

ALGEBRAIC OPERATIONS ON PENTAPARTITIONED NEUTROSOPHIC VAGUE SETS

Abstract

In this paper, we present some algebraic operations for Pentapartitioned Neutrosophic Vague Set as an extension of single valued Neutrosophic vague sets. Specifically, we introduce addition, multiplication, scalar multiplication and power for the Pentapartitioned Neutrosophic Vague Sets also the compact spaces with suitable examples and a theorem with proofs.

Keywords: Neutrosophic Set, Vague Set, Neutrosophic Vague Set, Pentapartitioned Neutrosophic Vague Set, Compact space.

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I. INTRODUCTION

In order to solve uncertainty in decision making problems, some theories like *fuzzy set theory* [10] by Lofti A. Zadeh (1965), *Intuitionistic fuzzy set theory* [10] by Atanassov (1986), vague set theory [2] by Gau & Beuhrer (1993) were developed.

In fuzzy set theory, the uncertainty of a set is defined by membership function that fix values called membership degrees to the universal set elements within the range $[0, 1]$. As an extension, in *Intuitionistic fuzzy set theory*, non – membership degree is also taken into account. Later, to solve the problems involving incomplete, indeterminate and inconsistent information, Smarandache developed a new theory called *Neutrosophic set theory* [7] as it includes new membership called *indeterminacy – membershi* . In this pap , We introduced a new concept called *Pentapartitioned Neutrosophic Vague Set(PNVS)* by which includes four membership functions namely *truth membershi* , *contradiction membershi* , *ignorance membership*, unknown membership and *false membership functions*.

In 2010 Wang et al., [9] developed *single valued neutrosophic set (SVNS)* and he defined some basic operations like subset, equality, complement, union and intersection on *SVNS* . Meanwhile Liu and Wang [2] introduced relations such as *addition, multiplication, scalar multiplication and power between two SVNSs*.

Many extensions of *Neutrosophic set* were developed and areas like used in decision making, aggregation operators, image processing etc.,[5].

Following these developments, we present some algebraic operations for *Pentapartitioned Neutrosophic Vague Set* and defined some algebraic operations on *PNVSs* with examples.

II. PRELIMINARIES

Definition 2.1 [7]: Let U be a universe . A **Neutrosophic set** A on X can be defined as follows:

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$$

where $T, I, F: X \rightarrow [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Here $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy and $F_A(x)$ is the degree of non – membership.

For application in real scientific and engineering areas, Wang et al., proposed the concept of a *single valued neutrosophic set* as follows:

Operations between two *SVN – numbers* are defined by [6]. It is recalled as follows:

Let $x = (T_1, I_1, F_1)$ and $y = (T_2, I_2, F_2)$ be two *SVNS* , then the operations are defined as follows:

- a) $x \oplus y = (T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2)$
 b) $x \otimes y = (T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2)$
 c) $\lambda x = ((1 - (1 - T_1)^\lambda), I_1^\lambda, F_1^\lambda)$
 d) $x^\lambda = (T_1^\lambda, (1 - (1 - I_1)^\lambda), (1 - (1 - F_1)^\lambda))$

Definition 2.2 [8]: A *Neutrosophic vague set* ANV (NVS) on X written as

$A = \{ \langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x) \rangle; x \}$, whose truth membership, indeterminacy membership and false membership functions is defined as:

$$\hat{T}_A(x) = [T^-, T^+], \quad \hat{I}_A(x) = [I^-, I^+], \quad \hat{F}_A(x) = [F^-, F^+],$$

Where, (1) $T^+ = 1 - F^-$ (2) $F^+ = 1 - T^-$ and (3) $0 \leq T^- + I^- + F^- \leq 2^+$ when X is continuous, a NVS A can be written as

$$A = \int \langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x) \rangle / x, x \in X$$

When X is discrete, a NVS A can be written as

$$A = \sum_{i=1}^n \langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x) \rangle / x_i, \quad x_i \in X$$

Definition 2.3 [6]: A *Pentapartitioned Neutrosophic Vague Set* A_{PNV} on the universe of discourse written as

$$A_{PNV} = \{ \langle x; \hat{T}_{A_{PNV}}(x); \hat{C}_{A_{PNV}}(x); \hat{G}_{A_{PNV}}(x); \hat{U}_{A_{PNV}}(x); \hat{F}_{A_{PNV}}(x) \rangle; x \in X \}$$

whose truth membership, contradiction membership, ignorance membership, unknown membership, and false membership function is defined as:

$$\begin{aligned} \hat{T}_{A_{PNV}}(x) &= [T^-, T^+], \quad \hat{C}_{A_{PNV}}(x) = [C^-, C^+], \quad \hat{G}_{A_{PNV}}(x) = [G^-, G^+], \\ \hat{U}_{A_{PNV}}(x) &= [U^-, U^+], \quad \hat{F}_{A_{PNV}}(x) = [F^-, F^+], \end{aligned}$$

Where (1) $T^+ = 1 - F^-$ (2) $F^+ = 1 - T^-$ (3) $C^+ = 1 - U^-$ (4) $U^+ = 1 - C^-$
 (5) $0 \leq T^- + C^- + G^- + U^- + F^- \leq 4^+$

Definition 2.4 [6]: Let A_{PNV} and B_{PNV} be two of the universe U . If $\forall x \in U$, $\hat{T}_{A_{PNV}}(x) \leq \hat{T}_{B_{PNV}}(x)$; $\hat{C}_{A_{PNV}}(x) \leq \hat{C}_{B_{PNV}}(x)$;

$\hat{G}_{A_{PNV}}(x) \geq \hat{G}_{B_{PNV}}(x)$; $\hat{U}_{A_{PNV}}(x) \geq \hat{U}_{B_{PNV}}(x)$; $\hat{F}_{A_{PNV}}(x) \geq \hat{F}_{B_{PNV}}(x)$ then the $PNVS$ A_{PNV} is included by B_{PNV} , denoted by $A_{PNV} \subseteq B_{PNV}$.

Definition 2.5 [6]: The complement of **PNVS** A_{PNV} is denoted by A_{PNV}^C and is defined by $\hat{T}_{A_{PNV}}^C(x) = [1 - T^+, 1 - T^-]$, $\hat{C}_{A_{PNV}}^C(x) = [1 - C^+, 1 - C^-]$, $\hat{G}_{A_{PNV}}^C(x) = [1 - G^+, 1 - G^-]$, $\hat{U}_{A_{PNV}}^C(x) = [1 - U^+, 1 - U^-]$, $\hat{F}_{A_{PNV}}^C(x) = [1 - F^+, 1 - F^-]$.

Definition 2.6 [6]: Let A_{PNV} be a **PNVS** on the universe of discourse U where $\forall x \in U$, $\hat{T}_{A_{PNV}}(x) = [1, 1]$, $\hat{C}_{A_{PNV}}(x) = [1, 1]$,

$\hat{G}_{A_{PNV}}(x) = [0, 0]$, $\hat{U}_{A_{PNV}}(x) = [0, 0]$, $\hat{F}_{A_{PNV}}(x) = [0, 0]$. Then A_{PNV} is called unit **PNVS** (1_{PNV} in short).

Definition 2.7 [6]: Let A_{PNV} be a **PNVS** on the universe of discourse U where $\forall x \in U$,

$\hat{T}_{A_{PNV}}(x) = [0, 0]$, $\hat{C}_{A_{PNV}}(x) = [0, 0]$, $\hat{G}_{A_{PNV}}(x) = [1, 1]$, $\hat{U}_{A_{PNV}}(x) = [1, 1]$, $\hat{F}_{A_{PNV}}(x) = [1, 1]$

Then A_{PNV} is called zero **PNVS** (0_{PNV} in short).

Definition 2.8 [6]: Let A_{PNV} and B_{PNV} be two **PNVS** of the universe U . If $\forall x \in U$,

$\hat{T}_{A_{PNV}}(x) = \hat{T}_{B_{PNV}}(x)$; $\hat{C}_{A_{PNV}}(x) = \hat{C}_{B_{PNV}}(x)$; $\hat{G}_{A_{PNV}}(x) = \hat{G}_{B_{PNV}}(x)$; $\hat{U}_{A_{PNV}}(x) =$

$\hat{U}_{B_{PNV}}(x)$; $\hat{F}_{A_{PNV}}(x) = \hat{F}_{B_{PNV}}(x)$ then the **PNVS** A_{PNV} and B_{PNV} are equal

Definition 2.9 [6]: The union of **PNVSs** two A_{PNV} and B_{PNV} is a **PNVS** K_{PNV} written as $K_{PNV} = A_{PNV} \cup B_{PNV}$ whose truth – membership, contradiction – membership, ignorance membership, unknown membership and false membership functions are related to those of A_{PNV} and B_{PNV} by

$$\hat{T}_{K_{PNV}}(x) = [\max(\hat{T}_{A_{PNV}}^-, \hat{T}_{B_{PNV}}^-), \max(\hat{T}_{A_{PNV}}^+, \hat{T}_{B_{PNV}}^+)]$$

$$\hat{C}_{K_{PNV}}(x) = [\max(\hat{C}_{A_{PNV}}^-, \hat{C}_{B_{PNV}}^-), \max(\hat{C}_{A_{PNV}}^+, \hat{C}_{B_{PNV}}^+)]$$

$$\hat{G}_{K_{PNV}}(x) = [\min(\hat{G}_{A_{PNV}}^-, \hat{G}_{B_{PNV}}^-), \min(\hat{G}_{A_{PNV}}^+, \hat{G}_{B_{PNV}}^+)]$$

$$\hat{U}_{K_{PNV}}(x) = [\min(\hat{U}_{A_{PNV}}^-, \hat{U}_{B_{PNV}}^-), \min(\hat{U}_{A_{PNV}}^+, \hat{U}_{B_{PNV}}^+)]$$

$$\hat{F}_{K_{PNV}}(x) = [\min(\hat{F}_{A_{PNV}}^-, \hat{F}_{B_{PNV}}^-), \min(\hat{F}_{A_{PNV}}^+, \hat{F}_{B_{PNV}}^+)]$$

Definition 2.10 [6]: The intersection of two **PNVSs** A_{PNV} and B_{PNV} is a **PNVS** H_{PNV} , written as $H_{PNV} = A_{PNV} \cap B_{PNV}$ whose truth – membership, contradiction – membership, ignorance membership, unknown membership and false – membership functions are related to those of A_{PNV} and B_{PNV} by

$$\hat{T}_{H_{PNV}}(x) = [\min(\hat{T}_{A_{PNV}}^-, \hat{T}_{B_{PNV}}^-), \min(\hat{T}_{A_{PNV}}^+, \hat{T}_{B_{PNV}}^+)]$$

$$\hat{C}_{H_{PNV}}(x) = [\min(\hat{C}_{A_{PNV}}^-, \hat{C}_{B_{PNV}}^-), \min(\hat{C}_{A_{PNV}}^+, \hat{C}_{B_{PNV}}^+)]$$

$$\hat{G}_{HPNV}(x) = \left[\max(\hat{G}_{APNV}^-, \hat{G}_{BPNV}^-), \max(\hat{G}_{APNV}^+, \hat{G}_{BPNV}^+) \right]$$

$$\hat{U}_{HPNV}(x) = \left[\max(\hat{U}_{APNV}^-, \hat{U}_{BPNV}^-), \max(\hat{U}_{APNV}^+, \hat{U}_{BPNV}^+) \right]$$

$$\hat{F}_{HPNV}(x) = \left[\max(\hat{F}_{APNV}^-, \hat{F}_{BPNV}^-), \max(\hat{F}_{APNV}^+, \hat{F}_{BPNV}^+) \right]$$

Definition 2.11 [6]: Let $\{A_{iPNV} : i \in J\}$ be an arbitrary family of $PNVS$ s. Then

$$\bigcup A_{iPNV} = \left\{ \left\langle x; \left(\max_{i \in J}(\hat{T}_{A_{iPNV}}^-), \max_{i \in J}(\hat{T}_{A_{iPNV}}^+) \right), \left(\max_{i \in J}(\hat{C}_{A_{iPNV}}^-), \max_{i \in J}(\hat{C}_{A_{iPNV}}^+) \right), \left(\min_{i \in J}(\hat{G}_{A_{iPNV}}^-), \min_{i \in J}(\hat{G}_{A_{iPNV}}^+) \right) \right\rangle; x \right. \\ \left. \left(\min_{i \in J}(\hat{U}_{A_{iPNV}}^-), \min_{i \in J}(\hat{U}_{A_{iPNV}}^+) \right), \left(\min_{i \in J}(\hat{F}_{A_{iPNV}}^-), \min_{i \in J}(\hat{F}_{A_{iPNV}}^+) \right) \right\} \\ \in X$$

$$\bigcap A_{iPNV} = \left\{ \left\langle x; \left(\max_{i \in J}(\hat{T}_{A_{iPNV}}^-), \max_{i \in J}(\hat{T}_{A_{iPNV}}^+) \right), \left(\max_{i \in J}(\hat{C}_{A_{iPNV}}^-), \max_{i \in J}(\hat{C}_{A_{iPNV}}^+) \right), \left(\min_{i \in J}(\hat{G}_{A_{iPNV}}^-), \min_{i \in J}(\hat{G}_{A_{iPNV}}^+) \right) \right\rangle; x \right. \\ \left. \left(\min_{i \in J}(\hat{U}_{A_{iPNV}}^-), \min_{i \in J}(\hat{U}_{A_{iPNV}}^+) \right), \left(\min_{i \in J}(\hat{F}_{A_{iPNV}}^-), \min_{i \in J}(\hat{F}_{A_{iPNV}}^+) \right) \right\} \\ \in X$$

III. OPERATIONS ON PENTAPARTITIONED NEUTROSOPHIC VAGUE SETS

This section introduces various algebraic operations on PNVS based on SVNS operations. The following are the definitions of addition, multiplication, scalar multiplication, and power operations:

Definition 3.1:

Let

$$A_{PNV} = \left\{ \left\langle x; [\hat{T}_{APNV}^-(x), \hat{T}_{APNV}^+(x)]; [\hat{C}_{APNV}^-(x), \hat{C}_{APNV}^+(x)]; [\hat{G}_{APNV}^-(x), \hat{G}_{APNV}^+(x)]; \right. \right. \\ \left. \left. [\hat{U}_{APNV}^-(x), \hat{U}_{APNV}^+(x)]; [\hat{F}_{APNV}^-(x), \hat{F}_{APNV}^+(x)] \right\rangle; x \right\} \\ \in X$$

$$B_{PNV} = \left\{ \left\langle x; [\hat{T}_{BPNV}^-(x), \hat{T}_{BPNV}^+(x)]; [\hat{C}_{BPNV}^-(x), \hat{C}_{BPNV}^+(x)]; [\hat{G}_{BPNV}^-(x), \hat{G}_{BPNV}^+(x)]; \right. \right. \\ \left. \left. [\hat{U}_{BPNV}^-(x), \hat{U}_{BPNV}^+(x)]; [\hat{F}_{BPNV}^-(x), \hat{F}_{BPNV}^+(x)] \right\rangle; x \in X \right\}$$

be two $PNVSS$. Then the addition of A_{PNV} and B_{PNV} denoted as $(A \oplus B)_{PNV}$ defined by

$$(A \oplus B)_{PNV} = \left\langle \begin{array}{l} x; [\hat{T}^-_{A_{PNV}}(x) + \hat{T}^-_{B_{PNV}}(x) - \hat{T}^-_{A_{PNV}}(x)\hat{T}^-_{B_{PNV}}(x), \hat{T}^+_{A_{PNV}}(x) + \hat{T}^+_{B_{PNV}}(x) - \hat{T}^+_{A_{PNV}}(x)\hat{T}^+_{B_{PNV}}(x)]; \\ [\hat{C}^-_{A_{PNV}}(x) + \hat{C}^-_{B_{PNV}}(x) - \hat{C}^-_{A_{PNV}}(x)\hat{C}^-_{B_{PNV}}(x), \hat{C}^+_{A_{PNV}}(x) + \hat{C}^+_{B_{PNV}}(x) - \hat{C}^+_{A_{PNV}}(x)\hat{C}^+_{B_{PNV}}(x)]; \\ [\hat{G}^-_{A_{PNV}}(x)\hat{G}^-_{B_{PNV}}(x), \hat{G}^+_{A_{PNV}}(x)\hat{G}^+_{B_{PNV}}(x)]; [\hat{U}^-_{A_{PNV}}(x)\hat{U}^-_{B_{PNV}}(x), \hat{U}^+_{A_{PNV}}(x)\hat{U}^+_{B_{PNV}}(x)]; \\ [\hat{F}^-_{A_{PNV}}(x)\hat{F}^-_{B_{PNV}}(x), \hat{F}^+_{A_{PNV}}(x)\hat{F}^+_{B_{PNV}}(x)] \end{array} \right\rangle$$

Example 3.2:

Let $U = \{m, n\}$ be a set of universe and let

$$A_{PNV} = \left\langle \begin{array}{l} \frac{m}{\{[0.3,0.55],[0.45,0.5],[0.2,0.75],[0.55,0.78],[0.65,0.7]\}} \\ \frac{n}{\{[0.4,0.85],[0.35,0.6],[0.45,0.8],[0.15,0.76],[0.37,0.9]\}} \end{array} \right\rangle,$$

$$B_{PNV} = \left\langle \begin{array}{l} \frac{m}{\{[0.43,0.65], [0.56,0.7], [0.37,0.68], [0.59,0.8], [0.4,0.65]\}} \\ \frac{n}{\{[0.32,0.65], [0.5,0.86], [0.25,0.5], [0.4,0.65], [0.6,0.95]\}} \end{array} \right\rangle$$

Then $(A \oplus B)_{PNV}$

$$= \left\langle \begin{array}{l} \frac{m}{\{[0.601,0.8425], [0.758,0.85], [0.074,0.51], [0.3245,0.624], [0.26,0.455]\}} \\ \frac{n}{\{[0.592,0.9475], [0.675,0.944], [0.1125,0.4], [0.06,0.494], [0.18,0.855]\}} \end{array} \right\rangle$$

which is again a $PNVS$.

Definition 3.3:

Let $A_{PNV} = \left\langle \begin{array}{l} x; [\hat{T}^-_{A_{PNV}}(x), \hat{T}^+_{A_{PNV}}(x)]; [\hat{C}^-_{A_{PNV}}(x), \hat{C}^+_{A_{PNV}}(x)]; [\hat{G}^-_{A_{PNV}}(x), \hat{G}^+_{A_{PNV}}(x)]; \\ [\hat{U}^-_{A_{PNV}}(x), \hat{U}^+_{A_{PNV}}(x)]; [\hat{F}^-_{A_{PNV}}(x), \hat{F}^+_{A_{PNV}}(x)] \end{array} \right\rangle; x \in X$

$B_{PNV} = \left\langle \begin{array}{l} x; [\hat{T}^-_{B_{PNV}}(x), \hat{T}^+_{B_{PNV}}(x)]; [\hat{C}^-_{B_{PNV}}(x), \hat{C}^+_{B_{PNV}}(x)]; [\hat{G}^-_{B_{PNV}}(x), \hat{G}^+_{B_{PNV}}(x)]; \\ [\hat{U}^-_{B_{PNV}}(x), \hat{U}^+_{B_{PNV}}(x)]; [\hat{F}^-_{B_{PNV}}(x), \hat{F}^+_{B_{PNV}}(x)] \end{array} \right\rangle; x \in X$

be two $PNVSS$. Then the addition of A_{PNV} and B_{PNV} denoted as $(A \otimes B)_{PNV}$ defined by

$$(A \otimes B)_{PNV} = \left\langle \begin{array}{l} x; [\hat{T}^-_{APNV}(x)\hat{T}^-_{BPNV}(x), \hat{T}^+_{APNV}(x)\hat{T}^+_{BPNV}(x)]; [\hat{C}^-_{APNV}(x)\hat{C}^-_{BPNV}(x), \hat{C}^+_{APNV}(x)\hat{C}^+_{BPNV}(x)]; \\ [\hat{G}^-_{APNV}(x) + \hat{G}^-_{BPNV}(x) - \hat{G}^-_{APNV}(x)\hat{G}^-_{BPNV}(x), \hat{G}^+_{APNV}(x) + \hat{G}^+_{BPNV}(x) - \hat{G}^+_{APNV}(x)\hat{G}^+_{BPNV}(x)]; \\ [\hat{U}^-_{APNV}(x) + \hat{U}^-_{BPNV}(x) - \hat{U}^-_{APNV}(x)\hat{U}^-_{APNV}(x), \hat{U}^+_{APNV}(x) + \hat{U}^+_{BPNV}(x) - \hat{U}^+_{APNV}(x)\hat{U}^-_{BPNV}(x)]; \\ [\hat{F}^-_{APNV}(x) + \hat{F}^-_{BPNV}(x) - \hat{F}^-_{APNV}(x)\hat{F}^-_{APNV}(x), \hat{F}^+_{APNV}(x) + \hat{F}^+_{BPNV}(x) - \hat{F}^+_{APNV}(x)\hat{F}^-_{BPNV}(x)] \end{array} \right\rangle$$

Example 3.4:

Let $U = \{m, n\}$ be a set of universe and let

$$A_{PNV} = \left\langle \begin{array}{l} \overline{\{[0.25,0.43],[0.36,0.48],[0.35,0.62],[0.47,0.6],[0.45,0.55]\}}^m \\ \overline{\{[0.53,0.55],[0.4,0.85],[0.26,0.64],[0.5,0.75],[0.45,0.76]\}}^n \end{array} \right\rangle,$$

$$B_{PNV} = \left\langle \begin{array}{l} \overline{\{[0.35,0.59],[0.43,0.55],[0.15,0.4],[0.29,0.65],[0.52,0.71]\}}^m \\ \overline{\{[0.45,0.8],[0.37,0.9],[0.25,0.5],[0.25,0.4],[0.47,0.5]\}}^n \end{array} \right\rangle$$

Then $(A \otimes B)_{PNV}$

$$= \left\langle \begin{array}{l} \overline{\{[0.0875,0.2537],[0.1548,0.264],[0.4475,0.772],[0.6237,0.86],[0.736,0.8695]\}}^m \\ \overline{\{[0.2385,0.44],[0.148,0.765],[0.445,0.82],[0.625,0.85],[0.7085,0.88]\}}^n \end{array} \right\rangle$$

Definition 3.5:

Let

$$A_{PNV} = \left\langle \begin{array}{l} x; [\hat{T}^-_{APNV}(x), \hat{T}^+_{APNV}(x)]; [\hat{C}^-_{APNV}(x), \hat{C}^+_{APNV}(x)]; [\hat{G}^-_{APNV}(x), \hat{G}^+_{APNV}(x)]; \\ [\hat{U}^-_{APNV}(x), \hat{U}^+_{APNV}(x)]; [\hat{F}^-_{APNV}(x), \hat{F}^+_{APNV}(x)] \end{array} \right\rangle; x \in X$$

be a $PNVS$. Then the scalar (λ) multiplication of A_{PNV} denoted as λA_{PNV} defined by

$$\lambda A_{PNV} = \left\langle \begin{array}{l} x; \left[1 - (1 - \hat{T}^-_{APNV}(x))^\lambda, 1 - (1 - \hat{T}^+_{APNV}(x))^\lambda \right]; \\ \left[1 - (1 - \hat{C}^-_{APNV}(x))^\lambda, 1 - (1 - \hat{C}^+_{APNV}(x))^\lambda \right]; \\ \left[(\hat{G}^-_{APNV}(x))^\lambda, (\hat{G}^+_{APNV}(x))^\lambda \right]; \\ \left[(\hat{U}^-_{APNV}(x))^\lambda, (\hat{U}^+_{APNV}(x))^\lambda \right]; \\ \left[(\hat{F}^-_{APNV}(x))^\lambda, (\hat{F}^+_{APNV}(x))^\lambda \right] \end{array} \right\rangle; x \in X$$

Example 3.6:

Let $U = \{m, n\}$ be a set of universe, let

$$A_{PNV} = \left\{ \frac{m}{\{[0.3,0.5], [0.4,0.6], [0.2,0.7], [0.3,0.6], [0.5,0.8]\}}, \frac{n}{\{[0.1,0.6], [0.6,0.7], [0.3,0.4], [0.5,0.7], [0.6,0.9]\}} \right\}$$

be a $PNVS$ and if $\lambda = 3$, then

$$\lambda A_{PNV} = \left\{ \frac{m}{\{[1 - (1 - 0.3)^3, 1 - (1 - 0.5)^3], [1 - (1 - 0.4)^3, 1 - (1 - 0.6)^3], [(0.2)^3, (0.7)^3], [(0.3)^3, (0.6)^3], [(0.5)^3, (0.8)^3]\}}, \frac{n}{\{[1 - (1 - 0.1)^3, 1 - (1 - 0.6)^3], [1 - (1 - 0.6)^3, 1 - (1 - 0.7)^3], [(0.3)^3, (0.4)^3], [(0.5)^3, (0.7)^3], [(0.6)^3, (0.9)^3]\}} \right\}$$

$$\lambda A_{PNV} = \left\{ \frac{m}{\{[0.657,0.875], [0.784,0.936], [0.008,0.343], [0.027,0.216], [0.125,0.512]\}}, \frac{n}{\{[0.271,0.936], [0.936,0.973], [0.027,0.064], [0.125,0.343], [0.216,0.729]\}} \right\}$$

Definition 3.7:

Let $A_{PNV} = \left\{ x; \left[\hat{T}^-_{APNV}(x), \hat{T}^+_{APNV}(x) \right]; \left[\hat{C}^-_{APNV}(x), \hat{C}^+_{APNV}(x) \right]; \left[\hat{G}^-_{APNV}(x), \hat{G}^+_{APNV}(x) \right]; \left[\hat{U}^-_{APNV}(x), \hat{U}^+_{APNV}(x) \right]; \left[\hat{F}^-_{APNV}(x), \hat{F}^+_{APNV}(x) \right] \right\}; x \in X$ be a $PNVS$.

Then, $PNVS$ power denoted by A_{PNV}^λ is defined by

$$A_{PNV}^\lambda = \left\{ \left\langle x; \left[\left(\hat{T}^-_{APNV}(x) \right)^\lambda, \left(\hat{T}^+_{APNV}(x) \right)^\lambda \right]; \left[\left(\hat{C}^-_{APNV}(x) \right)^\lambda, \left(\hat{C}^+_{APNV}(x) \right)^\lambda \right]; \left[1 - \left(1 - \hat{G}^-_{APNV}(x) \right)^\lambda, 1 - \left(1 - \hat{G}^+_{APNV}(x) \right)^\lambda \right]; \left[1 - \left(1 - \hat{U}^-_{APNV}(x) \right)^\lambda, 1 - \left(1 - \hat{U}^+_{APNV}(x) \right)^\lambda \right]; \left[1 - \left(1 - \hat{F}^-_{APNV}(x) \right)^\lambda, 1 - \left(1 - \hat{F}^+_{APNV}(x) \right)^\lambda \right] \right\rangle; x \in X \right\}$$

Example 3.8:

Let $U = \{m, n\}$ be a set of universe, let

$$A_{PNV} = \left\{ \frac{m}{\{[0.3,0.5], [0.4,0.6], [0.2,0.7], [0.3,0.6], [0.5,0.8]\}}, \frac{n}{\{[0.1,0.6], [0.6,0.7], [0.3,0.4], [0.5,0.7], [0.6,0.9]\}} \right\}$$

be a $PNVS$ and if $\lambda = 2$, then

$$A_{PNV}^\lambda = \left\{ \frac{\left\{ [(0.3)^2, (0.5)^2], [(0.4)^2, (0.6)^2], [1 - (1 - 0.2)^2, 1 - (1 - 0.7)^2], [1 - (1 - 0.3)^2, 1 - (1 - 0.6)^2], [1 - (1 - 0.5)^2, 1 - (1 - 0.8)^2] \right\}}{n} \right\}^m$$

$$A_{PNV}^\lambda = \left\{ \frac{\left\{ [0.09, 0.25], [0.16, 0.36], [0.36, 0.91], [0.51, 0.84], [0.75, 0.96] \right\}}{n} \right\}^m$$

Theorem 3.9:

Let A_{PNV} and B_{PNV} be two PNVS and $\lambda, \lambda_1, \lambda_2 > 0$. Then

- i) $(A \oplus B)_{PNV} = (B \oplus A)_{PNV}$
- ii) $(A \otimes B)_{PNV} = (B \otimes A)_{PNV}$
- iii) $\lambda(A \oplus B)_{PNV} = \lambda A_{PNV} \oplus \lambda B_{PNV}$
- iv) $\lambda_1 A_{PNV} \oplus \lambda_2 A_{PNV} = (\lambda_1 \oplus \lambda_2) A_{PNV}$
- v) $A_{PNV}^{\lambda_1} \otimes A_{PNV}^{\lambda_2} = A_{PNV}^{\lambda_1 + \lambda_2}$
- vi) $A_{PNV}^\lambda \otimes B_{PNV}^\lambda = (A \otimes B)_{PNV}^\lambda$

Proof:

The results (i), (ii), (iv) and (vi) are obvious, thus we prove the others.

Proof of (iii)

By definition, we have

$$(A \oplus B)_{PNV} = \left\{ \begin{array}{l} x; [\hat{T}^-_{A_{PNV}}(x) + \hat{T}^-_{B_{PNV}}(x) - \hat{T}^-_{A_{PNV}}(x)\hat{T}^-_{B_{PNV}}(x), \hat{T}^+_{A_{PNV}}(x) + \hat{T}^+_{B_{PNV}}(x) - \hat{T}^+_{A_{PNV}}(x)\hat{T}^+_{B_{PNV}}(x)]; \\ [\hat{C}^-_{A_{PNV}}(x) + \hat{C}^-_{B_{PNV}}(x) - \hat{C}^-_{A_{PNV}}(x)\hat{C}^-_{B_{PNV}}(x), \hat{C}^+_{A_{PNV}}(x) + \hat{C}^+_{B_{PNV}}(x) - \hat{C}^+_{A_{PNV}}(x)\hat{C}^+_{B_{PNV}}(x)]; \\ [\hat{G}^-_{A_{PNV}}(x)\hat{G}^-_{B_{PNV}}(x), \hat{G}^+_{A_{PNV}}(x)\hat{G}^+_{B_{PNV}}(x)]; [\hat{U}^-_{A_{PNV}}(x)\hat{U}^-_{B_{PNV}}(x), \hat{U}^+_{A_{PNV}}(x)\hat{U}^+_{B_{PNV}}(x)]; \\ [\hat{F}^-_{A_{PNV}}(x)\hat{F}^-_{B_{PNV}}(x), \hat{F}^+_{A_{PNV}}(x)\hat{F}^+_{B_{PNV}}(x)] \end{array} \right\}$$

$$\lambda(A \oplus B)_{PNV} = \left\{ \begin{array}{l} x; \left[1 - \left(1 - (\hat{T}^-_{A_{PNV}}(x) + \hat{T}^-_{B_{PNV}}(x) - \hat{T}^-_{A_{PNV}}(x)\hat{T}^-_{B_{PNV}}(x))^\lambda \right), 1 - \left(1 - (\hat{T}^+_{A_{PNV}}(x) + \hat{T}^+_{B_{PNV}}(x) - \hat{T}^+_{A_{PNV}}(x)\hat{T}^+_{B_{PNV}}(x))^\lambda \right) \right]; \\ \left[1 - \left(1 - (\hat{C}^-_{A_{PNV}}(x) + \hat{C}^-_{B_{PNV}}(x) - \hat{C}^-_{A_{PNV}}(x)\hat{C}^-_{B_{PNV}}(x))^\lambda \right), 1 - \left(1 - (\hat{C}^+_{A_{PNV}}(x) + \hat{C}^+_{B_{PNV}}(x) - \hat{C}^+_{A_{PNV}}(x)\hat{C}^+_{B_{PNV}}(x))^\lambda \right) \right]; \\ \left[(\hat{G}^-_{A_{PNV}}(x)\hat{G}^-_{B_{PNV}}(x))^\lambda, (\hat{G}^+_{A_{PNV}}(x)\hat{G}^+_{B_{PNV}}(x))^\lambda \right]; \\ \left[(\hat{U}^-_{A_{PNV}}(x)\hat{U}^-_{B_{PNV}}(x))^\lambda, (\hat{U}^+_{A_{PNV}}(x)\hat{U}^+_{B_{PNV}}(x))^\lambda \right]; \\ \left[(\hat{F}^-_{A_{PNV}}(x)\hat{F}^-_{B_{PNV}}(x))^\lambda, (\hat{F}^+_{A_{PNV}}(x)\hat{F}^+_{B_{PNV}}(x))^\lambda \right] \end{array} \right\}; x \quad xx \in X$$

We solve truth membership functions

$$= 1 - \left(1 - \left(\hat{T}^-_{A_{PNV}}(x) + \hat{T}^-_{B_{PNV}}(x) - \hat{T}^-_{A_{PNV}}(x)\hat{T}^-_{B_{PNV}}(x) \right)^\lambda \right)$$

$$\begin{aligned}
 &= 1 - \left(1 - \hat{T}_{APNV}^-(x) - \hat{T}_{BPNV}^-(x) + \hat{T}_{APNV}^-(x)\hat{T}_{BPNV}^-(x)\right)^\lambda \\
 &= 1 - \left(\left(1 - \hat{T}_{APNV}^-(x)\right) - \hat{T}_{BPNV}^-(x)\left(1 - \hat{T}_{APNV}^-(x)\right)\right)^\lambda \\
 &= 1 - \left(\left(1 - \hat{T}_{APNV}^-(x)\right)\left(1 - \hat{T}_{BPNV}^-(x)\right)\right)^\lambda
 \end{aligned}$$

By similar calculation, we get

$$\lambda(A \oplus B)_{PNV} = \left\langle x; \begin{array}{l} \left[1 - \left(\left(1 - \hat{T}_{APNV}^-(x)\right)\left(1 - \hat{T}_{BPNV}^-(x)\right)\right)^\lambda, 1 - \left(\left(1 - \hat{T}_{APNV}^+(x)\right)\left(1 - \hat{T}_{BPNV}^+(x)\right)\right)^\lambda\right]; \\ \left[1 - \left(\left(1 - \hat{T}_{APNV}^-(x)\right)\left(1 - \hat{T}_{BPNV}^-(x)\right)\right)^\lambda, 1 - \left(\left(1 - \hat{T}_{APNV}^+(x)\right)\left(1 - \hat{T}_{BPNV}^+(x)\right)\right)^\lambda\right]; \\ \left[\left(\hat{G}_{APNV}^-(x)\hat{G}_{BPNV}^-(x)\right)^\lambda, \left(\hat{G}_{APNV}^+(x)\hat{G}_{BPNV}^+(x)\right)^\lambda\right]; \\ \left[\left(\hat{U}_{APNV}^-(x)\hat{U}_{BPNV}^-(x)\right)^\lambda, \left(\hat{U}_{APNV}^+(x)\hat{U}_{BPNV}^+(x)\right)^\lambda\right]; \\ \left[\left(\hat{F}_{APNV}^-(x)\hat{F}_{BPNV}^-(x)\right)^\lambda, \left(\hat{F}_{APNV}^+(x)\hat{F}_{BPNV}^+(x)\right)^\lambda\right] \end{array} \right\rangle_{x \in X} \rightarrow (1)$$

Now

$$\lambda A_{PNV} \oplus \lambda B_{PNV} = \left\langle x; \begin{array}{l} \left[\left(\left(1 - \left(1 - \hat{T}_{APNV}^-(x)\right)^\lambda\right) + \left(1 - \left(1 - \hat{T}_{BPNV}^-(x)\right)^\lambda\right) - \left(1 - \left(1 - \hat{T}_{APNV}^-(x)\right)^\lambda\right)\left(1 - \left(1 - \hat{T}_{BPNV}^-(x)\right)^\lambda\right)\right), \right. \\ \left.\left[\left(\left(1 - \left(1 - \hat{T}_{APNV}^+(x)\right)^\lambda\right) + \left(1 - \left(1 - \hat{T}_{BPNV}^+(x)\right)^\lambda\right) - \left(1 - \left(1 - \hat{T}_{APNV}^+(x)\right)^\lambda\right)\left(1 - \left(1 - \hat{T}_{BPNV}^+(x)\right)^\lambda\right)\right)\right]; \\ \left[\left(\left(1 - \left(1 - \hat{C}_{APNV}^-(x)\right)^\lambda\right) + \left(1 - \left(1 - \hat{C}_{BPNV}^-(x)\right)^\lambda\right) - \left(1 - \left(1 - \hat{C}_{APNV}^-(x)\right)^\lambda\right)\left(1 - \left(1 - \hat{C}_{BPNV}^-(x)\right)^\lambda\right)\right), \right. \\ \left.\left[\left(\left(1 - \left(1 - \hat{C}_{APNV}^+(x)\right)^\lambda\right) + \left(1 - \left(1 - \hat{C}_{BPNV}^+(x)\right)^\lambda\right) - \left(1 - \left(1 - \hat{C}_{APNV}^+(x)\right)^\lambda\right)\left(1 - \left(1 - \hat{C}_{BPNV}^+(x)\right)^\lambda\right)\right)\right]; \\ \left[\left(\hat{G}_{APNV}^-(x)\right)^\lambda \left(\hat{G}_{BPNV}^-(x)\right)^\lambda, \left(\hat{G}_{APNV}^+(x)\hat{G}_{BPNV}^+(x)\right)^\lambda\right]; \left[\left(\hat{U}_{APNV}^-(x)\right)^\lambda \left(\hat{U}_{BPNV}^-(x)\right)^\lambda, \left(\hat{U}_{APNV}^+(x)\hat{U}_{BPNV}^+(x)\right)^\lambda\right]; \\ \left[\left(\hat{F}_{APNV}^-(x)\hat{F}_{BPNV}^-(x)\right)^\lambda, \left(\hat{F}_{APNV}^+(x)\hat{F}_{BPNV}^+(x)\right)^\lambda\right] \end{array} \right\rangle_{x \in X}$$

Consider the truth membership function

$$\begin{aligned}
 &= \left(1 - \left(1 - \hat{T}_{APNV}^-(x)\right)^\lambda\right) + \left(1 - \left(1 - \hat{T}_{BPNV}^-(x)\right)^\lambda\right) - \left(1 - \left(1 - \hat{T}_{APNV}^-(x)\right)^\lambda\right)\left(1 - \left(1 - \hat{T}_{BPNV}^-(x)\right)^\lambda\right) \\
 &= 2 - \left[\left(1 - \hat{T}_{APNV}^-(x)\right)^\lambda + \left(1 - \hat{T}_{BPNV}^-(x)\right)^\lambda\right] - \left[1 - \left(1 - \hat{T}_{APNV}^-(x)\right)^\lambda - \left(1 - \hat{T}_{BPNV}^-(x)\right)^\lambda + \left(1 - \hat{T}_{APNV}^-(x)\right)^\lambda \left(1 - \hat{T}_{BPNV}^-(x)\right)^\lambda\right] \\
 &= 2 - \left(1 - \hat{T}_{APNV}^-(x)\right)^\lambda - \left(1 - \hat{T}_{BPNV}^-(x)\right)^\lambda - 1 + \left(1 - \hat{T}_{APNV}^-(x)\right)^\lambda + \left(1 - \hat{T}_{BPNV}^-(x)\right)^\lambda - \left(1 - \hat{T}_{APNV}^-(x)\right)^\lambda \left(1 - \hat{T}_{BPNV}^-(x)\right)^\lambda \\
 &= 1 - \left(\left(1 - \hat{T}_{APNV}^-(x)\right)^\lambda \left(1 - \hat{T}_{BPNV}^-(x)\right)^\lambda\right) \\
 &= 1 - \left[\left(1 - \hat{T}_{APNV}^-(x)\right)\left(1 - \hat{T}_{BPNV}^-(x)\right)\right]^\lambda
 \end{aligned}$$

In the similar manner, we have

$$\lambda A_{PNV} \oplus \lambda B_{PNV} = \left\{ x; \left[1 - \left((1 - \hat{T}_{APNV}^-(x))(1 - \hat{T}_{BPNV}^-(x)) \right)^\lambda, 1 - \left((1 - \hat{T}_{APNV}^+(x))(1 - \hat{T}_{BPNV}^+(x)) \right)^\lambda \right]; \right. \\ \left. \left[1 - \left((1 - \hat{C}_{APNV}^-(x))(1 - \hat{C}_{BPNV}^-(x)) \right)^\lambda, 1 - \left((1 - \hat{C}_{APNV}^+(x))(1 - \hat{C}_{BPNV}^+(x)) \right)^\lambda \right]; \right. \\ \left. \left[\left(\hat{G}_{APNV}^-(x)\hat{G}_{BPNV}^-(x) \right)^\lambda, \left(\hat{G}_{APNV}^+(x)\hat{G}_{BPNV}^+(x) \right)^\lambda \right]; \left[\left(\hat{U}_{APNV}^-(x)\hat{U}_{BPNV}^-(x) \right)^\lambda, \left(\hat{U}_{APNV}^+(x)\hat{U}_{BPNV}^+(x) \right)^\lambda \right] \right. \\ \left. \left[\left(\hat{F}_{APNV}^-(x)\hat{F}_{BPNV}^-(x) \right)^\lambda, \left(\hat{F}_{APNV}^+(x)\hat{F}_{BPNV}^+(x) \right)^\lambda \right] \right\}; x \in X \rightarrow (2)$$

From (1) and (2), we have $\lambda(A \oplus B)_{PNV} = \lambda A_{PNV} \oplus \lambda B_{PNV}$

Proof of (v):

The proof is similar as the proof of (iii).

IV. PENTAPARTITIONED NEUTROSOPHIC VAGUE COMPACT SPACE

Definition 4.1:

Let (X_{PNV}, τ_{PNV}) be *PNVTS*.

i) If a family $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle; i \in J\}$ of *PNVOS* in X satisfies the condition $\cup \{\langle x, T_A, C_A, G_A, U_A, F_A \rangle; i \in J\} = 1_{PNV}$, then it is called *Pentapartitioned Neutrosophic vague open cover* of X . A finite subfamily of *Pentapartitioned neutrosophic vague open cover* $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle; i \in J\}$ of X , which is also a *pentapartitioned neutrosophic vague cover* of X , is called *Pentapartitioned neutrosophic vague finite subcover* of $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle; i \in J\}$.

ii) A family $\{\langle x, T_B, C_B, G_B, U_B, F_B \rangle; i \in J\}$ of *PNVCS* in X satisfies finite intersection property iff every finite subfamily $\{\langle x, T_B, C_B, G_B, U_B, F_B \rangle; i = 1, 2, \dots, n\}$ of the family satisfies the condition $\cap_{i=1}^n \{\langle x, T_{B_i}, C_{B_i}, G_{B_i}, U_{B_i}, F_{B_i} \rangle\} \neq 0_{QNV}$.

Definition 4.2: A *PNVTS* (X_{PNV}, τ_{PNV}) is *Pentapartitioned neutrosophic vague compact* iff every *Pentapartitioned neutrosophic vague open cover* of X has a *Pentapartitioned neutrosophic vague finite subcover*.

Corollary 4.3: A *PNVTS* (X_{PNV}, τ_{PNV}) is *Pentapartitioned neutrosophic vague compact* iff every family $\{\langle x, T_B, C_B, G_B, U_B, F_B \rangle; i \in J\}$ of *PNVCSs* in X having the *FIP* has nonempty intersection.

Corollary 4.4: Let (X_{PNV}, τ_{PNV}) , (Y_{PNV}, σ_{PNV}) be PNVTSS and $f: (X_{PNV}, \tau_{PNV}) \rightarrow (Y_{PNV}, \sigma_{PNV})$ a Pentapartitioned neutrosophic vague continuous surjection. If (X_{PNV}, τ_{PNV}) is Pentapartitioned neutrosophic vague compact, then so is (Y_{PNV}, σ_{PNV}) .

Definition 4.5: Let (X_{PNV}, τ_{PNV}) be PNVTSS and A_{PNV} a PNVS in X .

- i) If a family $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle : i \in J\}$ of PNVS in X satisfies the condition $A_{PNV} \subseteq \bigcup \{\langle x, T_A, C_A, G_A, U_A, F_A \rangle : i \in J\}$, then it is called Pentapartitioned neutrosophic vague open cover of A_{PNV} . A finite subfamily of Pentapartitioned neutrosophic vague open cover $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle : i \in J\}$ of A_{PNV} , which is also a pentapartitioned neutrosophic vague cover of A_{PNV} , is called Pentapartitioned neutrosophic vague finite subcover of $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle : i \in J\}$.
- ii) A PNVS in a PNVTSS (X_{PNV}, τ_{PNV}) is called Pentapartitioned neutrosophic vague compact iff every Pentapartitioned neutrosophic vague cover A_{PNV} of has finite subcover.

Corollary 4.6: Let (X_{PNV}, τ_{PNV}) , (Y_{PNV}, σ_{PNV}) be PNVTSS and $f: (X_{PNV}, \tau_{PNV}) \rightarrow (Y_{PNV}, \sigma_{PNV})$ a Pentapartitioned neutrosophic vague continuous function. If A_{PNV} is Pentapartitioned neutrosophic vague compact in (X_{PNV}, τ_{PNV}) , then so is $f(A_{PNV})$ in (Y_{PNV}, σ_{PNV}) .

V. CONCLUSION

In this paper, we defined some new operations on Pentapartitioned Neutrosophic Vague Set under neutrosophic environment. The basic algebraic operations on Pentapartitioned neutrosophic vague sets namely addition, multiplication, scalar multiplication and power also compact space along with illustrative examples were presented. Subsequently, the basic properties of these operations such as commutative law and relevant laws are mathematically proven. This new extension will broaden the fundamental knowledge of existing set theories and subsequently could be applied to real life experiments where truthness, indeterminacy and falsity could be dealt with.

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