ENHANCING MEDICAL IMAGE DENOISING AND SEGMENTATION USING NEUTROSOPHIC DOMAIN ANALYSIS

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Abstract

Image segmentation is crucial for machine learning, pattern recognition, and image processing applications. A common imaging technique for identifying brain tumours and gathering detailed data on tumour type, location, size, identification, and detection is magnetic resonance imaging (MRI). In order to obtain a clean and easier to analyse image, several segmentation techniques are available. This work proposes a novel three-stage image processing approach that focuses on anomaly detection and denoising in particular. Three different filters are used in the first step to reduce image noise. Concurrently, the image is converted into the neutrosophic domain, which effectively removes intrinsic uncertainties from the data. A thorough performance study using the Root Mean Square Error (RMSE) and Peak Signal-to-Noise Ratio (PSNR) formulas is conducted after the subsequent neutrosophic domain processing. Two distinct approaches, namely, Fuzzy C Means and Fuzzy K Means, are used to segment the affected area. The research highlights the possibility of additional expansions. Finally, the region of interest is extracted using morphological techniques, producing improved segmentation. The suggested approach shows how neutrosophic domain processing can be improved for abnormality identification and image denoising. The study highlights the effectiveness of the Gaussian filter in the neutrosophic domain and establishes a foundation for future improvements in image segmentation and classification techniques that will be useful in the healthcare industry and other relevant fields.

Keywords: Image segmentation, neutrosophic sets, clustering, fuzzy c means, fuzzy k means**.**

I. INTRODUCTION

Mathematics is a body of knowledge that covers topics such as numbers, formulae and associated structures, shapes and the spaces in which they are contained, and quantities and their variations. These topics are covered by the four main branches of modern mathematics, namely number theory, algebra, geometry, and analysis, respectively. Regarding a standard definition for their academic field, mathematicians are divided. The natural sciences, engineering, medicine, economics, computer science, and the social sciences all require mathematics. The fundamental principles of mathematics are independent of any scientific experimentation, despite the fact that it is frequently used to model phenomena.

Images are vital to our lives in our visually-driven, modern society. Images are used in everything from taking pictures of special moments to using medical scans to assist doctors diagnose illnesses. It's similar to imparting human-like vision and comprehension of the visual world on machines. The magic of this technology allows for the creation of art, object recognition, and even better image quality. Picture correction is only one aspect of picture processing. It serves as the foundation for other cutting-edge technologies, including facial recognition. Image processing saves lives in the medical field. Consider MRIs, CT scans, and X-rays. These pictures can be extremely complex, yet they provide important information about your health. Image processing gives medical professionals a clearer view of your condition by helping them identify problems like tumours or fractured bones. Image processing and mathematics work together to produce more intelligent and better photos. This paper will examine the fascinating intersection of mathematics and images and how it affects our perception and comprehension of the surrounding visual environment.

Medical imaging [1,3,16] techniques play a major role in the diagnosis and understanding of diseases in the modern medical world. Images from CT scans, MRIs, X-rays, and ultrasounds offer crucial information about a patient's health. Making sense of these pictures, though, can be a difficult endeavour, particularly when attempting to locate and isolate certain structures or areas of interest.

This is the situation in which the idea of the neutrosophic domain [5] is useful. The study of indeterminacy, ambiguity, and uncertainty is known as neutrosophy, a relatively recent field in both philosophy and mathematics. It provides an innovative method for addressing the difficulties associated with segmentation in the context of medical picture analysis.

The process of dividing an image[9,14] into useful areas, such tumours in a brain scan or blood arteries in an angiography, is called segmentation. When addressing the inherent fuzziness and uncertainty in medical imaging, traditional approaches have their limits.

A more adaptable and sophisticated method [2] of addressing this issue is introduced by the "neutrosophic domain". It helps us deal with the ambiguity that appears when attempting to discern between various tissues or structures in a medical imaging by accepting indeterminacy. An interesting new path towards more precise and dependable medical image segmentation [10] is opened up by this innovative method.

The notion of neutrosophic domain will be examined in greater detail in this journal article, along with its potential uses in the field of medical image segmentation. the drawbacks of conventional techniques, the advantages of adding neutrosophy, and specific instances from actual applications where this strategy has worked well are highlighted. Our ultimate goal is to provide insight into a potentially fruitful area of medical image analysis that could enhance patient care and outcomes. In image processing three are different stages. The stages include pre-processing, image segmentation, feature extraction and Classification. In this proposed methodology, two segmentation process are being used. The article is divided into three sections: pre-processing, conversion of image to neutrosophic domain and finally segmentation.

II. NEUTROSOPHIC SET

The idea of uncertainty is expanded upon by neutrosophic sets. They address partial membership, hesitancy, and a third criterion known as the "neutrosophic component." An element can have an ambiguous status, which is neither full membership nor full non-membership, and also belong to a set to some extent or not at all. This component symbolises indeterminacy.

Take U to be a neutrosophic set and a Universe of discourse [7]. In U, A is contained. As T, I, and F are referred to as the neutrosophic components, an element x in set A is written as x (F, I, T). This is the manner in which element x (F, I, T) is a part of A. With respect to t varying in T, i varying in I, and f varying in F, the set has t% true, i% indeterminate, and f % false values [8].

1. Convert The Image Into Neutrosophic

Let U be a Universe of discourse and A is a set of U, which is composed by bright pixels. A neutrosophic image P_{NS} is characterized by three membership sets T, I, F. A pixel P in the image is defined as P (T, I, F). It belongs to A as the following way: It is t true in the set, i indeterminate in the set, and f false in the set, where t varies in T, i varies in I and f varies in F. Then the pixel P(i , j) in the image domain is converted into the neutrosophic set P_{NS}= {T(i , j), I(i , j).F(i , j)} . T(i , j), I(i , j).F(i , j) are the probabilities belong to white pixels set, indeterminate set and non-white pixels set respectively These are defined as:

$$T(i,j) = \frac{\overline{K}(i,j) - \overline{K}_{min}}{\overline{K}_{max} - \overline{K}_{min}} \tag{1}$$

$$\overline{K}_{max} = \max \overline{K}(i,j) \tag{2}$$

$$\overline{K}_{min} = \min \overline{K}(i,j) \tag{3}$$

$$\overline{K}(i,j) = \frac{1}{w \, X \, w} \sum_{m=i-\frac{w}{2}}^{i+\frac{w}{2}} \sum_{n=j-\frac{w}{2}}^{j+\frac{w}{2}} K(m,n) \tag{4}$$

$$I(i,j) = \frac{\delta(i,j) - \delta_{min}}{\delta_{max} - \delta_{min}} \tag{5}$$

$$\delta(i,j) = \text{abs}\,(K(i,j) - \overline{K}(i,j)) \tag{6}$$

$$\delta_{max} = \max \delta(i, j) \tag{7}$$

$$\delta_{min} = \min \delta(i, j) \tag{8}$$

$$F(i, j) = 1 - T(i, j) \tag{9}$$

where $\bar{K}(i, j))$ is the local mean value of the pixel of the image. δ (i, j) is the absolute value of difference between intensity K (i, j) and its local mean value $\bar{K}(i, j)$. Hence the image is converted from spatial domain to neutrosophic domain and the image becomes a 3D matrix such as $I_{ND} = [\ T_{ij} \quad I_{ij} \quad F_{ij}]$ with (m x n x 3) dimensions.

III. FILTERATION

This is the first stage in image processing. Denoising [4,6] is a process of eliminating noises or disturbances present in an image. This noise can appear as graininess, sporadic spots, or other flaws that detract from the image's clarity and sharpness. Mathematical ideas are used in the denoising process to identify and remove these defects. In this work three filters, namely, Median, Wiener and Gaussian [19,20] are being used and their performance is being calculated using some statistical values.

1. Median Filter

A non-linear digital filtering method called the median filter is frequently used to eliminate noise from images. One common pre-processing technique to enhance the outcomes of subsequent processing is noise reduction. In digital image processing, median filtering is particularly popular because, in some cases, it removes noise while maintaining edges. The image's useable detail is maintained using the median filter. The median filter looks at each pixel individually and at its close neighbours to decide if it is representative of its surroundings. The median of those values is used in place of the pixel value. After sorting all of the neighbouring pixel values numerically, the pixel under consideration is compared to the middle pixel value to determine the median.

2. Wiener Filter

Inverse filtering, also known as generalised inverse filtering, is a restoration technique for deconvolution that can be used to restore a picture that has been blurred by a well-known low pass filter. However, additive noise is especially prone to affecting inverse filtering. Because the restoration algorithm lowers one degradation at a time, it is possible to design it for each type of deterioration and then combine them with ease. Wiener filtering achieves an ideal compromise between noise smoothing and inverse filtering. Simultaneously, the extra noise and the blurring are reversed.

3. Gaussian Filter

A popular image processing filter for noise reduction and blurring is the Gaussian filter. It functions by using a Gaussian kernel, a 2D bell-shaped curve, to convolve the image. The image's level of blurring is determined by the Gaussian distribution's standard deviation (σ). It is mathematically denoted as:

$$D(p,n) = \frac{1}{2\pi\sigma^2} e^{-(p^2+n^2)/(2\sigma^2)}$$

Where D (p , n) denotes Gaussian kernel at position (p , n) , p and n denotes the co-ordinates of kernel and σ is the standard deviation which controls the amount of blurring.

IV. ERROR ESTIMATES

Metrics for evaluating and preserving image fidelity are crucial for a number of applications, including super-resolution, denoising, image compression, and other tasks. They offer an objective assessment of the perceived quality of the images and aid in identifying the degree of distortion or mistake that has occurred during image processing or compression. In this area, two of the most often used metrics are Root Mean Square (RMSE) and Peak Signal Noise Ratio (PSNR). RMSE gives an absolute error value, whereas PSNR conveys the error in decibels with respect to the highest potential signal power. These metrics are used to measure and objectively assess image quality, including the degree to which an image is accurately reproduced or processed from its original source.

1. Root Mean Square

The average difference between the pixel values in the original and processed images is quantified by RMSE. It represents the "average" distortion or error between the two pictures. It is mathematically represented as:

$$RMSE = \sqrt{\frac{1}{P\times N}\sum_{i=1}^{P}\sum_{j=1}^{N}(Original\ image(i,j) - Processed\ Image(i,j))^2}$$

....................(10)

2. Peak Signal-To-Noise Ratio

PSNR is the ratio of a signal's maximal potential strength to the amount of corrupting noise that degrades an image's quality. It measures how well an image can be reproduced and how much of a deviation from the original there is without compromising quality. Its mathematical formula is denoted as:

$$PSNR = 10.log_{10}\left(\frac{(Maximum\ possible\ pixel\ value)^2}{\frac{1}{P\times N}\sum_{i=1}^{P}\sum_{j=1}^{N}(Original\ Image(i,j) - Processed\ Image(i,j))^2}\right)$$

................(11)

Better image quality is indicated by a greater PSNR value, which also shows a smaller discrepancy between the original and processed image. In order to minimize quality loss and achieve larger compression ratios, it is frequently employed in image and video compression.

V. IMAGE SEGMENTATION

The process of dividing a digital image into several image segments [18] , often referred to as image regions or image objects (sets of pixels), is known as image segmentation in digital image processing and computer vision. Segmentation is to transform an image's representation into something more understandable and simpler to examine. Finding boundary (lines, curves, etc.) and objects in images typically requires image segmentation. More specifically, image segmentation is the process of giving each pixel in an image a label so that those pixels that have the same label have similar properties.

Image segmentation yields a series of contours taken from the image or a set of segments that together encompass the full image. Regarding any characteristics or computed property, such as colour, intensity, or texture, every pixel in a region is comparable to every other pixel. Regarding the same characteristic, adjacent patches differ greatly in colour. Applying image segmentation to a stack of photos, common in medical imaging, yields contours that, when combined with geometry reconstruction methods such as marching cubes, can be used to generate 3D reconstructions.

There are three types of segmentation

- **Semantic segmentation** is a method that determines the belonging class of each pixel. For instance, in an image with multiple people, the background pixels will be labelled as backdrop and all the pixels that belong to humans will have the same class id.

- A technique called **instance segmentation** locates the precise instance of an item that belongs to each pixel. Every unique object of interest in the picture is recognised by it. For instance, when every person in a figure is represented as a separate item.

- Segmentation by both semantic and instance is combined in panoptic segmentation. **Panoptic segmentation** is a method that determines the belonging class for each pixel, just like semantic segmentation. Panoptic segmentation additionally distinguishes between several instances of the same class, much like instance segmentation does.

In this paper two types of segmentation are used namely Fuzzy C Means (FCM) and Fuzzy K Means.

1. Fuzzy K Means

An enhancement on the classic K-means clustering technique [17] that permits fuzziness in cluster assignments is called fuzzy K-means. It is applied to image segmentation in medical image processing, where the objective is to divide an image into distinct areas or objects. Fuzzy K-means is helpful in situations where pixels aren't sure they belong to a single cluster because it allocates each pixel to many clusters with differing degrees of membership.
Algorithm:

Step 1: Initialization: Determine each pixel's degree of belonging to each cluster centre by comparing its similarities to each cluster centre. A membership function, usually a fuzzy membership function, is used to do this.

- Choose the number of clusters
- Randomly initialize the clusters centres
- Assign an initial degree of membership (between 0 and 1) for each pixel in the image for each clusters.

Step 2: Update Membership:

- Determine each pixel's degree of belonging to each cluster centre by comparing its similarities to each cluster centre. A membership function, usually a fuzzy membership function, is used to do this.

Step 3: Update Cluster Centres:

- Recalculate the cluster centres based on the degree of membership of all pixels.

Step 4: Repeat Steps 2 and 3 until the cluster centres and memberships no longer change significantly.

Mathematical Representation: In Fuzzy K- Means, each pixel w and cluster x, the degree of membership $I_F(w, x)$ is represented as,

$$I_F(w,x) = \frac{1}{\sum_{r=1}^{G}\left[\left(\frac{Distance\ from\ mpixel\ w\ to\ cluster\ centre}{Distance\ from\ pixel\ w\ to\ all\ clusters\ centres}\right)^{(2/(h-1))}\right]}$$

Where, h be the weighted exponent that controls the degree of fuzziness, G denotes number of clusters, Distance means the Euclidean distance is used as a measure of dissimilarity.

2. Fuzzy C Means

A clustering technique called fuzzy C-means (FCM) [15] is utilised for segmentation of images in a variety of domains, including medical image processing. It is a development of the classic K-means clustering technique, which places data points in clusters according to the similarity they have in common with cluster centres. By enabling "fuzziness" or degrees of membership, FCM, on the other hand, permits each data point to be a part of many clusters with different degrees of membership. FCM is used in the field of medical image processing for segmenting pictures, like CT or MRI scans, in order to distinguish between various tissue types or structures.

Algorithm:

Step 1: Initialization:

- Choose number of Clusters
- Randomly initialize the cluster centres and the degree of membership for each pixel in the image.

Step 2: Update Membership:

- Determine each pixel's degree of belonging to each cluster centre by comparing its similarities to each cluster centre. Fuzzy membership functions, in particular, are commonly used for this purpose.

Step 3: Update Cluster Centres:

- Recalculate the cluster centres by considering the degree of membership of all pixels for each cluster.

Step 4: Repeat Steps 2 and 3 until the cluster centres and membership no longer change significantly.

Mathematical formula:

- **Fuzzy membership**

$$\mu_{wx} = 1 \Big/ \sum_{t=1}^{c} (d_{wx}/d_{wt})^{(2lm-1)}$$

d_{wx} represents the distance between pixel w and the cluster centre x, d_{wt} denotes distance from pixel I to all cluster centres.

- Compute the fuzzy centre’s,

$$v_j = \left(\sum_{i=1}^{n} (\mu_{ij})^m x_i\right) \Big/ \left(\sum_{i=1}^{n} (\mu_{ij})^m\right), \forall\, j = 1,2, \dots .c$$

- Objective function

$$J(U,V) = \sum_{i=1}^{n} \sum_{j=1}^{c} (\mu_{ij})^m \|x_i - v_j\|^2$$

VI. MORPHOLOGICAL OPERATORS

A group of image processing methods known as morphological operators [11,12,13] are derived from the mathematical theory of morphology, which addresses the structure and form of things. The shapes and structures found in images can be altered and examined using these operators. In computer vision and image processing, they are especially helpful for tasks like object analysis, feature extraction, and image preparation. There are basically four basic operators namely: Opening, Closing, Dilation and Erosion. In this paper two basic operators are used namely dilation and erosion.

1. Dilation

Dilation thickens or enlarges an object's border in an image. The input image is moved across a small binary image that serves as a structural element. The matching pixel in the output image is set to 1 if any portion of the structuring element overlaps with an object in the image. Dilation is used to close small gaps or holes in things, connect adjacent objects, and make objects more noticeable. Let F_1 be the input image and S_1 be the structuring element then dilation is mathematically notated as

$$(I_1 \oplus F_1)(h,k) = \vee \{I_1(h-h', k-k') \big| (h',k') \text{ } is \text{ } in \text{ } F_1\}$$

2. Erosion

Erosion causes an image's object boundaries to become smaller or more eroded. A structuring element is moved across the input image in this process. All of the structuring element's components must overlap with the matching object in the input image for a pixel to be set to 1 in the output image. Erosion is used to isolate particular features, separate items that are touching or overlapping, and eliminate tiny noise. Let F_1 be the input image and S_1 be the structuring element then erosion is mathematically notated as

$$(I_1 \ominus F_1)(h,k) = \wedge \{I_1(h+h', k+k') \big| (h',k') \text{ } is \text{ } in \text{ } F_1\}$$

VII. WORK PROCEDURE

This paper is divided into four major stages. In this paper, MR image is taken. The main objective is to segment the affected region that is to highlight the tumor affected area separately.

Stage 1: The first stage also known as the pre-processing in image processing. In this, the disturbances or noises present in an image is refined. In order to refine an image an appropriate filter has to be chosen. In this proposed work three types of filters are being used.

Stage 2: The next stage is converting an image into neutrosophic domain using the formula given in equation (1), (5) and (9).

Stage 3: The performance of filters is calculated with the help of statistical formulas given in equation (10) and (11).

Stage 4: The final stage is segmentation. In this work two different process namely, Fuzzy C Means and Fuzzy K Means algorithm are applied. Both are applied for segmenting the affected region in MR Image

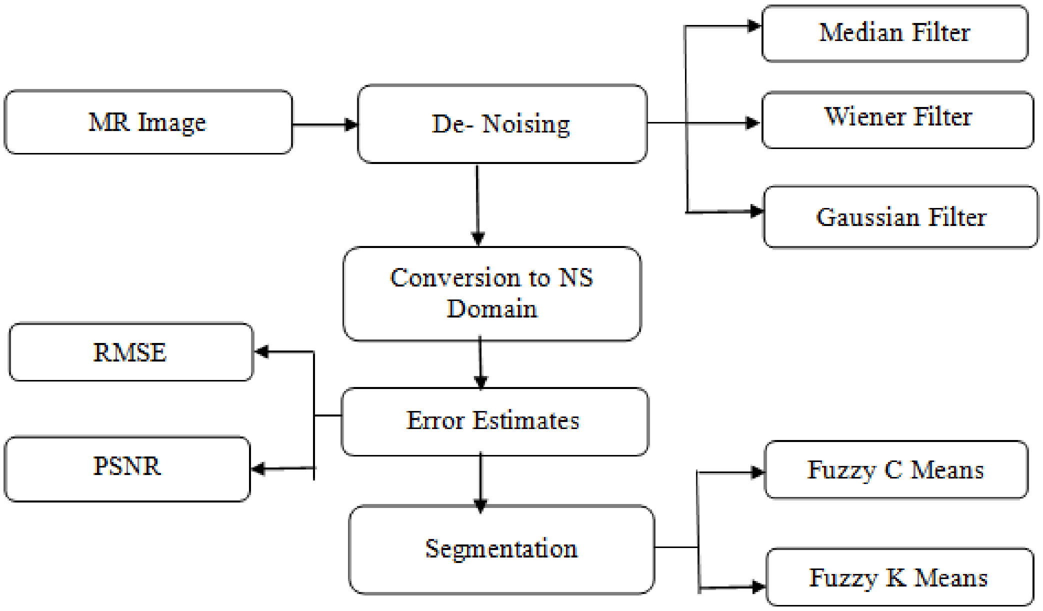
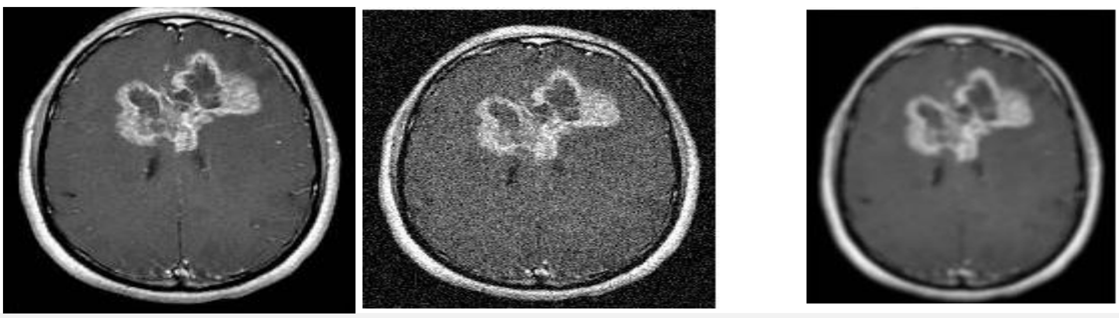


Figure 1: Work Flow of the Paper

VIII. RESULT AND ANALYSIS

The proposed methodology is implemented in MATLAB and following results are observed. For this proposed methodology MR Image is considered. Table 1 and Table 2 represents the performance of different filters. The performance is calculated using the statistical formulas RMSE and PSNR. Also, from the analysis it is obtained that Gaussian filter performance is better when compared to other two filters. The PSNR value is high when compared to other filters and also the RMSE values are low when compared to other two filters. Once the pre-processing stage is completed, the image is converted to neutrosophic domain and the results are shown in Fig. 2. Also, in fig. 3. The conversion to neutrosophic domain images and the concatenated image is shown.

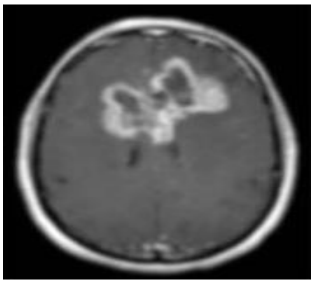


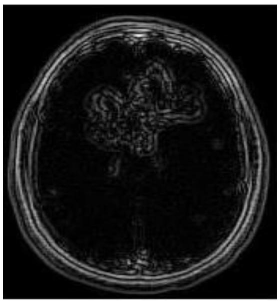
(a) Original Image

(b) Noise Image with 0.04 density

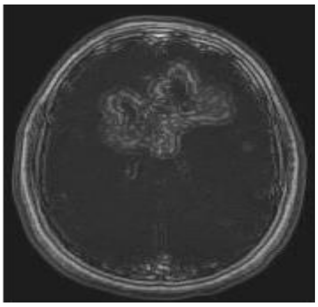
(c) Denoised Image (Gaussian Filter)

Figure 2: Represents Pre-Processing Stage





(a) True (b) False (c) Indeterminacy



(d) Final Image in Neutrosophic Domain

Figure 3: Represents Image in NS Domain

Table 1: PSNR Value of Different Filters

Noise Density	PSNR		
	Median	**Wiener**	**Gaussian**
0.01	13.4897	13.2953	**13.8851**
0.02	13.2985	13.0649	**13.7182**
0.03	13.1383	12.8742	**13.5173**
0.04	12.9069	12.7028	**13.2967**
0.05	12.6726	12.47	**13.1137**
0.06	12.518	12.2655	**12.9035**
0.07	12.2953	12.0568	**12.7094**
0.08	12.0728	11.8413	**12.4843**
0.09	11.8979	11.6424	**12.2973**
0.1	11.7136	11.4393	**12.0873**

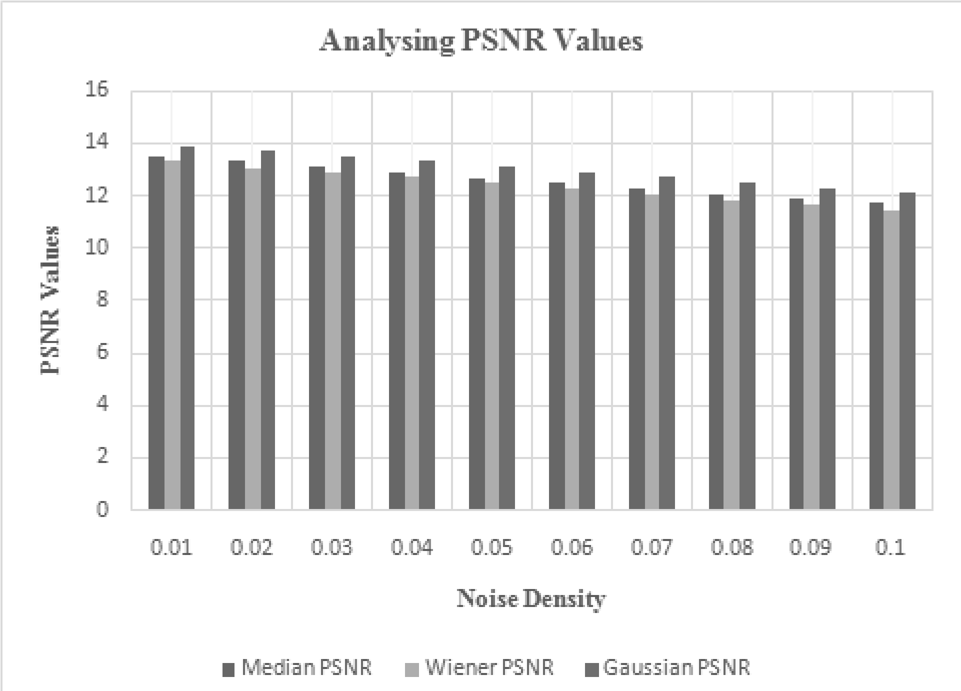


Figure 4: Graphical Representation of PSNR Values

Table 2: PSNR Value of Three Different Filters

Noise Density	RMSE		
	Median	Wiener	Gaussian
0.01	0.2116	0.2164	**0.2022**
0.02	0.2163	0.2222	**0.2061**
0.03	0.2203	0.2271	**0.2109**
0.04	0.2263	0.2317	**0.2164**
0.05	0.2325	0.238	**0.221**
0.06	0.2366	0.2436	**0.2264**
0.07	0.2428	0.2496	**0.2315**
0.08	0.2491	0.2558	**0.2376**
0.09	0.2542	0.2617	**0.2427**
0.1	0.2596	0.2679	**0.2487**

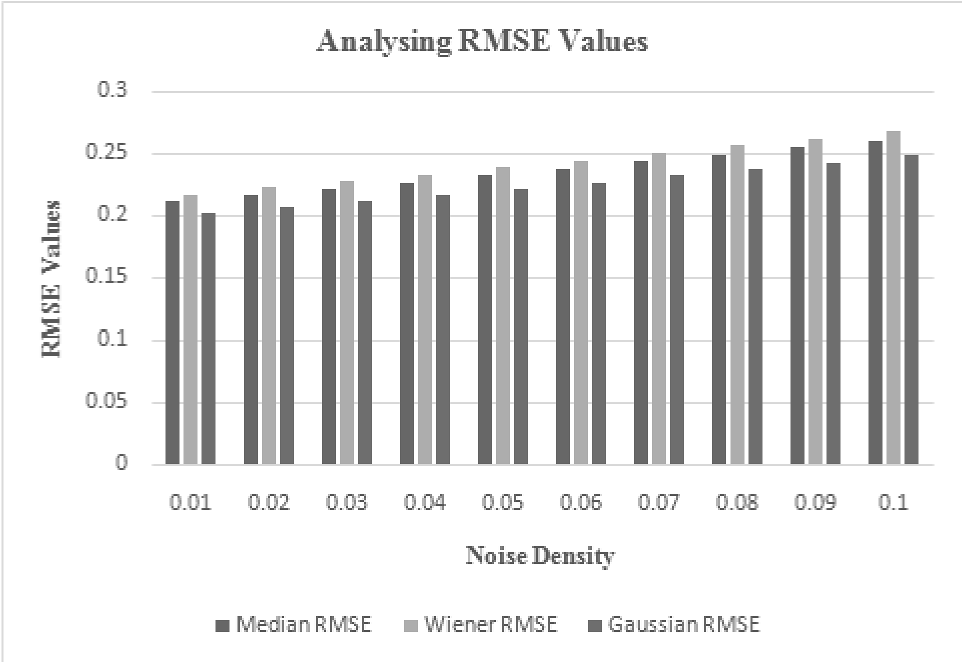
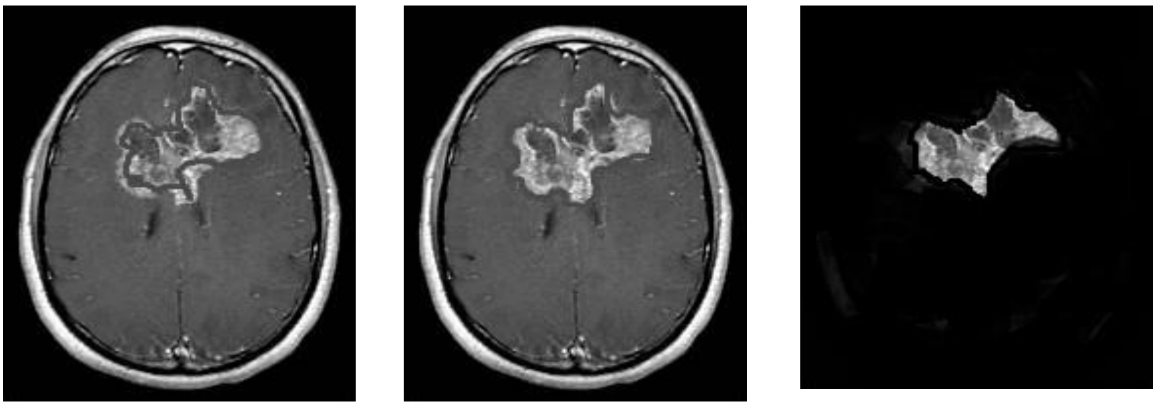


Figure 5: Graphical Representation of RMSE Values

The image is converted into neutrosophic domain and then the segmentation is performed. In this article, two segmentation methodologies are proposed. The resultant processed images are shown in Fig.6.



(a) Fuzzy K Means (b) Fuzzy C Means (c) Tumor region

Figure 6: Segmented Image

IX. CONCLUSION AND FUTURE WORK

In proposed methodology is divided into three stages. Firstly, the image is denoised using three different types of filters. Also, the image is converted to neutrosophic domain in order to remove uncertainties present in the images. After the image is processed in neutrosophic domain the performance analysis is performed using the formulas RMSE and PSNR. The original image and the reconstructed image are compared and concluded that the best reconstructed image is obtained when gaussian filtered image is processed in neutrosophic domain. Thus, gaussian filter works better when compared to median and wiener filter. the next stage is to segment the affected region. Two methodologies were adapted and the affected region is segmented. This work can be further extended by analysis the performance of different segmentation process by some statistical values. Further, it is possible to classify the image as normal or abnormal images using some classifier techniques.

REFERENCES

[1] Ahmad El Alla Oui, M'barek Nasri,"Medical image segmentation by marker controlled watershed and mathematical morphology",The International Journal of Multimedia & Its Applications (IJMA), vol. 4,No.3, June 2012. http://doi.org/10.5121/ijma.2012.4301

[2] Ashutosh Kumar Chaubey,"Comparision of the Local and global thresholding methods in image segmentation", World Journal of Research and Review (WJRR)ISSN:2455-3956, Vol. 2, Issue 1,January 2016,pages 01-04. https://www.wjrr.org/download_data/WJRR0201009.pdf

[3] Ayushdogra, Parvinder Bhalla, "Image sharpening by gaussian and butterworth high pass filter", Biomedical & Pharmacology Journal. Vol7(2),707-713(2014). https://dx.doi.org/10.13005/bpj/545

[4] Farah H.Zawaideh,Oais M.Yusuf,Firas H. Zawaideh,"Comparison between butterworth and gaussian high-pass filters using an enhanced method", ILCSNS International Journal of Computer Science and Network Security, Vol.17, No. 7,July 2017. http://paper.ijcsns.org/07_book/201707/20170716.pdf

[5] F. Samarandache," A unifying field in logics neutrosophic logic, in neutrosophy, neutrosophic Set, neutrosophic probability", 3rd edition, American Research Press, 2003. https://arxiv.org/pdf/math/0101228

[6] Jiying Yuan, Guojin He,"Application of an anisotropic diffusion based preprocessing filtering algorithm for high resolution remote sensing image segmentation",IEEE 2008. https://doi.org/10.1109/CISP.2008.318

[7] J.Mohan, Krishnaveni , Yanhui Guo," A neutrosophic approach of MRI denoising", ICIIP,(2011). https://doi.org/10.1109/ICIIP.2011.6108880

[8] .J.Mohan,J,Thilaga Shri, A.P Chandra, V. Krishnaveni,., Yanhui Guo, "Evaluation of neutrosophic set approach filtering technique for image denoising " ,IJMA , Vol.4, No.4, August (2012). http://doi.org/10.5121/ijma.2012.4407

[9] K.Parvati, B.S.Prakasa Rao,M.Mariya Das," Image segmentation using gray-scale morphology and marked-controlled watershed transformation", Hindawi Publishing Corporation Discrete Dynamics in Nature and Society, vol. 2008. https://doi.org/10.1155/2008/384346

[10] M.A.Dorairangaswamy,"A novel invisible and blind watermarking scheme for copyright protection of digital images", IJCSNS International Journal of Computer Science and Network Security, vol. 9, no. 4, (2009). http://paper.ijcsns.org/07_book/200904/20090411.pdf

[11] Shweta Pandav," Brain Tumor extraction using marker controlled watershed segmentation", International Journal of Engineering Research & Technology (IJERT),vol. 3 Issue 6, June 2014. http://doi.org/10.17577/IJERTV3IS061690

[12] P.Soille,"Morphological image analysis: principles and applications", Second Edition,Springer,2004.

[13] S.Ravi,A.M Khan, "Morphological operations for image processing:understanding and its applications",National Conferece on VLSI, Signal Processing & Communications, December 2013.

[14] Vincent.L, Soille P "Watersheds in Digital Spaces: an efficient algorithm based on immersion simulations", IEEE Transactions on Pattern Analysis and Machine Intelligence, 13(6),1991,pp 583-598.

[15] Weiling Cai, Songcan Chen, Daoqiang Zhang,"Fast and robust fuzzy c-means clustering algorithms incorporating local information for image segmentation", Elseveir, Vol 40, March 2007.

[16] S. Santhosh Kumar, James Albert," Medical image segmentation using kernel weighted fuzzy clustering (KWFC) algorithm", International Journal of Inventions in Compuer Science and Engineering, Vol 1 Issue 9, October 2014.

[17] Maria Brigida Ferraro, "Fuzzy k-means: history and applications", Elsevier Journal of Econometrics and Statistics, November 2021. https://doi.org/10.1016/j.ecosta.2021.11.008

[18] Zhaobin Wang, E.Wang, Ying Zhu, "Image segmentation evaluation: a survey of methods", Journal of Artificial Intelligence Review, April 2020.

[19] Esam M.A. Hussein, “9-Preprocessing of Measurements”, Journal of Computed Radiation Imaging, pp 97 -123, 2011. https://doi.org/10.1016/B978-0-12-387777-2.00009-4
[20] Horst Haubecker, Hamid R, Tizhoosh, “16- Fuzzy Image Processing”, Journal of Computer Vision and Applications, pp 541-576, 2000. https://doi.org/10.1016/B978-012379777-3/50017-0