC}[A \cap cl_{NC}(int_{NC}(A)]) \subseteq sint_{NC}(A) \cup int_{NC}[cl_{NC}(A) \cap cl_{NC}(cl_{NC}(int_{NC}(A)))] = sint_{NC}(A) \cup int_{NC}[cl_{NC}(int_{NC}(A))]$

To establish the opposite inclusion we observe that,
$scl_{NC}(sint_{NC}(A)) = sint_{NC}(A) \cup int_{NC}(cl_{NC}(sint_{NC}(A)) \supseteq sint_{NC}(A) \cup int_{NC}(cl_{NC}(int_{NC}(A)))$.

Therefore we have $scl_{NC}(sint_{NC}(A)) = sint_{NC}(A) \cup int_{NC}(cl_{NC}(int_{NC}(A)))$.
This proves (i).

The proof for (ii) is analogous.

3.18 Theorem Let A be a neu- chaotic set in neu-chaotic topological space. Then

i) $pcl_{NC}(pint_{NC}(A)) = pint_{NC}(A) \cup cl_{NC}(int_{NC}(A))$

ii) $pint_{NC}(pcl_{NC}(A)) = pcl_{NC}(A) \cap int_{NC}(cl_{NC}(A))$

Proof: We have $pcl_{NC}(pint_{NC}(A)) = pint_{NC}(A) \cup cl_{NC}(int_{NC}(pint_{NC}(A))) = pint_{NC}(A) \cup cl_{NC}(int_{NC}[A \cap int_{NC}(cl_{NC}(A))] = pint_{NC}(A) \cup cl_{NC}[int_{NC}(A) \cap int_{NC}(int_{NC}(cl_{NC}(A)))] = pint_{NC}(A) \cup cl_{NC}(int_{NC}(A))$

To establish the opposite inclusion we observe that,

$pcl_{NC}(pint_{NC}(A)) = pint_{NC}(A) \cup int_{NC}(cl_{NC}(pint_{NC}(A)) \supseteq pint_{NC}(A) \cup int_{NC}(cl_{NC}(int_{NC}(A)))$.

Therefore we have $pcl_{NC}(pint_{NC}(A)) = pint_{NC}(A) \cup cl_{NC}(int_{NC}(A))$

This proves (i).

Analogously (ii) can be proved.

3.19 Theorem Let $(X, \tau, \mathfrak{C})$ be a neu- chaotic topological space. If A is a neu- chaotic open set and B is a neu- chaotic b- open set in X. Then A∩B is a neu- chaotic b- open set in X.

Proof: Let A be a neu- chaotic open set and B is a neu- chaotic b- open set.
Now, $M= A\cap B = int_{NC}(A) \cap bint_{NC}(A) \subseteq bint_{NC}(A) \cap bint_{NC}(A)= bint_{NC}(A \cap B)= bint_{NC}(M)$

(i.e) $M\subseteq bint_{NC}(M)$. But $bint_{NC}(M) \subseteq M$. Hence, $M= bint_{NC}(M)$. (i.e) M= A∩B is a neu- chaotic b- open set.

3.20 Theorem Let $(X, \tau, \mathfrak{C})$ be a neu-chaotic topological space. If A is a neu- chaotic α-open set and B is a neu- chaotic b- open set in X. Then A∩B is a neu- chaotic b- open set in X.

Proof: Let A be a neu- chaotic $\mathfrak{C}$- open set and B is a neu- chaotic b- open set.
Now, $M= A \cap B = \alpha\, int_{NC}(A) \cap bint_{NC}(A) \subseteq bint_{NC}(A) \cap bint_{NC}(A)= bint_{NC}(A \cap B)= bint_{NC}(M)$

(i.e) $M\subseteq bint_{NC}(M)$. But $bint_{NC}(M) \subseteq M$. Hence, $M= bint_{NC}(M)$. (i.e) M= A∩B is a neu- chaotic b- open set.

3.21 Theorem If A be a subset of a space (X, τ, 𝔈), then $bint_{NC}(bcl_{NC}(A))= bcl_{NC}(bint_{NC}(A))$.

Proof: LetA be a subset of a space (X, τ, 𝔈), Now, $bint_{NC}(bcl_{NC}(A))=sint_{NC}(bcl_{NC}(A))\cup pint_{NC}(bcl_{NC}(A)) = bcl_{NC}(sint_{NC}(A))\cup pint_{NC}(bcl_{NC}(A)) = scl_{NC}(sint_{NC}(A))\cup pint_{NC}(pcl_{NC}(A))$ ----------(1)

And $bcl_{NC}(bint_{NC}(A))=bcl_{NC}(sint_{NC}(A) \cup pint_{NC}(A)) = bcl_{NC}(sint_{NC}(A))\cup bcl_{NC}(pint_{NC}(A)) = scl_{NC}(sint_{NC}(A))\cup pint_{NC}(pcl_{NC}(A))$ --------------(2)

Hence from (1) and (2) we get $bint_{NC}(bcl_{NC}(A))= bcl_{NC}(bint_{NC}(A))$.

Hence the theorem.

IV. CONCLUSION

By introducing the aforementioned definitions into topological spaces that are chaotic and neutrosophic. By making this new idea more general, we can increase its reach. This would create new research opportunities within the current neutrosophic topological framework.

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