TRANSIENT STABILITY ANALYSIS OF INDUCTION MOTOR DRIVE USING NONLINEAR MODEL

Abstract

The stability analysis of induction motor normally uses conventional steady state torque-speed characteristics. Study of stability during transient conditions is rather rare in literature. Rigorous analysis for stability based on full nonlinear dynamical model is lacking. For this purpose, stability analysis using the Lyapunov's theorem is essentially required. Global asymptotic stability for induction motor drive using Lyapunov criteria is analyzed using the full nonlinear dynamical model. The transient model is considered in stationary α-β reference frame about steady state operating point. Equations are derived for energy and power balance. The equations can be used to obtain an appropriate candidate for Lyapunov function for stability analysis. Global asymptotic stability condition in the sense of Lyapunov is derived at any possible speed, with load and without load, with variations in parameters and frequency. These generalized conditions of stability for any operating speed, load, frequency and parameters with a case study for confirmation are the outcomes.

Keywords: Stationary reference frame, Induction motor, Decoupling control, Global stability, Lyapunov theorem, Energy balance equation, Power balance equation

Author

Dr. Kanungo Barada Mohanty
Senior Member IEEE (USA), FIE(I), FIETE
Professor
Department of Electrical Engineering
National Institute of Technology Rourkela
Rourkela, India.
E-mail: kbmohanty@nitrkl.ac.in

I. INTRODUCTION

Induction motor offers a nonlinear, coupled and multivariable dynamics. Induction motor control has advanced from scalar control techniques to vector control [1]-[3], and decoupling control through state feedback linearization [4]-[10]. On the basis of these relatively new control strategies, a lot of modern controllers and estimation techniques, [8] have been applied. But, most of the researchers try, experiment and succeed to apply these modern control and estimation techniques without sufficient theoretical study of stability. The stability analysis of induction motor normally uses conventional steady state torque-speed characteristics [9]-[10]. Study of stability during transient conditions is rare in literature. Rigorous analysis for stability based on full nonlinear dynamical model is essentially required.

The state feedback linearization and vector control techniques are successfully implemented. They assure complete decoupling between motor speed and flux with certain motor physical parameters. But, some of the control techniques are sensitive to variations in speed, frequency, parameter and load. So, it is pertinent to analyze motor sustaining capability at steady state and transient durations. For this reason stability analysis using the Lyapunov's criteria is essentially required. Such a work is reported [11], with induction motor model in synchronously rotating reference frame. But, main drawback of synchronously rotating reference frame model, is requirement of synchronous angle, θ_e, which is obtained from a phase locked loop (PLL) and integrating the synchronous speed, ω_e. This means additional cost and complexity of the system, for predicting the stability on-line. This drawback is not present, with induction motor model in stationary reference frame. Such type of global asymptotic stability analysis using Lyapunov's theorem for induction motor drive is presented in this paper. Global asymptotic stability using Lyapunov's theorem for induction motor drive is discussed taking the full nonlinear transient model in stationary *α-β* reference frame, about a steady state operating point. Considering frequency, synchronous speed, load and motor parameters, conditions of stability are derived. The stability using Lyapunov approach is studied considering variations in frequency, speed, load and motor parameters. This work presents a theoretical demonstration of the stability analysis of the induction motor drive system utilizing the Lyapunov's stability theorem [4].

II. STATE-SPACE MODEL NEAR STEADY STATE OPERATING POINT

Many control schemes are developed for the induction motor drive using its model in stationary (α-β) reference frame with stator current components ($i_{\alpha s}$, $i_{\beta s}$), rotor flux components ($\psi_{\alpha r}$, $\psi_{\beta r}$) and speed (ω_r) as variables [6]-[10]. The mathematical model is presented below.

$$\dot{x} = f(x) + bu \tag{1}$$

where, $x = [i_{\alpha s}, i_{\beta s}, \psi_{\alpha r}, \psi_{\beta r}, \omega_r]^T$, $u = [u_{\alpha s}, u_{\beta s}, T_l]^T$, where

$$f(x)=\begin{bmatrix} -\frac{1}{\sigma L_s}\left(R_s+\frac{L_m^2}{L_r^2}R_r\right)i_{\alpha s}+\frac{1}{\sigma L_s}\frac{L_m R_r}{L_r^2}\psi_{\alpha r}+\frac{n_p L_m}{\sigma L_s L_r}\omega_r\psi_{\beta r} \\ -\frac{1}{\sigma L_s}\left(R_s+\frac{L_m^2}{L_r^2}R_r\right)i_{\beta s}+\frac{1}{\sigma L_s}\frac{L_m R_r}{L_r^2}\psi_{\beta r}-\frac{n_p L_m}{\sigma L_s L_r}\omega_r\psi_{\alpha r} \\ -\frac{R_r}{L_r}\psi_{\alpha r}-n_p\omega_r\psi_{\beta r}+\frac{L_m R_r}{L_r}i_{\alpha s} \\ -\frac{R_r}{L_r}\psi_{\beta r}+n_p\omega_r\psi_{\alpha r}+\frac{L_m R_r}{L_r}i_{\beta s} \\ -\frac{B}{J}\omega_r+\frac{K_T}{J}(i_{\beta s}\psi_{\alpha r}-i_{\alpha s}\psi_{\beta r}) \end{bmatrix} \qquad b=\begin{bmatrix} \frac{1}{\sigma L_s} & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{J} \end{bmatrix}$$

where, suffixes (α, β) denote the direct and quadrature axis equivalent components in the stator mounted reference frame. L_s, and R_s are are the stator inductance and resistance, respectively. L_r, and R_r are the the rotor inductance and resistance, respectively. L_m is the mutual inductance between stator and rotor. n_p is the number of pole pair. J is the moment of inertia and B is the viscous friction coefficient. T_l is the load torque. Input stator voltage components in the stator mounted reference frame are $u_{\alpha s}$ and $u_{\beta s}$. The leakage coefficient, σ is defined as $\sigma=\left(1-L_m^2/L_r L_s\right)$. The torque coefficient, K_T is defined as $K_T=n_p\, L_m/L_r$.

The motor speed is ω_r. The state feedback linearization decoupling and control algorithm for motor speed and rotor flux is expressed in [10] as:

$$\dot{\omega}_r=-\frac{B}{J}\omega_r+\frac{K_T}{J}\left(\psi_{\alpha s}i_{\beta s}-\psi_{\beta r}i_{\beta s}\right)-\frac{T_L}{J} \tag{2}$$

$$\dot{\psi}_r=-\frac{R_r}{L_r}\psi_r+\frac{L_m}{L_r\psi_r}\left(i_{\alpha s}\psi_{\alpha s}+i_{\beta s}\psi_{\beta s}\right) \tag{3}$$

Induction motor is stable at steady state condition for rated slip operation. The feedback linearization and decoupling control also ensures the stable motor operation near rated slip. In this research work, any possible operating point near steady state condition is assumed.

The steady state point state variable set is x_o.

where, $x_0=[i_{\alpha so},i_{\beta so},\psi_{\alpha ro},\psi_{\beta ro},\omega_{ro}]^T$

The steady state point is a fixed point. So, the system response about this point is given by the equations:

$$\left(R_s+\frac{L_m^2}{L_r^2}R_r\right)i_{\alpha so}-\frac{L_m R_r}{L_r^2}\psi_{\alpha ro}-\frac{n_p L_m}{L_r}\omega_{ro}\psi_{\beta ro}=u_{\alpha s}$$

$$\left(R_s+\frac{L_m^2}{L_r^2}R_r\right)i_{\beta so}-\frac{L_mR_r}{L_r^2}\psi_{\beta ro}+\frac{n_pL_m}{L_r}\omega_{ro}\psi_{\alpha ro}=u_{\beta s}$$

$$\frac{R_r}{L_r}\psi_{\alpha ro}+n_p\omega_{ro}\psi_{\beta ro}-\frac{L_mR_r}{L_r}i_{\alpha so}=0$$

$$\frac{R_r}{L_r}\psi_{\beta ro}-n_p\omega_{ro}\psi_{\alpha ro}-\frac{L_mR_r}{L_r}i_{\beta so}=0$$

$$\frac{B}{J}\omega_{ro}-\frac{K_T}{J}(i_{\beta so}\psi_{\alpha ro}-i_{\alpha so}\psi_{\beta ro})+\frac{T_l}{J}=0 \qquad (4)$$

When the variables and parameters at operating condition of motor drive system change, motor dynamic model deviates from its known model at steady state position. If error in the variables converge to zero, then after sometime motor drive system operates at steady state in another stable position. Theoretical analysis of this drive system error variable set is presented below.

The set of error variables for induction motor drive system is defined as given in (5).

$$e=(e_1,e_2,e_3,e_4,e_5)=(i_{\alpha s}-i_{\alpha so},i_{\beta s}-i_{\beta so},\psi_{\alpha r}-\psi_{\alpha ro},\psi_{\beta r}-\psi_{\beta ro},\omega_r-\omega_{ro}) \qquad (5)$$

The state space model of this drive system with the errors as variables is obtained from (1) as:

$$\dot{e}=A(x_o)e+g(e) \qquad (6)$$

$$A(x_o)=\begin{bmatrix} -\left[\frac{1}{\sigma L_s}\left(R_s+\frac{L_m^2}{L_r^2}R_r\right)\right] & 0 & \left(\frac{1}{\sigma L_s}\frac{L_mR_r}{L_r^2}\right) & \left(\frac{n_pL_m}{\sigma L_sL_r}\omega_{ro}\right) & \left(\frac{n_pL_m}{\sigma L_sL_r}\psi_{\beta ro}\right) \\ 0 & -\left[\frac{1}{\sigma L_s}\left(R_s+\frac{L_m^2}{L_r^2}R_r\right)\right] & -\left(\frac{n_pL_m}{\sigma L_sL_r}\omega_{ro}\right) & \left(\frac{1}{\sigma L_s}\frac{L_mR_r}{L_r^2}\right) & -\left(\frac{n_pL_m}{\sigma L_sL_r}\psi_{\alpha ro}\right) \\ \frac{L_mR_r}{L_r} & 0 & -\frac{R_r}{L_r} & -n_p\omega_{ro} & -n_p\psi_{\beta ro} \\ 0 & \frac{L_mR_r}{L_r} & n_p\omega_{ro} & -\frac{R_r}{L_r} & n_p\psi_{\alpha ro} \\ -\left(\frac{K_T}{J}\psi_{\beta ro}\right) & \left(\frac{K_T}{J}\psi_{\alpha ro}\right) & \left(\frac{K_T}{J}i_{\beta so}\right) & -\left(\frac{K_T}{J}i_{\alpha so}\right) & -\left(\frac{B}{J}\right) \end{bmatrix} \qquad g(e)=\begin{bmatrix} \frac{n_pL_m}{\sigma L_sL_r}e_4e_5 \\ -\frac{n_pL_m}{\sigma L_sL_r}e_3e_5 \\ -n_pe_4e_5 \\ n_pe_3e_5 \\ \frac{K_T}{J}(e_2e_3-e_1e_4) \end{bmatrix}$$

III. POWER AND ENERGY BALANCE EQUATIONS

In the stationary reference axes with the induction motor rotor and stator current α-β components as variables, the magnetic energy (w_f) and mechanical energy equations are expressed as in equation (7), [11]. Then total motor energy defined as (w_p) in equation (9).

$$w_f=\frac{1}{2}L_s\left(i_{\alpha s}^2+i_{\beta s}^2\right)+\frac{1}{2}L_r\left(i_{\alpha r}^2+i_{\beta r}^2\right)+L_m\left(i_{\alpha s}i_{\alpha r}+i_{\beta s}i_{\beta r}\right) \qquad (7)$$

$$w_J=\frac{1}{2}J\omega_r^2 \qquad (8)$$

$$w_p = w_f + w_J \tag{9}$$

The total motor stored energy in terms of stator fixed α-β axes variables like stator current i_s components ($i_{\alpha s}, i_{\beta s}$), rotor flux $\psi_{\alpha r}, \psi_{\beta r}$ components and motor speed, ω_r is given by [11].

$$w_p = \frac{1}{2}\sigma L_s\left(i_{\alpha s}^2 + i_{\beta s}^2\right) + \frac{1}{2L_r}\left(\psi_{\alpha r}^2 + \psi_{\beta r}^2\right) + \frac{1}{2}J\omega_r^2 \tag{10}$$

Taking the derivatives of (10) and substituting from (1) and simplifying leads to equation (11).

$$\frac{dw_p}{dt} = \sigma L_s\left(i_{\alpha s}\dot{i}_{\alpha s} + i_{\beta s}\dot{i}_{\beta s}\right) + \frac{1}{L_r}\left(\psi_{\alpha r}\dot{\psi}_{\alpha r} + \psi_{\beta r}\dot{\psi}_{\beta r}\right) + J\omega_r\dot{\omega}_r$$

$$\begin{aligned}
&= \sigma L_s i_{\alpha s}\left(-\frac{1}{\sigma L_s}\left(R_s + \frac{L_m^2}{L_r^2}R_r\right)i_{\alpha s} + \frac{1}{\sigma L_s}\frac{L_m R_r}{L_r^2}\psi_{\alpha r} + \frac{n_p L_m}{\sigma L_s L_r}\omega_r\psi_{\beta r} - \frac{u_{\alpha s}}{\sigma L_s}\right) \\
&+ \sigma L_s i_{\beta s}\left(-\frac{1}{\sigma L_s}\left(R_s + \frac{L_m^2}{L_r^2}R_r\right)i_{\beta s} + \frac{1}{\sigma L_s}\frac{L_m R_r}{L_r^2}\psi_{\beta r} - \frac{n_p L_m}{\sigma L_s L_r}\omega_r\psi_{\alpha r} - \frac{u_{\beta s}}{\sigma L_s}\right) \\
&+ \frac{1}{L_r}\left(\psi_{\alpha r}\left(-\frac{R_r}{L_r}\psi_{\alpha r} - n_p\omega_r\psi_{\beta r} + \frac{L_m R_r}{L_r}i_{\alpha s}\right) + \psi_{\beta r}\left(-\frac{R_r}{L_r}\psi_{\beta r} + n_p\omega_r\psi_{\alpha r} + \frac{L_m R_r}{L_r}i_{\beta s}\right)\right) \\
&+ J\omega_r\left(-\frac{B}{J}\omega_r + \frac{K_T}{J}(i_{\beta s}\psi_{\alpha r} - i_{\alpha s}\psi_{\beta r}) - \frac{T_l}{J}\right)
\end{aligned}$$

$$\begin{aligned}
&= \left(-\left(R_s + \frac{L_m^2}{L_r^2}R_r\right)\left(i_{\alpha s}^2 + i_{\beta s}^2\right) + \frac{L_m R_r}{L_r^2}\left(i_{\alpha s}\psi_{\alpha r} + i_{\beta s}\psi_{\beta r}\right) + \frac{n_p L_m}{L_r}\omega_r\left(i_{\alpha s}\psi_{\beta r} - i_{\beta s}\psi_{\alpha r}\right) + u_{\alpha s}i_{\alpha s} + u_{\beta s}i_{\beta s}\right) \\
&+ \left(\left(-\frac{R_r}{L_r^2}\psi_{\alpha r}^2 - \frac{n_p\omega_r\psi_{\alpha r}\psi_{\beta r}}{L_r} + \frac{L_m R_r\psi_{\alpha r}i_{\alpha s}}{L_r^2}\right) + \left(-\frac{R_r}{L_r^2}\psi_{\beta r}^2 + \frac{n_p\omega_r\psi_{\alpha r}\psi_{\beta r}}{L_r} + \frac{L_m R_r\psi_{\beta r}i_{\beta s}}{L_r^2}\right)\right) \\
&+ \left(-B\omega_r^2 + \frac{n_p L_m\omega_r}{L_r}(i_{\beta s}\psi_{\alpha r} - i_{\alpha s}\psi_{\beta r}) - T_l\omega_r\right)
\end{aligned}$$

$$\frac{dw_p}{dt} = u_{\alpha s}i_{\alpha s} + u_{\beta s}i_{\beta s} - B\omega_r^2 - T_l\omega_r - \left(R_s + \frac{L_m^2}{L_r^2}R_r\right)\left(i_{\alpha s}^2 + i_{\beta s}^2\right) + \frac{2L_m R_r}{L_r^2}\left(i_{\alpha s}\psi_{\alpha r} + i_{\beta s}\psi_{\beta r}\right) - \frac{R_r}{L_r^2}\left(\psi_{\alpha r}^2 + \psi_{\beta r}^2\right) \tag{11}$$

Equation (11) is for the power balance of induction motor as in [12]. Equation (11) states that the time rate of change of stored energy is the difference of input power and sum of mechanical power output with power loss.

Power loss in stator and rotor windings is expressed as in (12).

$$P_{loss} = \left(i_{\alpha s}^2 + i_{\beta s}^2\right)R_s + \left(i_{\alpha r}^2 + i_{\beta r}^2\right)R_r \tag{12}$$

The power loss equation using the stator current and the rotor flux α-β components is given by substituting (13) and (14) in (12), [12].

$$i_{\alpha r} = -\frac{L_m}{L_r}i_{\alpha s} + \frac{\psi_{\alpha r}}{L_r} \tag{13}$$

$$i_{\alpha r} = -\frac{L_m}{L_r} i_{\alpha s} + \frac{\psi_{\alpha r}}{L_r} \quad (14)$$

The power loss equation is derived as given in (15).

$$P_{loss} = \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(i_{\alpha s}^2 + i_{\beta s}^2\right) + \frac{R_r}{L_r^2}\left(i_{\alpha r}^2 + i_{\beta r}^2\right) - \frac{2L_m R_r}{L_r^2}\left(i_{\alpha s}\psi_{\alpha r} + i_{\beta s}\psi_{\beta r}\right) \quad (15)$$

For the stability analysis using Lyapunov approach, above power and energy balance equations are used. The total stored energy w_p in terms of error variables is given by (16).

$$\begin{aligned} & w_p(i_{\alpha so} + e_1, i_{\beta so} + e_2, \psi_{\alpha ro} + e_3, \psi_{\beta ro} + e_4, \omega_{ro} + e_5) \\ & = \frac{\sigma L_s}{2}\left(\left(i_{\alpha so} + e_1\right)^2 + \left(i_{\beta so} + e_2\right)^2\right) + \left(\frac{1}{2}\frac{\left(\psi_{\alpha ro} + e_3\right)^2 + \left(\psi_{\beta ro} + e_4\right)^2}{L_r}\right) \\ & + \left(\frac{1}{2} J(\omega_{ro} + e_5)^2\right) \end{aligned} \quad (16)$$

The total stored energy at steady state point, w_p is given by (17) using the steady state point variables.

$$\begin{aligned} & w_p(i_{\alpha so}, i_{\beta so}, \psi_{\alpha ro}, \psi_{\beta ro}, \omega_{ro}) \\ & = \frac{\sigma L_s}{2}\left(\left(i_{\alpha so}\right)^2 + \left(i_{\beta so}\right)^2\right) + \left(\frac{1}{2}\frac{\left(\psi_{\alpha ro}\right)^2 + \left(\psi_{\beta ro}\right)^2}{L_r}\right) + \left(\frac{1}{2} J(\omega_{ro})^2\right) \end{aligned} \quad (17)$$

Difference of (16) from (17) gives, where, $w_p(e)$ using the error variables, where, $w_p(e) = w_p - w_p(0)$

$$w_p(e) = \frac{\sigma L_s}{2}\left(e_1^2 + 2e_1 i_{\alpha so} + e_2^2 + 2e_2 i_{\beta so}\right) + \left(\frac{1}{2}\frac{e_3^2 + 2e_3\psi_{\alpha ro} + e_4^2 + 2e_4\psi_{\beta ro}}{L_r}\right) + \left(\frac{1}{2} J\left(e_5{}^2 + 2e_5\omega_{ro}\right)\right) \quad (18)$$

Arranging (18) in the error product form as in (19):

$$w_p(e) = e^T K e + d^T e \quad (19)$$

where,

$$K = \frac{1}{2}\begin{bmatrix} \sigma L_s & 0 & 0 & 0 & 0 \\ 0 & \sigma L_s & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_r} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{L_r} & 0 \\ 0 & 0 & 0 & 0 & J \end{bmatrix}, d = \begin{bmatrix} \sigma L_s i_{\alpha so} \\ \sigma L_s i_{\beta so} \\ \frac{\psi_{\alpha ro}}{L_r} \\ \frac{\psi_{\beta ro}}{L_r} \\ J\omega_{ro} \end{bmatrix}$$

The derivative of w_p is the same as the right-hand side of the power balance equation (11), as rewritten below [11].

$$\frac{dw_p(e)}{dt} = u_{\alpha s}e_1 + u_{\beta s}e_2 - B(e_5^2 + 2e_5\omega_{ro}) - T_l e_5 - \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(e_1^2 + 2e_1 i_{\alpha so} + e_2^2 + 2e_2 i_{\beta so}\right)$$
$$+\frac{2L_m R_r}{L_r^2}\left(i_{\alpha so}e_3 + \psi_{\alpha ro}e_1 + e_1e_3 + i_{\beta so}e_4 + \psi_{\beta ro}e_2 + e_2e_4\right) - \frac{R_r}{L_r^2}\left(e_3^2 + 2e_3\psi_{\alpha ro} + e_4^2 + 2e_4\psi_{\beta ro}\right) \tag{20}$$

Equation (20) is expressed in error vector product form as in (21).

$$\frac{dw_p(e)}{dt} = -e^T M_w e - s^T e \tag{21}$$

where,

$$M_w = \begin{bmatrix} \left(R_s + \frac{R_r L_m^2}{L_r^2}\right) & 0 & -\left(\frac{L_m R_r}{L_r^2}\right) & 0 & 0 \\ 0 & \left(R_s + \frac{R_r L_m^2}{L_r^2}\right) & 0 & -\left(\frac{L_m R_r}{L_r^2}\right) & 0 \\ -\left(\frac{L_m R_r}{L_r^2}\right) & 0 & \frac{R_r}{L_r^2} & 0 & 0 \\ 0 & -\left(\frac{L_m R_r}{L_r^2}\right) & 0 & \frac{R_r}{L_r^2} & 0 \\ 0 & 0 & 0 & 0 & B \end{bmatrix} \qquad s = \begin{bmatrix} 2\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)i_{\alpha so} - \frac{2L_m R_r}{L_r^2}\psi_{\alpha ro} - u_{\alpha s} \\ 2\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)i_{\beta so} - \frac{2L_m R_r}{L_r^2}\psi_{\beta ro} - u_{\beta s} \\ \frac{2R_r}{L_r^2}\psi_{\alpha ro} - \frac{2L_m R_r}{L_r^2}i_{\alpha so} \\ \frac{2R_r}{L_r^2}\psi_{\beta ro} - \frac{2L_m R_r}{L_r^2}i_{\beta so} \\ 2B\omega_{ro} + T_l \end{bmatrix}$$

IV. INDUCTION MOTOR STABILITY ANALYSIS USING LYAPUNOV APPROACH

1. **The Lyapunov Function Selection:** The first term of equation (19) is taken as a Lyapunov function candidate, *V*.

$$V = e^T K e \tag{22}$$

Derivative of the Lyapunov function using (19), (21) and (22) is obtained as:

$$\dot{V} = \frac{dw_p(e)}{dt} - d^T\dot{e} = -e^T M e - s^T e - d^T\left(A(x_o) + g(e)\right)$$

As the last two terms in above equation cancel each other, this gives equation (23).

$$\dot{V} = -e^T M e \tag{23}$$

where,

$$M=\begin{bmatrix} \left(R_s+\frac{R_rL_m^2}{L_r^2}\right) & 0 & -\left(\frac{L_mR_r}{L_r^2}\right) & -\left(\frac{n_pL_m\omega_{ro}}{2\times L_r}\right) & 0 \\ 0 & \left(R_s+\frac{R_rL_m^2}{L_r^2}\right) & \left(\frac{n_pL_m}{2\times L_r}\omega_{ro}\right) & \left(-\frac{L_mR_r}{L_r^2}\right) & 0 \\ -\left(\frac{L_mR_r}{L_r^2}\right) & \left(\frac{n_pL_m}{2\times L_r}\omega_{ro}\right) & \left(\frac{R_r}{L_r^2}\right) & 0 & \frac{n_p}{2L_r}\left(\psi_{\beta ro}-L_mi_{\beta so}\right) \\ -\left(\frac{n_pL_m\omega_{ro}}{2\times L_r}\right) & -\left(\frac{L_mR_r}{L_r^2}\right) & 0 & \left(\frac{R_r}{L_r^2}\right) & \frac{n_p}{2L_r}\left(L_mi_{\alpha so}-\psi_{\alpha ro}\right) \\ 0 & 0 & \frac{n_p}{2L_r}\left(\psi_{\beta ro}-L_mi_{\beta so}\right) & \frac{n_p}{2L_r}\left(L_mi_{\alpha so}-\psi_{\alpha ro}\right) & B \end{bmatrix}$$

2. **Global Asymptotic Stability of The Induction Motor Drive:** For the Global asymptotic stability theorem ([4], pp.65) the scalar function *V* of the state error (*e*) should have continuous first order derivative and satisfy the followings.

(a) *V(e)* is positive definite $\forall e \neq 0$, and $V(0)=0$

(b) $\frac{dV(e)}{dt} \leq 0 \forall e \neq 0$

(c) $\frac{dV(e)}{dt} \equiv 0 \Rightarrow e = 0$

(d) $V(e) \to \infty \, as \, \|e\| \to \infty$ (24)

For the first condition to be satisfied, the leading principal minors of *K* need to be positive definite. These are verified and mentioned below.

$$K_1=\frac{\sigma L_s}{2}>0,\ K_2=\frac{(\sigma L_s)^2}{4}>0,\ K_3=\frac{(\sigma L_s)^2}{8L_r}>0,\ K_4=\frac{(\sigma L_s)^2}{16L_r^2}>0\ \ K_5=\frac{(\sigma L_s)^2 J}{32L_r^2}>0 \quad (25)$$

The principal minors of *K* are positive definite. The verification confirms that all principal minors are positive. For the Lyapunov function defined in equation (22), conditions (a) and (d) hold good. Further conditions (b) and (c) are checked as follows.

3. **Global Asymptotic Stability of the Induction Motor Without Load:** In this case, motor load torque is zero. If viscous friction coefficient, *B* is *0*, then developed torque, T_e is also 0. So, the current components $i_{\alpha ro}$ and $i_{\beta ro}$ become zero. So, equations (13) and (14) lead to (26) and (27).

$$i_{\alpha ro}=-\frac{L_m}{L_r}i_{\alpha so}+\frac{\psi_{\alpha ro}}{L_r}=0 \quad (26)$$

$$i_{\beta ro}=-\frac{L_m}{L_r}i_{\beta ro}+\frac{\psi_{\beta ro}}{L_r}=0 \quad (27)$$

Substituting (26) and (27) in (23), matrix *M* converts to M_o, as given below.

$$M_0 = \begin{bmatrix} \left(R_s + \frac{R_r L_m^2}{L_r^2}\right) & 0 & -\left(\frac{L_m R_r}{L_r^2}\right) & -\left(\frac{n_p L_m \omega_{ro}}{2\times L_r}\right) & 0 \\ 0 & \left(R_s + \frac{R_r L_m^2}{L_r^2}\right) & \left(\frac{n_p L_m}{2\times L_r}\omega_{ro}\right) & \left(-\frac{L_m R_r}{L_r^2}\right) & 0 \\ -\left(\frac{L_m R_r}{L_r^2}\right) & \left(\frac{n_p L_m}{2\times L_r}\omega_{ro}\right) & \left(\frac{R_r}{L_r^2}\right) & 0 & 0 \\ -\left(\frac{n_p L_m \omega_{ro}}{2\times L_r}\right) & -\left(\frac{L_m R_r}{L_r^2}\right) & 0 & \left(\frac{R_r}{L_r^2}\right) & 0 \\ 0 & 0 & 0 & 0 & B \end{bmatrix} \tag{28}$$

This matrix M_o needs to be positive semidefinite. For this all the principal minors of M_0 are derived and mentioned in (29) to (33) below.

$$M_{0(1\times1)} = \left(R_s + \frac{R_r L_m^2}{L_r^2}\right), \left(\frac{R_r}{L_r^2}\right), B \tag{29}$$

$$M_{0(2\times2)} = \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)^2, \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(\frac{R_r}{L_r^2}\right) - \left(\frac{n_p L_m \omega_{ro}}{2L_r}\right)^2, \left(\frac{R_r}{L_r^2}\right)^2, B\left(\frac{R_r}{L_r^2}\right), B\left(R_s + \frac{R_r L_m^2}{L_r^2}\right) \tag{30}$$

$$M_{0(3\times3)} = \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(\left(\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(\frac{R_r}{L_r^2}\right) - \left(\frac{n_p L_m \omega_{ro}}{2L_r}\right)^2\right) - \left(\frac{L_m R_r}{L_r^2}\right)^2\right), \quad \left(\frac{R_r}{L_r^2}\right)\left(\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(\frac{R_r}{L_r^2}\right) - \left(\frac{n_p L_m \omega_{ro}}{2L_r}\right)^2 - \left(\frac{L_m R_r}{L_r^2}\right)^2\right), B\left(\frac{R_r}{L_r^2}\right)^2, B\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)^2 \tag{31}$$

$$M_{0(4\times4)} = \begin{pmatrix} \left(\left(\frac{L_m R_r}{L_r^2}\right)^2 - \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(\frac{R_r}{L_r^2}\right)\right)^2 + \left(\left(\frac{n_p L_m \omega_{ro}}{2L_r}\right)^2 - \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(\frac{R_r}{L_r^2}\right)\right)^2 \\ +2\left(\frac{L_m R_r}{L_r^2}\right)^2\left(\frac{n_p L_m \omega_{ro}}{2L_r}\right)^2 - \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(\frac{L_m R_r}{L_r^2}\right) \end{pmatrix}, B\left(\frac{R_r}{L_r^2}\right)\left(\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(\frac{R_r}{L_r^2}\right) - \left(\frac{n_p L_m \omega_{ro}}{2L_r}\right)^2 - \left(\frac{L_m R_r}{L_r^2}\right)^2\right) \tag{32}$$

$$M_{0(5\times5)} = B\begin{pmatrix} \left(\left(\frac{L_m R_r}{L_r^2}\right)^2 - \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(\frac{R_r}{L_r^2}\right)\right)^2 \\ + \left(\left(\frac{n_p L_m \omega_{ro}}{2L_r}\right)^2 - \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(\frac{R_r}{L_r^2}\right)\right)^2 \\ + \left(2\left(\frac{L_m R_r}{L_r^2}\right)^2\left(\frac{n_p L_m \omega_{ro}}{2L_r}\right)^2 - \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)^2\left(\frac{R_r}{L_r^2}\right)^2\right) \end{pmatrix} \tag{33}$$

Here it is noted that M_0 is positive semidefinite if it satisfies the following condition, which is derived from (31) and (32).

$$\left(\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(\frac{R_r}{L_r^2}\right) - \left(\frac{n_p L_m \omega_{ro}}{2L_r}\right)^2 - \left(\frac{L_m R_r}{L_r^2}\right)^2\right) \geq 0 \tag{34}$$

The left hand side of inequality (34) is dependent on the motor parameters. Inequality (34) gives positive value in case of small induction motors [11] and at less values of speed. For larger induction motors, the left hand side of inequality (34) becomes negative. So, at the time of starting large induction motor it is necessary to increase the rotor and stator resistances. This concludes the fact that, smaller induction motors can be started directly online at no load without loosing stability. In larger induction motors, rotor and stator resistances have to be increased for stability during starting acceleration.

Condition (c) in (24) leads to: $\frac{dV(e)}{dt} \equiv -e^T Me \equiv 0 \forall e = 0$

From (28) what follows is (35).

$$\frac{dV(e)}{dt} = \left(R_s + \frac{R_r L_m^2}{L_r^2}\right)\left(e_1^2 + e_2^2\right) + \frac{R_r}{L_r^2}\left(e_3^2 + e_4^2\right) - \frac{2L_m R_r}{L_r^2}\left(e_1 e_3 + e_2 e_4\right) - \frac{n_p \omega_{ro} L_m}{L_r}\left(e_1 e_4 - e_2 e_3\right) + Be_5 \equiv 0 \quad (35)$$

Here, an expression in terms motor parameters and error variables is considered as given in left hand side of (36) as given below. This expression should be positive or zero.

$$\left(\sqrt{\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)} e_1 \pm \sqrt{\frac{R_r}{L_r^2}} e_3\right)^2 + \left(\sqrt{\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)} e_2 \pm \sqrt{\frac{R_r}{L_r^2}} e_4\right)^2 \geq 0 \quad (36)$$

On expanding the above:

$$\left(R_s + \frac{R_r L_m^2}{L_r^2}\right) e_1^2 + \left(R_s + \frac{R_r L_m^2}{L_r^2}\right) e_2^2 + \frac{R_r}{L_r^2} e_3^2 + \frac{R_r}{L_r^2} e_4^2 \pm 2\sqrt{\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)}\sqrt{\frac{R_r}{L_r^2}}\left(e_1 e_3 + e_2 e_4\right) \geq 0 \quad (37)$$

Subtracting (35) from (37):

$$\frac{2L_m R_r}{L_r^2}\left(e_2 e_4 + e_1 e_3\right) \pm 2\sqrt{\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)}\sqrt{\frac{R_r}{L_r^2}}\left(e_1 e_3 + e_2 e_4\right) + \frac{n_p \omega_{ro} L_m}{L_r}\left(e_2 e_3 - e_1 e_4\right) - Be_5 \geq 0 \quad (38)$$

Finally,

$$\left(\pm 2\sqrt{\left(R_s + \frac{R_r L_m^2}{L_r^2}\right)}\sqrt{\frac{R_r}{L_r^2}} + \frac{2L_m R_r}{L_r^2}\right)\left(e_1 e_3 + e_2 e_4\right) + \frac{n_p \omega_{ro} L_m}{L_r}\left(e_2 e_3 - e_1 e_4\right) - Be_5 \geq 0 \quad (39)$$

The left side of (38) will be equal to zero, only when *e=0.*

4. **Global Asymptotic Stability of Induction Motor Drive With Load:** When the induction motor is loaded the positive definiteness of matrix *M* is evaluated. This positive definiteness of matrix *M* will be fulfilled if the leading principal minors are positive. The derived expressions of leading principal minors are given in equations (40) to (44).

$$M_1 = \left(R_s + \left(\frac{L_m}{L_r}\right)^2 R_r\right) \quad (40)$$

$$M_2 = \left(R_s + \left(\frac{L_m}{L_r}\right)^2 R_r\right)^2 \quad (41)$$

$$M_3 = \left(R_s + \frac{R_r L_m^2}{L_r^2} \right) \left(\left(R_s + \frac{R_r L_m^2}{L_r^2} \right) \left(\frac{R_r}{L_r^2} \right) - \left(\frac{n_p L_m \omega_{ro}}{L_r} \right)^2 - \left(\frac{L_m R_r}{L_r^2} \right)^2 \right), \tag{42}$$

$$M_4 = \left(\left(R_s + \frac{R_r L_m^2}{L_r^2} \right) \left(\frac{R_r}{L_r^2} \right) - \left(\left(\frac{L_m R_r}{L_r^2} \right)^2 + \left(\frac{n_p L_m \omega_{ro}}{L_r} \right)^2 \right) \right)^2 \tag{43}$$

$$\begin{aligned} M_5 = & \left(\left(\frac{n_p L_m \omega_{ro}}{2L_r} \right)^2 - \left(R_s + \frac{R_r L_m^2}{L_r^2} \right) \left(\frac{R_r}{L_r^2} \right) \right) \left(\left(\frac{n_p}{2L_r} \left(\psi_{\beta ro} - L_m i_{\beta so} \right) \right)^2 + \left(\frac{n_p}{2L_r} \left(L_m i_{\alpha so} - \psi_{\alpha ro} \right) \right)^2 - B \frac{R_r}{L_r^2} \right) - B \frac{R_r}{L_m^2} \left(\left(\frac{L_m R_r}{L_r^2} \right)^2 + \left(\frac{n_p L_m \omega_m}{2L_r} \right)^2 \right) \\ & + \left(\frac{L_m R_r}{L_r^2} \right) \left(\frac{n_p}{2L_r} \left(\psi_{\beta ro} - L_m i_{\beta so} \right) \right) \left(\left(\frac{L_m R_r}{L_r^2} \right) \left(\frac{n_p}{2L_r} \left(\psi_{\beta ro} - L_m i_{\beta so} \right) \right) + \left(\frac{n_p L_m \omega_m}{2L_r} \right) \left(\frac{n_p}{2L_r} \left(L_m i_{\alpha so} - \psi_{\alpha ro} \right) \right) \right) \end{aligned} \tag{44}$$

5. **Case Study:** Above conditions for positive definiteness of matrix *M* are verified by taking induction motor with specifications as follows. 5 Hp (3.7Kw), 6pole, Δ-connected, 415V, 1445 rpm. The motor parameters are as follows. R_s=7.5Ω, L_m=0.5H, L_s=0.52H, L_r=0.52H, R_r=5.4Ω, J=0.16 kg-m^2, B=0.035 kg-m^2/s.

   Substituting the parameter values in the matrix *M*,

$$M = \begin{bmatrix} (12.5) & 0 & (-9.95) & -(0.4\omega_r) & 0 \\ 0 & (12.5) & (0.4\omega_r) & (-9.95) & 0 \\ (-9.95) & (0.4\omega_r) & (19.89) & 0 & (1.9\psi_{\beta ro} - 0.95 i_{\beta so}) \\ -(0.4\omega_r) & (-9.95) & 0 & (19.89) & (0.95 i_{\alpha so} - 1.9\psi_{\alpha ro}) \\ 0 & 0 & (1.9\psi_{\beta ro} - 0.95 i_{\beta so}) & (0.95 i_{\alpha so} - 1.9\psi_{\alpha ro}) & 0.035 \end{bmatrix} \tag{45}$$

   The minors of matrix, *M* are

$$M_1 = \left(R_s + \left(\frac{L_m}{L_r} \right)^2 R_r \right) = 12.5 \tag{46}$$

$$M_2 = \left(R_s + \left(\frac{L_m}{L_r} \right)^2 R_r \right)^2 = 156.3 \tag{47}$$

$$M_3 = \left(R_s + \frac{R_r L_m^2}{L_r^2} \right) \left(\left(R_s + \frac{R_r L_m^2}{L_r^2} \right) \left(\frac{R_r}{L_r^2} \right) - \left(\frac{n_p L_m \omega_{ro}}{L_r} \right)^2 - \left(\frac{L_m R_r}{L_r^2} \right)^2 \right) \tag{48}$$

   Solving for higher principal minors

$$M_4 = \begin{bmatrix} (12.5) & 0 & (-9.95) & (-0.4 v \omega_s) \\ 0 & (12.5) & (0.4 v \omega_s) & (-9.95) \\ (-9.95) & (0.4 v \omega_s) & (19.89) & 0 \\ (-0.4 v \omega_s) & (-9.95) & 0 & (19.89) \end{bmatrix}$$

$$M_{41} = 12.5\begin{bmatrix} 12.5 & (0.4\nu\omega_s) & (-9.95) \\ (0.4\nu\omega_s) & 19.89 & 0 \\ (-9.95) & 0 & 19.89 \end{bmatrix}$$

$$M_{41} = 12.5\left(12.5(19.89)^2 - 19.89(0.4\nu\omega_s)^2 - 19.8(9.95)^2\right)$$

$$M_{41} = 35711.88 - 980537.125\nu^2$$

$$M_{44} = -0.4\nu\omega_s \begin{bmatrix} 0 & (12.5) & (0.4\nu\omega_s) \\ (-9.95) & (0.4\nu\omega_s) & 19.89 \\ (-0.4\nu\omega_s) & -9.95 & 0 \end{bmatrix}$$

$$M_{44} = 0.4\nu\omega_s\left(-12.5(0.4\nu\omega_s \times 19.89) + 0.4\nu\omega_s\left((9.95)^2 + (0.4\nu\omega_s)^2\right)\right)$$

$$M_{44} = -590090.78\nu^2 + 15553873.82\nu^4$$

$$M_4 = M_{41} + M_{43} + M_{44}$$

$$M_4 = 35711.88 - 980537.125\nu^2 - 14812.5 + 390449.94\nu^2 - 590090.78\nu^2 + 15553873.82\nu^4$$

$$M_4 = 15553873.82\nu^4 - 2\times 590090.78\nu^2 + 20899.38$$

The foregoing equation is solved to find the real roots. If:

$$\nu^2 = x \Rightarrow 15553873.82x^2 - 2\times 590090.78x - 109843.12 = 0$$

$$x = \pm 0.13, \Rightarrow x = \nu^2 > 0.13$$

$$-0.36 < \nu > 0.36 \Rightarrow -0.36 < 1 - s > 0.36$$

$$s < 0.64$$

$$s < 1.36$$

The matrix M is related to motor current and flux values at steady state. So, for testing the positive definiteness the following three sets of observations (Table-I) have been considered [10].

- Induction motor running at 52.19 rad/s (500 rpm) under no load.
- Induction motor running at 52.19 rad/s (500 rpm) subjected to 10 N.m load torque.
- Induction motor running at 104.7 rad/s (1000 rpm) under no load.

During the stability study test following results are obtained for determinant of the matrix M_5 and principal minor M_4 as shown in Table-I.

Table 1: Stability Test Results for Three cases

ω_{ro} (rad/s)	T_l (N.s)	$i_{\alpha so}$(A) $i_{\beta so}$(A)	$\psi_{\alpha r0}$ (V.s) $\psi_{\beta r0}$ (V.s)	Principal Minor
52.1	0.575	2.17 1.947	0.45 -0.48	M_4=8.1e4 M_5=2.12e4
52.02	10.1	-5.415 -8.468	0.220 -0.427	M_4=8.1e4 M_5=2.72e5
104.7	0.4	-1.78 -3.965	-3.965 -0.48	M_4=1.16e8 M_5=5.9e6

V. CONCLUSION

The global asymptotic stability using Lyapunov's theorem for the perturbed induction motor drive near steady state operating point has been analyzed without load and with load. It has been noticed that stability depends on slip at the operating point and motor parameters at the operating condition. The rotor resistance has more predominant effect than other parameters. The fact that increase of rotor circuit resistance through addition of extra resistance increases the starting torque thereby making the motor capable of accelerating stably is confirmed and reestablished through stability study. The sufficient condition for stability is also derived. This stability analysis helps to understand the stability and instability of the induction motor drive.

REFERENCES

[1] W. Leonard, *Control of Electrical Drives* (Spinger-Verlag, Berlin, 1990).
[2] Blaschke, "The principle of field orientation as applied to the new TRANSVEKTOR closed-loop control system for rotating-field machines," *Siemens review, 39*, 1972, 217-220.
[3] K. Hasse, "On the dynamic behavior of induction machine driven by variable frequency and voltage source," *ETZ Arch. Bd. 89, H.4*, 1968, 77-81.
[4] J. J. E. Slotine and W. Li, *Applied Nonlinear Control* (Prentice Hall International, Englewood Cliffs, New Jersey, 1991).
[5] Isidori, *Nonlinear control Systems* (Springer Verlag, Berlin, 1995).
[6] R. Marino, S. Peresada, and P. Valigi, "Adaptive input-output linearizing control of induction motors," *IEEE Trans. AC, 38(2)*, 1993, 208-221.
[7] J. Chiasson, "Dynamic feedback linearization of the induction motor," *IEEE Trans. Auto. Control, 38(10)*, 1993, 1588-1594.
[8] P. Vas, *Sensorless Vector and Direct torque control* (Oxford University Press, 1998).
[9] K. B. Mohanty, "Sensorless control of a linearized induction motor drive," *International Review of Electrical Engineering, 2(3)*, 2007, 386-397.
[10] K. B. Mohanty, M. Singh, "Performance improvement of induction motor drive using feedback linearization and fuzzy torque compensator with RTDS implementation," *International Review of Electrical Engineering, 7(3)*, 2012, 4374-4382.
[11] A. Oteafy, J. Chaisson, "A study of the Lyapunov stability of an open-loop induction machine," *IEEE Trans. Control Systems Technology, 18(6)*, 2010, 1469-1476.
[12] R. Marino, P. Tomei, C. M. Verrelli, *Induction motor control design* (Springer-Verlag, London, 2010).

