NANO M_J OPEN MAP IN NANO TOPOLOGICAL SPACES

Abstract

In this paper we are going to establishing new class of function in Nano topological spaces named as Nano M_J-open map. The properties of Nano M_J -open map are explained. Also introduce Nano -M_J closed map and discussed their properties. Finally introduce Nano M_J-homeomorphism and analyze their properties with examples.

Keywords: Nano M_J open sets; Nano M_J Closed Sets; Nano M_J-open map; Nano M_J-closed map ; Nano M_J-homeomorphism.

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I. INTRODUCTION

In the year 2013 a new branch of topology called Nano topology established by Lellis Thivagar[].Nano topology became the important branch in topology that uses in data analysing and many real life situation.He also defined some weaker form of Nano-open sets[2] such as Nano α-open sets, Nano semi-open sets and Nano pre-open sets. He also defined Nano-continuity[3] and Nano-homeomorphism. Jackson and Gnanaselvam jude defined Nano M_J open set using the operators Nscl and Nint. In this paper a new class of function named as Nano M_J-open map established and their properties are discussed.

II. PRELIMNERIES

On this paper, W_1, W_2 and W_3 are non empty ,finite universes; $P \subseteq W_1, Q \subseteq W_2$ and $S \subseteq W_3$; $W_1/R, W_2/R'$ and W_3/R'' are the families of equivalence relations R, R' and R'' respectively on W_1, W_2 and W_3. $\left(W_1, \tau_R(P)\right), \left(W_2, \tau_{R'}(Q)\right)$ And $(W_3, \tau_{R''}(S))$ are the NTS with repect to P, Q and S respectively. $Nano\ M_J O(W_1, P)$, $Nano\ M_J O(W_2, Q)$ and $Nano\ M_J O(W_3, S)$ are the $Nano\ M_J\ open$ with repect to P, Q and S respectively. $Nano\ M_J C(W_1, P)$, $Nano\ M_J C(W_2, Q)$ and $Nano\ M_J C(W_3, S)$ are the $Nano\ M_J\ closed$ with repect to P, Q and S respectively.

Definition 2.1 [2]: Take the universe W_1 be a nonempty finite set object and R is known as the indiscernibility relation and is an equivalence relation on W_1. It is known that elements of the same equivalence class are indistinguishable from one another. The approximation space is the pair(W_1, R). Let $P \subseteq W_1$

1. The Lower approximation of P with respect to R is defined by $L_R(P) = \bigcup_{G \in U}\{R(P) : R(P) \subseteq P\}$
2. The upper approximation of P with respect to R is defined by $U_R(P) = \cup_{G \in U} \{R(P): R(P) \cap P \neq \oint\}$
3. The boundary region of X with respect to R classified by $B_R(P) = U_R(P) - L_R(P)$.

Proposition 2.2: [2]

As (W_1, R) is an approximation space and $J_1, J_2 \subseteq W_1$,
$L_R(J_1) \subseteq J_1 \subseteq U_R(J_1)$
$L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$
$U_R(J_1 \cup J_2) = U_R(J_1) \cup U_R(J_2)$
$U_R(J_1 \cap J_2) \subseteq U_R(J_1) \cap U_R(J_2)$
$L_R(J_1 \cup J_2) \supseteq L_R(J_1) \cup L_R(J_2)$
$L_R(J_1 \cap J_2) = L_R(J_1) \cap L_R(J_2)$
$L_R(J_1) \subseteq L_R(J_2)$ and $U_R(X) \subseteq U_R(J_2)$ whenever $J_1 \subseteq J_2$
$U_R({J_1}^c) = [L_R(J_1)]^c$ and $L_R({J_1}^c) = [U_R(J_1)]^c$
$U_R U_R(J_1) = L_R L_R(X) = U_R(J_1)$
$L_R L_R(J_1) = U_R L_R(J_1) = L_R(J_1)$

Definition 2.3 [2]: *Let W_1 be finite, non-empty universe of objects and R be an equivalence relation on W_1. Let $P \subseteq U$. Let $\tau_R(P) = \{W_1, \varphi, L_R(P), U_R(P), B_R(P)\}$. Then $\tau_R(P)$ a topology on W_1, called as the $Nano\ topology$ with respect to P. Elements of the $Nano\ topology$ are known as the $Nano - open$ sets in W_1 and $(W_1, \tau_R(P))$ is called the $Nano\ topological\ space$ (briefly NTS). $[\tau_R(P)]^c$ is called the $Dual\ Nano\ topology$ on $\tau_R(P)$. Elements of $\tau_R c(P)$ are called as $Nano - closed$ sets.*

Remark 2.4 [2]: The basis for the $Nano\ topology$ $\tau_R(P)$ with respect to P is given by $\beta_R(P) = \{U, L_R(P), B_R(P)\}$.

Definition 2.5 [2]: If $(W_1, \tau_R(P))$ is a NTS with respect to P where $P \subseteq W_1$ and if $S \subseteq W_1$ then the Nano interior of S is defined as the union of all $Nano - open$ subsets of S and it is denoted as $Nint(S)$. Nano interior is the largest $Nano - open$ subset of S.The $Nano\ Closure$ of S is defined as the intersection of all $Nano - closed$ sets containing S and it is denoted by $Ncl(S)$. It is the smallest $Nano - closed$ set containing S.

Definition 2.6 [7]: A subset G of a NTS, G is called $Nano\ M_J\ open$ if $G \subseteq (Nscl(Nint\ (G))$. The collection of all $Nano\ M_J\ open$ sets in (U, G) is denoted by $M_J\ O(U, G)$. The complement of $Nano\ M_J\ open$ set is called a $Nano\ M_J\ closed$ set. The collection of all $Nano\ M_J\ closed$ sets in (U, G) is denoted by $M_J C(U, G)$.

Definition 2.7[3]: Let $(W_1, \tau_R(P)))$ and $(W_2, \tau_{R'}(Q))$ be two NTS. Then a mapping $f: (W_1, \tau_R(P)) \to (W_2, \tau_{R'}(Q))$ is said to be

1. $Nano - continuous$ if $f^{-1}(M)$ is $Nano - open$ in W_1 for every $Nano - open$ set M in W_2.
2. $Nano\ \alpha - continuous$ if $f^{-1}(M)$ is $Nano\ \alpha - open$ in W_1 for every $Nano - open$ set M in W_2.
3. Nano $pre - continuous$ if $f^{-1}(M))$ is $Nano\ pre - open$ in W_1 for every $Nano - open$ set M in W_2.
4. $Nano\ semi - continuous$ if $f^{-1}(M)$ is Nano $semi - open$ in W_1 for every $Nano - open$ set M in W_2.
5. $Nano\ M_J\ continous$ if $f^{-1}(M)$ is $Nano\ M_J\ open$ in W_1 for every $Nano - open$ set M in W_2.

Definition 2.8[3]: A function $f: (W_1, \tau_R(P)) \to (W_2, \tau_{R'}(Q))$ is a $Nano - open\ map$ if the image of every $Nano - open$ set in W_1 is $Nano - open$ in W_2. The mapping f is said to be a $Nano - closed\ map$ if the image of every $Nano - closed$ set in W_1 is $Nano - closed$ in W_2.

Definition 2.9[3]: A function $f: (W_1, \tau_R(P)) \to (W_2, \tau_{R'}(Q))$ is said to be a $Nano\ homeomorphism$ if

1. f is $1 - 1$ and $onto$
2. f is $Nano\ continuous$
3. f is $Nano - open\ map$

III. $NANO\ M_J - OPEN\ MAP$

Definition 3.1: The map $f: (W_1, \tau_R(P)) \to (W_2, \tau_{R'}(Q))$ is said to be $Nano\ M_J - open\ map$ if the the image of every $Nano - open$ set in $(W_1, \tau_R(P))$ is $Nano\ M_J\ open$ in $(W_2, \tau_R(Q))$.

Example 3.2: Let $W_1 = \{x, y, z, w\}$ with $U/R = \{\{x\}, \{y, z\}, \{w\}\}$ and $P = \{x, w\}$. Then the topology $\tau_R(P) = \{W_1, \varphi, \{x, w\}\}$. Let $W_2 = \{a, b, c, d\}$ with $W_2/R' = \{a\}, \{b, d\}, \{c\}\}$ and $Q = \{a, d\}$. Then the topology $\tau_R(Q) = \{W_2, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$ and $Nano\ M_J\, O(W_2, Q) = \{W_2, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Define $f: W_1 \to W_2$ as $f(x) = b$; $f(y) = a$; $f(z) = c$; $f(w) = d$,then $f\{x, w\} = \{b, d\}$. Therefore f is $Nano\ M_J - open\ map$.

Remark 3.3: The composition of two Nano $M_J -$ open maps need not be Nano $M_J -$ open map as seen by the example below.

Example 3.4: Let $W_1 = \{a, b, c, d\}$ with $W_1/R = \{\{a, b, c\}, \{d\}\}$ and $P = \{a, c\}$. Then the topology $\tau_R(P) = \{W_1, \varphi, \{a, b, c\}\}$.Let $W_2 = \{a , b, c, d\}$ with $W_2/R' = \{\{a\}, \{b, c\}, \{d\}\}$ and $Q = \{b, c\}$. Then the topology $\tau_{R'}(Q) = \{W_2 , \varphi, \{b, c\}\}$, $Nano\ M_J\, O(W_2, Q) = \{W_2, \varphi, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. Let $W_3 = \{a, b, c, d\}$ with $W_3/R'' = \{\{a\}, \{d\}, \{b, c\}\}$ and $S = \{a, b, d\}$. Then the topology $\tau_{R''}(S) = Nano\ M_J\, O(W_3, S) = \{W_3, \varphi, \{a, d\}, \{b, c\}\}$.

Let $f: (W_1, \tau_R(P)) \to (W_2, \tau_{R'}(Q))$ and $g: (W_2, \tau_{R'}(Q)) \to (W_3, \tau_{R''}(S))$ be an identity maps then f and g are $Nano\ M_J - open\ map$s. But their composition is not $Nano\ M_J - open\ map$ since image of the $Nano - open$ set $\{a, b, c\}$ is not $Nano\ M_J\ open$ in $(W_3, \tau_{R''}(S))$.

Remark 3.5: Image of a $Nano\ M_J\ open$ set need not be a $Nano\ M_J\ open$ set under a $Nano\ M_J -\ open\ map$.

Example 3.6: Let $W_1 = \{a, b, c, d\}$ with $W_1/R = \{\{a, d\}, \{b\}, \{c\}\}$ and $P = \{b, c\}$ then $\tau_R(P) = \{W_1, \varphi, \{b, c\}\}$ and $Nano\ M_J(W_1, P) = \{ U, \varphi, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. Let $W_2 = \{h, p, u, f\}$ with $W_2/R' = \{\{h, f\}, \{p\}, \{u\}\}$ and $Q = \{p, f\}$ then $\tau_{R'}(Q) = Nano\ M_J\, O(W_2, Q) = \{W_2, \varphi, \{h, f\}, \{p\}, \{h, p, f\}\}$. Define $f: W_1 \to W_2$ as $f(a) = p$; $f(b) = h$; $f(c) = f$; $f(d) = u$. Then f is $Nano\ M_J - open\ map$ but the image of $Nano\ M_J\ open$ sets $\{a, b, c\}, \{b, c, d\}$ is not $Nano\ M_J\ open$ in $(W_2, \tau_{R'}(Q))$.

Theorem 3.7: Every $Nano -\ open\ map$ is $Nano\ M_J - open\ map$.

Proof. Let $f: (W_1, \tau_R(P)) \to (W_2, \tau_{R'}(Q))$ be $Nano - open\ map$. Let I be $Nano - open$ in the topological space $(W_1, \tau_R(P))$. Then the image of I under the map f is $Nano - open$ in the topological space $(W_2, \tau_{R'}(Q))$. Since every $Nano - open$ is $Nano\ M_J\ open,$ f is $Nano\ M_J - open\ map$.

Remark 3.8: The coverse of the theorem 3.7 is not true.

Example 3.9: Let $W_1 = \{a, b, c, d\}$ with $W_1/R = \{\{a, b, c\}, \{d\}\}$ and $P = \{a, c\}$. Then the topology $\tau_R(P) = \{W_1, \varphi, \{a, b, c\}\}$. Let $W_2 = \{ x, y, z, w \}$ with $W_2/R' = \{\{x\}, \{y\}, \{z, w\}\}$ and $Q = \{ z, w \}$. Then the topology $\tau_{R'}(Q) = \{W_2, \varphi, \{ z, w \}\}$ and $Nano\ M_J\, O(W_2, Q) = \{W_2, \varphi, \{ z, w \}, \{ y, z, w \}, \{x, z, w \}\}$. Define $f: W_1 \to W_2$ as $f(a) = x$; $f(b) =$

z; $f(c) = w$; $f(d) = y$. Then f is $Nano\ M_J - open\ map$ but not $Nano - open\ map$. Since the image of set $\{a, b, c\}$ is not $Nano - open$ in $(W_1, \tau_R(P))$.

Theorem 3.10: Let $f: (W_1, \tau_R(P)) \to (W_2, \tau_{R'}(Q))$ be $Nano - open\ map$ and $g: (W_2, \tau_{R'}(Q)) \to (W_3, \tau_{R''}(S))$ be $Nano\ M_J\ open\ map$. Then their composition is $Nano\ M_J - open\ map$.

Proof. Let I be $Nano - open$ set in $(W_1, \tau_R(P))$. Then $f(I)$ is $Nano - open$ in $(W_2, \tau_{R'}(Q))$ and $(g \circ f)(I) = g(f(I))$ is $Nano\ M_J\ open$ since g is $Nano\ M_J - open\ map$. Hence the composition is $Nano\ M_J - open\ map$.

Remark 3.11: Let $f: (W_1, \tau_R(P)) \to (W_2, \tau_{R'}(Q))$ be $Nano\ M_J - open\ map$ and $g: (W_2, \tau_{R'}(Q)) \to (W_3, \tau_{R''}(S))$ be $Nano - open\ map$. Then their composition is not $Nano\ M_J - open\ map$.

Example 3.12: In example 3.4 f is $Nano\ M_J - open\ map$ and g is $Nano - open\ map$ but their composition is not $Nano\ M_J - open\ map$.

IV. $NANO\ M_J - CLOSED\ MAP$

Definition 4.1: The map $f: (W_1, \tau_R c(P)) \to (W_2, \tau_R c(Q))$ is said to be $Nano\ M_J - closed\ map$ if the the image of every $Nano - closed$ set in $(W_1, \tau_R c(P))$ is $Nano\ M_J\ closed$ in $(W_2, \tau_R c(Q))$.

Example 4.2: Let $W_1 = \{x, y, z, w\}$ with $W_1/R = \{\{x\}, \{y, z\}, \{w\}\}$ and $P = \{x, w\}$. Then the topology $\tau_R c(P) = \{W_1, \varphi, \{y, z\}\}$. Let $W_2 = \{a, b, c, d\}$ with $W_2/R' = \{\{a\}, \{b, d\}, \{c\}\}$ and $Q = \{a, d\}$. Then the topology $\tau_{R'}(Q) = \{W_2, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$ and $Nano\ M_J C(W_2, Q) = \{W_2, \varphi, \{c\}, \{a, c\}, \{b, c, d\}\}$. Define $f: W_1 \to W_2$ as $f(x) = b; f(y) = a; f(z) = c; f(w) = d$,then f is $Nano\ M_J - closed\ map$.

Remark 4.3: The composition of two $Nano\ M_J - closed\ map$s need not be $Nano\ M_J - closed\ map$ as seen by the example below.

Example 4.4: Let $W_1 = \{a, b, c, d\}$ with $W_1/R = \{\{a, b, c\}, \{d\}\}$ and $P = \{a, c\}$. Then the topology $\tau_R(P) = \{W_1, \varphi, \{a, b, c\}\}, \tau_R c(P) = \{W_1, \varphi, \{d\}\}$. Let $W_2 = \{a, b, c, d\}$ with $W_2/R' = \{\{a\}, \{b, c\}, \{d\}\}$ and $Q = \{b, c\}$. Then the topology $\tau_{R'} c(Q) = \{W_2, \varphi, \{a, d\}\}$ and $Nano\ M_J C(W_2, Q) = \{W_2, \varphi, \{a\}, \{d\}, \{a, d\}\}$. Let $W_3 = \{a, b, c, d\}$ with $W_3/R'' = \{\{a\}, \{d\}, \{b, c\}\}$ and $S = \{a, b, d\}$. Then the topology $\tau_{R''} c(S) = Nano\ M_J C(W_3, S) = \{W_3, \varphi, \{a, d\}, \{b, c\}\}$. Let $f: (W_1, \tau_R c(P)) \to (W_2, \tau_{R'} c(Q))$ and $g: (W_2, \tau_{R'} c(Q)) \to (W_3, \tau_{R''} c(S))$ be an identity maps then f and g are $Nano\ M_J - closed\ map$s. But their composition is not $Nano\ M_J - closed\ map$ since image of the $Nano - closed$ set $\{d\}$ is not $Nano\ MJ\ open$ in $(W_3, \tau_{R''}(S))$.

Remark 4.5: Image of a $Nano\ M_J\ closed$ set need not be a $Nano\ M_J\ closed$ set under a $Nano\ M_J - closed\ map$.

Example 4.6: Let $W_1 = \{a, b, c, d\}$ with $W_1/R = \{\{a, d\}, \{b\}, \{c\}\}$ and $P = \{b, c\}$ then $\tau_R c(P) = \{W_1, \varphi, \{a, d\}\}$ and $Nano\ M_J C(W_1, P) = \{U, \varphi, \{a\}, \{d\}, \{a, d\}\}$. Let $W_2 = \{h, p, u, f\}$ with $W_2/R' = \{\{h, f\}, \{p\}, \{u\}\}$ and $Q = \{p, f\}$ then

$\tau_{R'}c(Q) = Nano\ M_J C(W_2, Q) = \{W_2, \varphi, \{u\}, \{p,u\}, \{h,f,u\}\}$. Define $f: W_1 \rightarrow W_2$ as $f(a) = p$; $f(b) = h$; $f(c) = f$; $f(d) = u$. Then f is $Nano\ M_J - closed\ map$ but the image of $Nano\ M_J\ closed$ set $\{a\}$ is not $Nano\ M_J\ closed$ in $(W_2, \tau_{R'}c(Q))$.

Theorem 4.7: Every $Nano - closed\ map$ is $Nano\ M_J - closed\ map$.
Proof: Let $f: (W_1, \tau_R c(P)) \rightarrow (W_2, \tau_{R'}c(Q))$ be $Nano - closed\ map$. Let I be $Nano - closed$ in the topological space $(W_1, \tau_R(P))$. Then the image of I under the map f is $Nano - closed$ in the topological space $(W_3, \tau_{R'}(Q))$. Since every $Nano - closed$ is $Nano\ M_J\ open$, f is $Nano\ M_J - closed\ map$.
Remark 4.8: The coverse of the theorem 4.7 is not true.

Example 4.9: Let $W_1 = \{a,b,c,d\}$ with $W_1/R = \{\{a,b,c\},\{d\}\}$ and $P = \{a,c\}$. Then the topology $\tau_R c(X) = \{W_1, \varphi, \{d\}\}$. Let $W_2 = \{x,y,z,w\}$ with $W_2/R' = \{\{x\},\{y\},\{z,w\}\}$ and $Q = \{z,w\}$. Then the topology $\tau_{R'}c(Q) = \{W_2, \varphi, \{x,y\}\}$ and $Nano\ M_J C(W_2, Q) = \{W_2, \varphi, \{x,y\}, \{x\}, \{y\}\}$. Define $f: W_1 \rightarrow W_2$ as $f(a) = x$; $f(b) = z$; $f(c) = w$; $f(d) = y$. Then f is $Nano\ M_J - closed\ map$ but not $Nano - closed\ map$. Since the image of set $\{d\}$ is not closed in $\big(W_1, \tau_R(P)\big)$.

Theorem 4.10: Let $f: (W_1, \tau_R c(P)) \rightarrow (W_2, \tau_{R'}c(Q))$ be $Nano - closed\ map$ and $g: (W_2, \tau_{R'}c(Q)) \rightarrow (W_3, \tau_{R''}c(S))$ be $Nano\ M_J - closed\ map$. Then their composition is $Nano\ M_J -\ closed\ map$.
Proof: Let I be $Nano - closed$ set in $(W_1, \tau_R(P))$. Then $f(I)$ is $Nano - closed$ in $(W_2, \tau_{R'}(Q))$ and $(g \circ f)(I) = g(f(I))$ is $Nano\ M_J\ closed$ since g is $Nano\ M_J\ closed\ map$. Hence the composition is $Nano\ M_J - closed\ map$.
Remark 4.11: Let $f: (W_1, \tau_R c(P)) \rightarrow (W_2, \tau_{R'}c(Q))$ be $Nano\ M_J - closed\ map$ and $g: (W_2, \tau_{R'}c(Q)) \rightarrow (W_3, \tau_{R''}c(S))$ be $Nano - closed\ map$. Then their composition is not $Nano\ M_J - closed\ map$.

Example 4.12: In example 3.4 f is $Nano\ M_J -\ closed\ map$ and g is $Nano - closed\ map$ but their composition is not $Nano\ M_J - closed\ map$.

V. $NANO\ M_J - HOMEOMORPHISM$

Definition 5.1: A function $f: (W_1, \tau_R(P)) \rightarrow (W_2, \tau_{R'}(Q))$ is said to be $Nano\ M_J - homeomorphism$ if f is $one - one$ and $onto$, $Nano\ M_J - Continous$ and $Nano\ M_J - open\ map$.

Example 5.2: Let $W_1 = \{x,y,z,w\}$ with $W_1/R = \{\{x,w\},\{y\},\{z\}\}$ and $P = \{y,w\}$ then $\tau_R(P) = Nano\ M_J O(W_1, \tau_R(P)) = \{W_1, \varphi, \{y\}, \{x,w\}, \{x,y,w\}\}$. Let $W_2 = \{a,b,c,d\}$ with $W_2/R' = \{\{a,c\},\{b\},\{d\}\}$ and $Q = \{a,d\}$. Then the topology $\tau_R(Q) = Nano\ M_J O(W_2, Q) = \{W_2, \varphi, \{d\}, \{a,c\}, \{a,c,d\}\}$. Define $f: W_1 \rightarrow W_2$ as $f(x) = a$; $f(y) = d$; $f(z) = b$; $f(w) = c$, then f is $Nano\ M_J - open\ map$, $Nano\ M_J - Continous\ function$ and also f is $one - one$ and $onto$.

Remark 5.3: A function $f: (W_1, \tau_R c(P)) \to (W_2, \tau_{R'} c(Q))$ is said to be $Nano\ M_J - homeomorphism$ if f is $one - one$ and $onto$, $Namo\ M_J - Continous$ and $Nano\ M_J - closed\ map$.

Theorem 5.4: Every $Nano - homeomorphism$ is $Nano\ M_J - homeomorphism$.

Proof: Let $f: (W_1, \tau_R(P)) \to (W_2, \tau_{R'}(Q))$ be a $Nano - homeomorphism$. Then f is $one - one$ and $onto$, $Nano - continous$ and $Nano - open\ map$. Since every $Nano - continous$ is $Nano\ M_J - Continous$ and every $Nano - open\ map$ is $Nano\ M_J - open\ map$, f is $Nano\ M_J - homeomorphism$.

Theorem 5.5: Let $f: (W_1, \tau_R(P)) \to (W_2, \tau_{R'}(Q))$ be a bijective $Nano\ M_J - Continous\ function$. Then the following are equivalent.

(1) f is an $Nano\ M_J - Open$ map.
(2) f is an $Nano\ M_J - homeomorphism$.
(3) f is an $Nano\ M_J - Closed$ map

Proof:

$(1) \to (2)$

Let f is an $Nano\ M_J\ Open$ map and given f is bijective and $Nano\ M_J - Continous$ function. Then by definition, f is $Nano\ M_J - homeomorphism$.

$(2) \to (3)$

Sice f is $Nano\ M_J - homeomorphism$, it is bijective, $Nano\ M_J - Continous$ and $Nano\ M_J - Open$ map. Let H be a $Nano - Closed$ set in $\left(W_1, \tau_R(P)\right)$. Then $U - H$ is $Nano\ \ Open$ in $\left(W_1, \tau_R(P)\right)$ and $f(U - H)$ is $Nano\ M_J\ Open$ in $\left(W_2, \tau_{R'}(Q)\right)$. $f(U - H) = f(U) - f(H) = V - f(H)$ is $Nano\ M_J\ open$. Hence $f(H)$ is $Nano\ M_J\ Closed$ set in $\left(W_2, \tau_{R'}(Q)\right)$.

$(3) \to (1)$

Let H be $Nano - Open$ set in $\left(W_1, \tau_R(P)\right)$. Then $f(U - H)$ is $Nano\ M_J - Closed$ in $\left(W_2, \tau_{R'}(Q)\right)$. (i.e) $f(H)$ is $Nano\ M_J\ Open$ in $\left(W_2, \tau_{R'}(Q)\right)$. Therefore f is $Nano\ M_J - Open\ map$.

VI. CONCLUSION

In this paper we delivered a $Nano\ M_J - open\ map$ and $Nano\ M_J - closed\ map$ as a weaker form of function in Nano Toplogical spaces and also defined $Nano\ M_J - homeomorphism$ and detailing thier properties with the suitable examples.

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