MAGNETO CONVECTION IN COUPLE STRESS NANOFLUID LAYER

Abstract

In this chapter, it has been analysed how the application of the horizontalmagnetic field affects convection inhorizontal layer of couple stress nanofluid. The effects of different parameters on stability of nanofluid have been studied.The comparison of results obtained has been done with the existing relevant studies.

Keywords: Nanofluid, magnetic field, critical Rayleigh number, couple stress.

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I. NOMENCLATURE

- c Specific heat of nanofluid
- **g** Gravitational Acceleration
- t^* Time
- t Dimensionless Time
- T^* Nanofluid Temperature
- (x^*, y^*, z^*) Space coordinates
- (X, Y, Z) Dimensionless space-coordinates
- μ viscosity
- $(\rho c)_M$ Medium's effective heat capacity
- $(\rho c)_F$ Fluid's effective heat capacity
- $(\rho c)_P$ Effective heat capacity of the material constituting nanoparticles
- B_d Diffusion coefficient due to Brownian motion
- B_t Diffusion coefficient due to Thermophoresis
- S_d Diffusion Coefficient
- β_t Thermal Volumetric Coefficient
- k_m Effective thermal conductivity
- α Wave number
- α_m Thermal diffusivity of the porous medium
- λ Relaxation time
- σ Heat capacity ratio
- ψ^* Volume fraction of Nanoparticle
- ψ_0^* Reference nanoparticle volume fraction
- $\in$ Porosity
- μ_e Magnetic permeability
- σ' Electrical conductivity of nanofluid
- K Permeability

II. INTRODUCTION

Because of increasing non-Newtonian fluid's utilization, various problems concerned with many non-Newtonian fluids attracted interest of researchers. The couple stress fluid is one such fluid. In fluids the presence of couple stress vector and body couple were first introduced by Stokes [1].

Walicki and Walicka [2] observed that synovial fluids with very large molecules in humanjoints can be termed as couple stress fluids. Cosserat and Cosserat [3] modelled the equations governing couple stress vector. Important results on stability of couple stresses binary fluids with vertical temperature and concentration gradients were discussed by Rachana and Agrawal [4]. Hiremath and Patil [5] studied the oscillatory convection of couple stress fluids in a porous medium and Hayat, Mustafa, Iqbal and Alsaedi [6] studied couple stress flow over a stretching surface.

Nanofluids are recent fluids that cause significantly enhanced thermophysical properties. Couple stress nanofluids have a significant importance in MHD power generators, for the arteries blockage-removal, cancer tumour treatment, hyperthermia etc.

It is well-known that the flow field of a conducting fluid is altered on introducing a magnetic field. As far as the stability is concerned, the magnetic field, in general, is having a stabilizing effect apart from few exceptions. Thermo-solutal convection in a couple stress fluids through a porous medium having vertical magnetic field and vertical rotation was studied by Kumar [7]. Rotation was observed to have a stabilizing effect but magnetic field and couple stress were observed to give both stabilizing and destabilizing effects.

Instability of Magneto Hydrostatic stellar interiors from magnetic buoyancy were studied by Gilman [8]. Normal mode instability was observed due to magnetic buoyancy in fluids having large heat diffusivity compared with viscosity and magnetic diffusivity, as in stellar interiors. However, the magnetic buoyancy instability was found to be non-axisymmetric which is different from those in that, in a star with toroidal magnetic field. Schatzman [9] also did formal analysis on magnetic buoyancy instability but only for a special case.

The influence of magnetic field in couple stress fluids was studied by Shankar, Kumar and Shivkumara [10]. It was observed by them that magnetic fieldslows down the onset of instability whereasin presence of increasing couple stress parameteran opposite kind of behaviour was observed. Sharma and Thakur [11] studied the effect of uniform magnetic field on convection in a couple stress fluid layers. The suspended particle effect in couple stress fluid layer heated from below was studied by Sharma and Sharma [12]. The magnetic field and couple stress both were found to have stabilizing and destabilizing effects in thermo-solutal convection problem for a couple stress studied by Kumar and Kumar [13].

The problem considering thermal radiation and heat generation in couple stress nanofluid in presence on magnetic field was studied by Sithole, Mondal, Goqo, Sibanda and Motsa [14]. Malashetty, Pop, Kollur and Sidram [15] studied double diffusive convection in a couple stress fluid and found significant effect of couple stress.

The literature survey indicates that no study has investigated the magnetic field effect on diffusive convection in a couple stress nanofluid layer. The present study examines the effect of horizontal magnetic field onconvection in a couple stress nanofluid horizontal layer.

III. MATHEMATICAL FORMULATION

We consider an infinite isotropic porous layer of incompressible Maxwellian couple stress viscoelastic fluid confined between two horizontal planes, where the temperatures at the lower and upper boundaries are T_l^* and T_u^* respectively, T_l^* being greater than T_u^*. A uniform horizontal magnetic field $\boldsymbol{F}^* = (F_0^*, 0, 0)$ acts on the system.

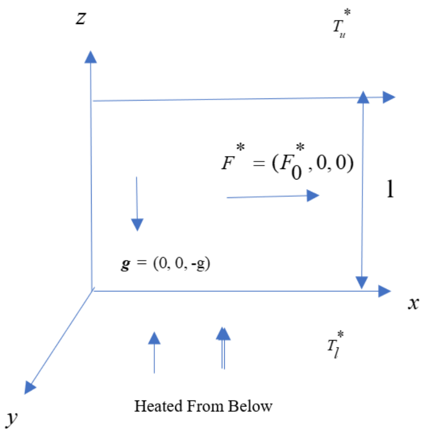

Figure 1: Physical Configuration of the Problem

The governing equations for conservation of mass, momentum, energy and concentration of nanoparticles are as follows:

$$\nabla^* . \boldsymbol{q}_d^* = 0 \tag{1}$$

$$\frac{1}{K}(\mu - \mu_c \nabla^{*2})\boldsymbol{q}_d^* = (1 + \lambda^* \frac{\partial}{\partial t^*})[\{-\nabla^* p^* + (\psi^* \rho_P + (1-\psi^*)\left\{\rho\left(1-\beta_t(T^* - T_u^*)\right)\right\})\boldsymbol{g}] + \frac{\mu_e}{4\pi}(\nabla^* \times \boldsymbol{F}^*) \times \boldsymbol{F}^*] \tag{2}$$

$$(\rho c)_M \frac{\partial T^*}{\partial t^*} + (\rho c)_F \, \boldsymbol{q}_d^* . \nabla^* T^* = k_m \nabla^{*2} T^* + \in (\rho c)_P \left[B_d \nabla^* \psi^* . \nabla^* T^* + (\frac{B_t}{T_c^*}) \nabla^* T^* . \nabla^* T^* \right] \tag{3}$$

$$\frac{\partial \psi^*}{\partial t^*} + \frac{1}{\in} \boldsymbol{q}_d^* . \nabla^* \psi^* = B_d \nabla^{*2} \psi^* + \frac{B_t}{T_r^*} \nabla^{*2} T^* \tag{4}$$

The modified Maxwell equations are

$$\left(\frac{\partial}{\partial t^*} + \frac{1}{\in} \left(\boldsymbol{q}_d^* . \nabla^* \right) \right) \boldsymbol{F}^* = \left(\boldsymbol{F}^* . \nabla^* \right) \frac{1}{\in} \boldsymbol{q}_d^* + \eta \nabla^{*2} \boldsymbol{F}^* \tag{5}$$

$$\nabla^* \cdot \boldsymbol{F}^* = 0, \quad \eta = \frac{1}{4\pi\mu_e \sigma'} \tag{6}$$

where $\boldsymbol{q}_d^* = (u_{1d}^*, u_{2d}^*, u_{3d}^*)$ is velocity.

The boundary conditions are

$$\boldsymbol{q}_d^* = 0,\ T^* = T_l^*,\ B_d \frac{\partial \psi^*}{\partial z^*} + \frac{B_t}{T_r^*}\frac{\partial T^*}{\partial z^*} = 0 \quad \text{at} \quad z^* = 0 \tag{7}$$

$$\boldsymbol{q}_d^* = 0,\ T^* = T_u^*,\ B_d \frac{\partial \psi^*}{\partial z^*} + \frac{B_t}{T_u^*}\frac{\partial T^*}{\partial z^*} = 0 \quad \text{at} \quad z^* = l \tag{8}$$

Taking following non-dimensional parameters

$$\left(X,Y,Z\right) = \frac{(x^*, y^*, z^*)}{l},\quad t = \frac{t^*\alpha_m}{\sigma l^2},\quad \left(u_{1d}, u_{2d}, u_{3d}\right) = \frac{\left(u_{1d}^*, u_{2d}^*, u_{3d}^*\right) l}{\alpha_m},\ p = \frac{p^* K}{\mu \alpha_m},\quad \psi = \frac{\psi^* - \psi_0^*}{\psi_0^*},$$

$$T = \frac{T^* - T_u^*}{T_l^* - T_u^*},\ \lambda = \frac{\lambda^* \alpha_m}{l^2},$$

where $\alpha_m \left(= \frac{k_m}{\left(\rho c\right)_F} \right)$ and $\sigma \left(= \frac{\left(\rho c\right)_M}{\left(\rho c\right)_F} \right)$.

On replacing $\boldsymbol{q}_d$ by $\boldsymbol{q}$, non-dimensional form of equations (1) to (5) together with boundary conditions (7) and (8) can be written as

$$\nabla . \boldsymbol{q} = 0 \tag{9}$$

$$(\boldsymbol{q} - S\nabla^2 \boldsymbol{q}) = \left(1 + \frac{\lambda}{\sigma}\frac{\partial}{\partial t}\right)\left[\left(-\nabla p - R_m \hat{\boldsymbol{e}}_{\boldsymbol{z}} - R_n \psi \hat{\boldsymbol{e}}_{\boldsymbol{z}} + R_a T \hat{\boldsymbol{e}}_{\boldsymbol{z}}\right) + \frac{P_1}{P_{1M}} Q D_a \left(\nabla \times \boldsymbol{F}\right) \times \boldsymbol{F}\right] \tag{10}$$

$$\frac{\partial T}{\partial t} + \left(\boldsymbol{q}.\nabla\right) T = \nabla^2 T + \frac{N_b}{Le}\nabla \psi . \nabla T + \frac{N_a N_b}{Le}\nabla T . \nabla T \tag{11}$$

$$\frac{1}{\sigma}\frac{\partial \psi}{\partial t} + \frac{1}{\in}\left(\boldsymbol{q}.\nabla\right)\psi = \frac{1}{Le}\nabla^2 \psi + \frac{N_a}{Le}\nabla^2 T \tag{12}$$

$$\frac{1}{\sigma}\frac{\partial \boldsymbol{F}}{\partial t} + \frac{1}{\in}\left(\boldsymbol{q}.\nabla\right)\boldsymbol{F} = \frac{1}{\in}(\boldsymbol{F}.\nabla)\boldsymbol{q} + \frac{\mathrm{P}_1}{P_{1M}}\nabla^2 \boldsymbol{F} \tag{13}$$

and boundary conditions are

$$\boldsymbol{q} = 0,\ \text{T=1},\ \frac{\partial \psi}{\partial Z} + N_a \frac{\partial T}{\partial Z} = 0 \ \text{at} \quad Z = 0 \tag{14}$$

$$\boldsymbol{q} = 0,\ \text{T=0},\ \frac{\partial \psi}{\partial Z} + N_a \frac{\partial T}{\partial Z} = 0 \ \text{at} \quad Z = 1 \tag{15}$$

Here $R_a (= \frac{\rho g \beta K d \left(T_l^* - T_u^*\right)}{\mu \alpha_m})$, $R_n (= \frac{\left(\rho_P - \rho\right)\psi_0^* gKd}{\mu \alpha_m})$, $R_m (= \frac{\rho_P \psi_0^* + \rho(1-\psi_0^*) gKd}{\mu \alpha_m})$, $R_s (= \frac{\rho \beta_c gdK (S_l^* - S_u^*)}{\mu S_d})$ are thermal, concentration, basic density and solutal Rayleigh Drcy number respectively, $S(= \frac{\mu_{cs}}{\mu l^2})$ is couple-Stress parameter, $P_1 (= \frac{\mu}{\rho \alpha_m})$ and $P_{1m} (= \frac{\mu}{\rho \eta})$ are Prandtl numbers, $Q(= \frac{\mu_e M_0^{*2} d^2}{4\pi\mu\eta})$ is Magnetic Chandrasekhar number, $D_a (= \frac{K}{l^2})$ is Darcy number, $N_a (= \frac{B_t (T_l^* - T_r^*)}{B_d T_r^* Q_0^*})$ and $N_b (= \frac{\left(\rho c\right)_P \in Q_0^*}{\left(\rho c\right)_F})$ are modified diffusivity ratio and modified particle density increment respectively, $Le(= \frac{\alpha_m}{B_d})$ is Lewis number.

1. **Neutral State:**Time independent neutral state of nanofluid is described as

$$\boldsymbol{q} = 0 \,,\, p = p_{bs}(Z) \,,\, T = T_{bs}\left(Z\right) ,\, \psi = \psi_{bs}\left(Z\right) ,\, \boldsymbol{F} = \hat{\boldsymbol{e}}_X \tag{16}$$

where the suffix "bs" indicates the neutral flow. Following Chandrasekhar [16], the neutral nanoparticle volume fraction and temperature satisfy the following equations

$$\frac{d^2\psi_{bs}}{dZ^2} + N_a \frac{d^2 T_{bs}}{dZ^2} = 0 \tag{17}$$

$$\frac{d^2 T_{bs}}{dZ^2} + \frac{N_{bs}}{Le}\frac{d\psi_{bs}}{dZ}\frac{dT_{bs}}{dZ} + \frac{N_a N_b}{Le}\left(\frac{dT_{bs}}{dZ}\right)^2 = 0 \tag{18}$$

The boundary conditions are

$$\boldsymbol{q} = 0 \,,\quad T_{bs}\left(Z\right) = 1 ,\; \frac{d\psi_{bs}}{dZ} + N_a \frac{dT_{bs}}{dZ} = 0 \quad \text{at} \quad Z = 0 \tag{19}$$

$$\boldsymbol{q} = 0 \,,\quad T_{bs}\left(Z\right) = 0 \;,\; \frac{d\psi_{bs}}{dZ} + N_a \frac{dT_{bs}}{dZ} = 0 \;\text{at} \quad Z = 1 \tag{20}$$

On solving, we get

$T_{bs} = 1 - Z$, $\psi_{bs} = \psi_0 + N_a Z$.

2. **Perturbed State:** On the neutral state,superimposing theperturbations as

$$\boldsymbol{q} = \boldsymbol{q}' \,,\; p = p_{bs} + p' \,,\; \psi = \psi_{bs} + \psi' \,,\; \boldsymbol{F} = \hat{\boldsymbol{e}}_X + \boldsymbol{F}'$$

Where the primes denote infinitesimal small quantities. Ignoring the products of primed quantities and their derivatives, the following linearised perturbation equations of Couple Stress nanofluid are obtained as

$$\left(\frac{1}{\sigma}\frac{\partial}{\partial t}-\frac{P_1}{P_{1M}}\nabla^2\right)\left[(\nabla^2-\square\,\nabla^4)u'_{3d}-\left(1+\frac{\lambda}{\sigma}\frac{\partial}{\partial t}\right)\left(R_a\nabla_H^2 T'-R_n\nabla_H^2\psi'+\frac{R_s}{Ln}\nabla_H^2 S'\right)\right] \\ =(1+\frac{\lambda}{\sigma}\frac{\partial}{\partial t})Q\frac{P_1}{P_{1M}}\frac{D_a}{\in}\nabla^2\frac{\partial^2 u'_{3d}}{\partial X^2} \tag{21}$$

$$\frac{\partial T'}{\partial t}-u'_{3d}=\nabla^2 T'-\frac{N_aN_b}{Le}\frac{\partial T'}{\partial Z}-\frac{N_b}{Le}\frac{\partial\psi'}{\partial Z} \tag{22}$$

$$\frac{1}{\sigma}\frac{\partial\psi'}{\partial t}+\frac{1}{\in}N_a u'_{3d}=\frac{1}{Le}\nabla^2\psi'+\frac{N_a}{Le}\nabla^2 T' \tag{23}$$

With the boundary conditions

$$u'_{3d}=0, T'=0, \frac{\partial\psi'}{\partial Z}+N_a\frac{\partial T'}{\partial Z}=0 \text{ at } \quad Z=0 \text{ and } Z=1 \tag{24}$$

IV. LINEAR STABILITY ANALYSIS

Following thestability theory in linear form by Chandrasekhar [16], the perturbations are taken of the form

$$\left(\psi', T', u'_{3d}\right)=\left[\Phi(Z),\Theta(Z),\Omega(Z)\right]e^{st+iLX+iMY}, \tag{25}$$

where L represents dimensionless wave numbers in X direction and M represents dimensionless wave numbers in Y direction.
On substituting the above values, we get

$$\left[\frac{s}{\sigma}\left(D^2-\alpha^2\right)-\frac{P_1}{P_{1M}}\left(D^2-\alpha^2\right)^2-Q\left(1+\frac{\lambda s}{\sigma}\right)\frac{P_1}{P_{1M}}\frac{D_a}{\in}L^2\left(D^2-\alpha^2\right)-\frac{sS}{\sigma}(D^2-\alpha^2)^2+\frac{SP_1}{P_{1M}}(D^2-\alpha^2)^3\right]\Omega \\ -R_a\alpha^2\left(1+\frac{\lambda s}{\sigma}\right)\left[\frac{P_1}{P_{1M}}\left(D^2-\alpha^2\right)-\frac{s}{\sigma}\right]\Theta \\ +R_n\alpha^2\left(1+\frac{\lambda s}{\sigma}\right)\left[\frac{P_1}{P_{1M}}\left(D^2-\alpha^2\right)-\frac{s}{\sigma}\right]\Phi=0 \tag{26}$$

$$\Omega+\left(D^2-\alpha^2-s-\frac{N_aN_b}{Le}D\right)\Theta-\frac{N_b}{Le}D\Phi=0 \tag{27}$$

$$\frac{N_a}{\in}\Omega-\frac{N_a}{Le}(D^2-\alpha^2)\Theta-\left[\frac{1}{Le}(D^2-\alpha^2)-\frac{s}{\sigma}\right]\Phi=0 \tag{28}$$

With the boundary conditions

$\Omega = 0 = \Theta$, $D\Phi + N_a D\Theta = 0$ at $Z = 0$ and $Z = 1$. (29)

1. **Solution:** For employing Galerkin-type weighted residuals method to get an estimated solution to the system, we takeapproximate functions as:

$$\Omega = \sum_{k=1}^{N} A_k W_k \, , \Theta = \sum_{k=1}^{N} B_k \Theta_k \, , \quad \Phi = \sum_{k=1}^{N} C_k \Phi_k \tag{30}$$

where A_k , B_k , C_k are unknown constants. Using boundary conditions given by equation (29), for first estimation, we get the following:
$\Omega = A_1 \sin \pi Z$, $\Theta = B_1 \sin \pi Z$, $\Phi = -N_a C_1 \sin \pi Z$

Putting these values in equations (27)-(29) and for non-trivial solution of above matrix equations, we take the determinant of coefficient matrix as zero. On putting s=0 the following value ofstationary Rayleigh numberis obtained

$$R_a^{st} = \frac{\delta^4}{\alpha^2} - \left(1 + \frac{Le}{\in}\right) R_n N_a + \frac{Q D_a L^2 \delta^2}{\in \alpha^2} + \frac{S \delta^6}{\alpha^2} \tag{31}$$

where $\delta^2 = \pi^2 + \alpha^2$

2. **Analysis:** The above relation shows that the stationary Rayleigh number depends on parameters S , Le , Q , N_a , R_n , $\in$, D_a and dimensionless wave number α .

To obtain critical Rayleigh number putting $\frac{dR_a^{st}}{d\alpha} = 0$, it is observed that critical value of wave number depends on Couple Stress factor, Darcy number, Porosity and Magnetic field.

V. RESULTS AND DISCUSSION

From equation (31), we have

$$\frac{\partial R_a^{st}}{\partial S} = \frac{(\pi^2 + \alpha^2)^3}{\alpha^2}$$

Which is same as obtained for a couple stress regular fluid by Bishnoi, Jawla and Kumar [17]. This shows that the impact of couple stress parameter for a nanofluid layer is to stabilize the stationary convection in same wayas for a regular fluid.

Further it is also clear from equation (31) that Q , D_a have stabilizing effect where as Le , N_a , R_n have stabilizing effect on stationary convection. Porosity has dual behaviour in stationary convection.

If $R_n = 0, Q = 0$, then

$$R_a^{st} = \frac{\delta^4}{\alpha^2} + \frac{S\delta^6}{\alpha^2} = \frac{(\pi^2+\alpha^2)^2}{\alpha^2}(1+S\delta^2)$$

Which is same as obtained by Shivkumara, Lee and Kumar[18].

VI. CONCLUSION

In this chapter, analysis has been done to show how the presence of the magnetic field affects convection in horizontal layer of couple stress nanofluid. The comparison of results obtained has been done with the existing relevant studies. The outcomes of the present analysis are summarised as follows:

- R_a^{st} has been observed to be function of parameters S , D_a, ε , Le, R_n , N_a , Q.
- The couple stress parameter has been found to stabilize the stationary convection as observed by Chand, Rana and Yadav [19] too while studying thermal instability in a layer of couple stress nanofluid in absence of magnetic field
- The effect of Lewis number Le is to decrease Rayleigh number.
- The impact of magnetic field is to stabilise the diffusive convection as was found by Yadav, Changhoon, Jinho and Hyung [22] in nanofluid convection induced by internal heating.
- In this convection under magnetic field, Darcy number also comes into play and has been observed to provide stabilizing effect on stationary modes.

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