BIANCHI TYPE-V INFLATIONARY COSMOLOGICAL MODEL WITH FLAT POTENTIAL FOR BAROTROPIC PERFECT FLUID DISTRIBUTION IN GENERAL RELATIVITY

Abstract

We have examined the Bianchi type-V inflationary world with flat potential for barotropic perfect fluid distribution under general relativity. Our assumption is that the isotropic pressure p, is proportional to the appropriate energy density ρ, in order to derive the deterministic solution of the model, which leads to $p = \gamma\rho$ and potential $V(\phi)$ as constant. The model's behavior based on geometric and physical aspects is also discussed.

Keywords: Bianchi Type-V, Inflationary Cosmological, Barotropic Perfect fluid, Flat Potential, Cosmology.

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I. INTRODUCTION

Over the past few years, there has been significant enthusiasm surrounding cosmological models of the universe, as they hold a key role in unraveling the enigmas associated with its initial development. Among these models, inflationary theories stand out for their crucial contribution to addressing various longstanding cosmological puzzles, such as the uniformity, isotropy, and flatness observed in the universe. The conventional explanation for the universe's flatness posits that it underwent an early phase of exponential expansion known as "inflation". The early cosmos expanded rapidly, by a factor of 10^{78}, due to a negative pressure vacuum energy density, a process known as inflation.

Self-interacting scalar fields are widely recognized as being essential to the study of inflationary cosmology. As a plausible natural explanation for the universal expansion's observed large-scale homogeneity and near-critical density (flatness), Guth [15] has discussed the inflationary cosmos. A universe with only a minor amount of anisotropy will quickly become isotropized by inflation. Several authors have explored various facets of the inflationary universe in general relativity, including Guth [15], Linde [17], Albrecht and Steinhardt [2], Abbott and Wise [1], Mijic et al. [18], Rothman and Ellis [23], Earman and Mosterin [12], and Ainsworth [3].

Numerous iterations of the inflationary scenario have been explored by various researchers, including La and Steinhardt [16], Bali [7], who delved into the significance of inflation in achieving isotropy within the universe. This inflationary framework has also garnered support from observations of the As mentioned by Bassett et al. [10], the Cosmic Microwave Background (CMB). In inflationary models, the universe goes through a phase change that is represented by the Higgs field's evolution. (ϕ). Inflationary expansion occurs when the potential $V(\phi)$ exhibits a flat region, where the ϕ field evolves slowly, while the universe undergoes exponential expansion driven by vacuum field energy, as originally proposed by Stein-Schabes [31]. The potential's flat region naturally maps to a vacuum energy, or an effective cosmological constant (Λ), which starts an inflationary era.

Various investigations into inflationary scenarios have been undertaken in diverse cosmological contexts. Bali and Goyal [8] investigated inflation during the radiation-dominated phase in a Bianchi Type-V space-time with variable bulk viscosity and dark energy. Poonia et al. [19] investigated a general relativistic Bianchi Type-VI inflationary cosmic model with a huge string source. A Bianchi Type-V inflationary universe with declining vacuum energy (λ) was discussed by Bali and Singh [4]. Bali and Kumari [5] examined a Bianchi Type-V inflationary universe characterized by a flat potential and a stiff fluid distribution within the framework of general relativity. Bali and Kumari [6] derived a chaotic inflation scenario within a spatially homogeneous Bianchi Type-V space-time. Reddy et al. [21] investigated an axially symmetric inflationary universe within the context of general relativity. Reddy [22] explored a Bianchi Type-V inflationary universe within the same framework.

Furthermore, Bali and Saraf [9] delved into a cosmological model incorporating bulk viscous creation field and a cosmological term in a Bianchi Type-I space-time. Sharma and Poonia [24] obtained a Bianchi Type-I inflationary cosmological model incorporating bulk viscosity within general relativity. Poonia and Sharma [20] discussed an inflationary scenario within a Bianchi Type-II space-time, considering bulk viscosity within the framework of general relativity. Additionally, other cosmological modes have been explored by researchers such as Brahma and Dewri [11], Elli et al. [13], Gron and Hervik [14], Shri Ram and Singh [25], Shri Ram et al. [26, 27], Singh et al. [28], Singh and Tiwari [29], Singh et al. [30], Tyagi and Singh [32], and Verma and Shri Ram [33, 34], among others.

Motivated by the conversation above, we have studied the general relativistic Bianchi type-V inflationary cosmological model with flat potential for barotropic perfect fluid distribution. We assume that V(ϕ) is constant and that the isotropic pressure (p), is proportional to the correct energy density (ρ) , for the full solution of the field equation. There is further discussion of the model's geometrical and physical properties.

II. THE METRIC AND FIELD EQUATIONS

We consider Bianchi type-V line element in the form as:

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2x}(B^2 dy^2 + C^2 dz^2) \qquad (1)$$

In which A(t), B(t) and C(t) are cosmic scale functions.

We assume the co-ordinate to be co-moving so that
$v^1 = 0 = v^2 = v^3,\ v^4 = 1$

In case of gravity minimally coupled to a scalar field with potential V(ϕ), is given by Stein-Schabes [31], we have

$$S = \int \sqrt{-g}\left[R - \frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - V(\phi)\right] \mathrm{d}^4 x \qquad (2)$$

The Einstein's field equation (in gravitational units $c = 8\pi G = 1$), in the case of massless scalar field ϕ with potential $V(\phi)$ are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \qquad (3)$$

with

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij} + \partial_i \phi \partial_j \phi - \left[\frac{1}{2} \partial_r \phi \partial^r \phi + V(\phi)\right] g_{ij} \qquad (4)$$

Here, p stands for the isotropic pressure and ρ for the energy density, v_i is the unit time like vector, $V(\phi)$ is the potential, and ϕ stands for the Higgs field.

The conservation relation leads to

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, \partial^\mu \phi\right) = -\frac{dV}{d\phi} \qquad (5)$$

The non-linear differential equations that follow are derived from field equation (3) of Einstein for the line element (1)

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}-\frac{1}{A^2}=-p-\frac{1}{2}\phi_4^2+V(\phi) \tag{6}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}-\frac{1}{A^2}=-p-\frac{1}{2}\phi_4^2+V(\phi) \tag{7}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{1}{A^2}=-p-\frac{1}{2}\phi_4^2+V(\phi) \tag{8}$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{3}{A^2}=\rho+\frac{1}{2}\phi_4^2+V(\phi) \tag{9}$$

$$\frac{2A_4}{A}-\frac{B_4}{B}-\frac{C_4}{C}=0 \tag{10}$$

Equation (10) leads to

$$A^2=mBC \tag{11}$$

where m is constant of integration.

The equation (5) for scalar field (ϕ) leads to

$$\phi_{44}+\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)\phi_4=-\frac{dV}{d\phi} \tag{12}$$

where suffix '4' indicates derivative with respect to time t.

III. SOLUTION OF FIELD EQUATIONS

Equations (6) to (10) constitute a set of five distinct equations involving the unknown parameters A, B, C, ρ, p, and ϕ. To arrive at a deterministic solution, we adopt the following conditions:

- V(ϕ) is constant
  i.e. V(ϕ) = K (13)
- The isotropic presence (p) is proportional to the proper energy density (ρ)
  i.e. p = γρ, $0\le\gamma\le 1$ and $\theta=3H$, $\rho=3H^2$ (14)

Equations (12) and (13) lead to

$$\phi_4=\frac{E}{ABC} \tag{15}$$

where E is the integration constant.

The scale factor R for line-element (1) is given by

$$R^3=A^3,\ m=1 \tag{16}$$

From equations (7) and (8), we get

$$C^2\left(\frac{B}{C}\right)_4=\frac{F}{\sqrt{BC}} \tag{17}$$

…(4.3.7)

where F is constant of integration.

We assume $BC = \mu$ and $\frac{B}{C} = \nu$ in equation (17), we get

$$\frac{\nu_4}{\nu} = \frac{F}{\mu^{\frac{3}{2}}} \tag{18}$$

Equations (6) and (9), lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{1}{2}\left(\frac{B_4}{B} + \frac{C_4}{C}\right)^2 - \frac{4}{A^2} = \rho - p + 2V(\phi) \tag{19}$$

From equation (19), we get

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + a\left(\frac{B_4}{B} + \frac{C_4}{C}\right)^2 = 2K + \frac{4}{A^2} \tag{20}$$

where $a = \frac{3\gamma - 1}{4}$.

From equation (20), we get

$$\mu_{44} + \frac{a}{\mu}\mu_4^2 = 2K\mu + 4 \tag{21}$$

Let us consider $\mu_4 = f$ and $\mu_{44} = ff'$, where $f' = \frac{df}{d\mu}$, in equation (21) which leads to

$$\frac{df^2}{d\mu} + \frac{2a}{\mu} f^2 = 4K\mu + 8 \tag{22}$$

After integrating equation (22) we get

$$f = \frac{d\mu}{dt} = \sqrt{b\mu^2 + h\mu + J\mu^{-2a}} \tag{23}$$

where $b = \left(\frac{2K}{a+1}\right)$, $h = \left(\frac{8}{2a+1}\right)$ and J is constant of integration.

From equation (23), we get

$$t = \int \frac{\mu^a d\mu}{\sqrt{b\mu^{2a+2} + h\mu^{2a+1} + J}} + M \tag{24}$$

where M is the integration constant.

From equation (18) and (23), we get

$$\nu = e^{\int \frac{FT^{a-\frac{3}{2}}}{\sqrt{bT^{2a+2} + hT^{2a+1} + J}} dT + N} \tag{25}$$

where $\mu = T$ and N are constant of integration.

Therefore,

$$A^2 = T, \tag{26}$$

$$B^2 = Te^{\int \frac{FT^{a-\frac{3}{2}}}{\sqrt{bT^{2a+2}+hT^{2a+1}+J}}dT+N} \tag{27}$$

$$C^2 = Te^{-\int \frac{FT^{a-\frac{3}{2}}}{\sqrt{bT^{2a+2}+hT^{2a+1}+J}}dT+N} \tag{28}$$

Following an appropriate transformation of coordinates, the metric (1) yields the following form:

$$ds^2 = -\left(\frac{T^{2a}}{bT^{2a+2}+hT^{2a+1}+J}\right)dT^2 + TdX^2 + e^{2X}T\left\{e^{\int \frac{FT^{a-\frac{3}{2}}}{\sqrt{bT^{2a+2}+hT^{2a+1}+J}}dT+N}dY^2 + e^{-\int \frac{FT^{a-\frac{3}{2}}}{\sqrt{bT^{2a+2}+hT^{2a+1}+J}}dT+N}dZ^2\right\} \tag{29}$$

Where $x = X$, $y = Y$ and $z = Z$

IV. PHYSICAL AND GEOMETRICAL ASPECTS

For the model (29), the rate of Higgs field

$$\phi = E\int \frac{1}{\sqrt{\frac{8K}{3(\gamma+1)}T^5 + \frac{16}{3\gamma+1}T^4 + JT^{\frac{7-3\gamma}{2}}}}dT + P \tag{30}$$

where P is constant of integration.

For the model (29), pressure (p), Energy density (ρ), the spatial volume (R^3), the expansion (θ), shear (σ), decelerating parameter (q) and Hubble parameter (H) are given by

$$p = K + \frac{3J(1-2\gamma)}{4T^{\frac{3(\gamma+1)}{2}}} + \frac{1}{T} - \frac{12}{(3\gamma+1)T} - \frac{2E^2+F^2}{4T^3} - \frac{14K}{3(\gamma+1)} \tag{31}$$

$$\rho = \frac{1}{\gamma}\left[K + \frac{3J(1-2\gamma)}{4T^{\frac{3(\gamma+1)}{2}}} + \frac{1}{T} - \frac{12}{(3\gamma+1)T} - \frac{2E^2+F^2}{4T^3} - \frac{14K}{3(\gamma+1)}\right] \tag{32}$$

$$R^3 = T^{\frac{3}{2}} \tag{33}$$

$$\theta = \left(\frac{3}{2}\right)\left(\sqrt{\frac{8K}{3(\gamma+1)} + \frac{16}{(3\gamma+1)T} + \frac{J}{T^{\frac{3(\gamma+1)}{2}}}}\right) \tag{34}$$

$$\sigma = \frac{F}{2T^{\frac{3}{2}}} \tag{35}$$

$$q = -\left[\frac{b - \frac{J(3\gamma+1)}{2T^{\frac{3(\gamma+1)}{2}}}}{b + \frac{h}{T} + \frac{J}{T^{\frac{3(\gamma+1)}{2}}}}\right] \tag{36}$$

$$q < 0 \text{ if } b > \frac{J(3\gamma+1)}{2T^{\frac{3(\gamma+1)}{2}}}$$

$$q > 0 \text{ if } b < \frac{J(3\gamma+1)}{2T^{\frac{3(\gamma+1)}{2}}}$$

and

$$H = \frac{1}{2}\sqrt{\frac{8K}{3(\gamma+1)} + \frac{16}{(3\gamma+1)T} + \frac{J}{T^{\frac{3(\gamma+1)}{2}}}} \tag{37}$$

From equations (34) and (35), we get

$$\frac{\sigma}{\theta} = \frac{F}{3\sqrt{\frac{8K}{3(\gamma+1)}T^5 + \frac{16}{3\gamma+1}T^4 + JT^{\frac{7-3\gamma}{2}}}} \tag{38}$$

V. CONCLUSION

At T = 0 the model (29) begins to expand with the Big Bang. As time advances and when $T\to\infty$ universe expands continuously, the expansion (θ) diminishes.

As time goes on, the Spatial Volume (R^3) rises. It depicts an inflationary cosmos with a flat potential and a massless scalar field.

Since $\lim_{T\to\infty} \frac{\sigma}{\theta} = 0$, for large values of time T, the model is not anisotropic.

The Hubble parameter (H) starts with a high value but gradually decreases with the passage of time, and as T approaches infinity, it remains constant. Additionally, the model's pressure and energy density start off with large values.

Since the deceleration parameter tends to -1 when $T \to \infty$, the model represents the universe's accelerating phase.

The rate of Higgs field (ϕ) is big at first, but gets smaller over time and stays the same for $T \to \infty$. The model has point type singularity at $T = 0$.

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