# SOME FIXED POINT RESULTS ON OWC MAPPINGS FOR IFMS(INTUITIONISTIC FUZZY METRIC SPACE)

**Abstract**

OWC (occasionally weakly compatible mapping), Implicit relations, Complete intuitionistic fuzzy metric spaces, and a common fixed point.

**Keywords:** Implicit relations, Complete intuitionistic fuzzy metric spaces.

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# I. INTRODUCTION

In 1965, Zadeh invented fuzzy set theory [17].Numerous writers have presented and analyzed diverse fuzzy metric space concepts in various ways [9], [4], and [5], also proved fixed point theorems with new findings in fuzzy metric spaces [6]. Park [12] introduced the idea of an intuitionistic fuzzy metric space and Alaca et al. [2] and Mohamad [10] have examined the fixed point outcomes in these spaces. The idea of OWC maps was first suggested by Al-Thagafi and N. Shahzad[3].

In this paper we prove some fixed point theorems for IFMS as an application of OWC mappings.

## 1. Preliminaries

**Definition 1.1**[2] If X is an arbitrary set, $\circ$ is a continuous t-norm[14], $\triangle$ is continuous t-conorm[14] and two fuzzy sets Φ, Ψ are defined on $X^2 \times (0, \infty)$ that holds:

(I-1) $\Phi(\nu, \omega, \tau) + \Psi(\nu, \omega, \tau) \leq 1$,(I-2) $\Phi(\nu, \omega, 0) = 0$,
(I-3) $\Phi(\nu, \omega, \tau) = 1$ if and only if $\nu = \omega$,(I-4) $\Phi(\nu, \omega, \tau) = \Phi(\omega, \nu, \tau)$,
(I-5) $\Phi(\nu, \omega, \tau) \circ \Phi(\omega, \lambda, s) \leq \Phi(\nu, \lambda, \tau + s)$,
(I-6) $\Phi(\nu, \omega, .) : (0, \infty) \rightarrow (0,1]$ satisfy left continuity,(I-7) $\lim_{\tau\rightarrow\infty}\Phi(\nu, \omega, \tau) = 1$,
(I-8) $\Psi(\nu, \omega, 0) = 1$,
(I-9) $\Psi(\nu, \omega, \tau) = 0$ iff $\nu = \omega$,(I-10) $\Psi(\nu, \omega, \tau) = \Psi(\omega, \nu, \tau)$,
(I-11) $\Psi(\nu, \omega, \tau) \triangle \Psi(\omega, \lambda, s) \leq \Psi(\nu, \lambda, \tau + s)$,
(I-12) $\Psi(\nu, \omega, .) : (0, \infty) \rightarrow (0,1]$ satisfy right continuity,(I-13) $\lim_{\tau\rightarrow\infty}\Psi(\nu, \omega, \tau) = 0$,
$\forall\ \nu, \omega, \lambda \in X$ and $s, \tau > 0$, then (Φ, Ψ) is an intuitionistic fuzzy metric on X, where mappings $\Phi(\nu, \omega, \tau)$ represent the degree of nearness of ν w.r.t. τ, and $N(\nu, \omega, \tau)$ represent the degree of non-nearness of ω w.r.t τ,. We say $(X, \Phi, \Psi, \circ, \triangle)$ is an IFMS (intuitionistic fuzzy metric space).

**Remark 1.2** Every fuzzy metric space $(X, \Phi, \circ)$ is an IFMS of the form $(X, \Phi, 1\text{-}\Phi, \circ, \triangle)$ such that triangular norm $\circ$ and triangular conorm $\triangle$ are associated i.e. $\nu \triangle \omega = 1- ((1-\nu) \circ (1-\omega))$ for all $\nu, \omega \in X$.

Alaca et. Al.[2] also defined convergence of a sequence and Cauchy sequence and completeness of an intuitionistic fuzzy metric space.Park [13] has been given one important result about Cauchy sequence with certain condition as a lemma and otherresult has been very important for our research area as follows:

**Lemma 1.3**[13]For an IFMS $(X, \Phi, \Psi, \circ, \triangle)$ , if $\Phi(\nu, \omega, \kappa\tau) \geq \Phi(\nu, \omega, \tau)$ & $\Psi(\nu, \omega, \kappa\tau)$

$\leq \Psi(\nu, \omega, \tau)$ are true $\forall \nu, \omega \in X$ , $\tau > 0$ and $\kappa \in (0, 1)$, then $\nu = \omega$.

Jungck [7, 8] defined compatible, weakly compatible mapping and coincidence points for two self mappings of an IFMS. We use the definition of OWC given by C. T. Aage and J. N. Salunke [1]. They give following lemma which is useful for our researchas follows:

**Lemma 1.4**[1] For an IFMS $(X, \Phi, \Psi, \circ, \triangle)$, two owc self mappings F and G have unique point of coincidence such that $\mu = A\nu = S\nu$, then F and G have unique common fixedpoint $\mu$.

## 2. Main Results

Following theorem is given by [12] for fuzzy metric space:

**Theorem.** For a complete fuzzy metric space $(X, \Phi, \circ)$ four self mappings F, G, U and Vhaving OWC in pairs{F, U} and {G, V} satisfying following condition such that
$\Phi(Fx, Gy, qt) \geq \alpha_1 \Phi(Ux,Vy, t) + \alpha_2 \Phi(Fx,Vy, t) + \alpha_3 \Phi(Gy, Ux, t)$
For $q \in (0, 1)$ and $\forall x, y \in X$, where $\alpha_1, \alpha_2, \alpha_3 > 0$, $\alpha_1+ \alpha_2+ \alpha_3 > 1$, then $\exists$ a unique point $u \in X$ such that $Fu = Uu = u$ and a $v \in X$ is unique point s.t. $Gv = Vv = v$. Moreover, $v = u$, so that F, G, U and V have a unique common fixed point.

Here we generalized above theorem for IFMS as follows:

**Theorem 2.1** For complete IFMS $(X, \Phi, \Psi, \circ, \triangle)$ let the pairs {A, S} and {B, T} are OWCself mappings on X, for any $\nu, \omega \in X$ and $\tau > 0$ with positive number $\kappa \in (0, 1)$ such that

$$\Phi (A\nu, B\omega, \kappa\tau) \geq a_1 \Phi (S\nu, T\omega, \tau) + a_2 \Phi (A\nu, T\omega, \tau) + a_3 \Phi (B\omega, S\nu, \tau) \quad \ldots(i)$$

and

$$\Psi (A\nu, B\omega, \kappa\tau) \leq b_1 \Psi (S\nu, T\omega, \tau) + b_2 \Psi (A\nu, T\omega, \tau) + b_3 \Psi (B\omega, S\nu, \tau), \quad \ldots(ii)$$

Inequalities (i) and (ii) are true for all $\nu, \omega \in X$, where $a_1, a_2, a_3, b_1, b_2 , b_3 > 0$ , $a_1 + a_2 +a_3 > 1$ and $b_1+ b_2 + b_3 < 1$ then there exist a unique point $\mu \in X$ such that $A\mu = S\mu = \mu$ and a unique point $\lambda \in X$ such that $B\lambda = T\lambda = \lambda$. Moreover, $\lambda = \mu$, is a unique common fixed point of all four self mappings on X.

**Proof:** It is given that the pairs {A, S} and {B, T} be OWC, so $\nu, \omega \in X$ s.t. $A\nu = S\nu$ and $B\omega = T\omega$ . We now show that, $A\nu = B\omega$. If not, by inequality (i)$\Phi(A\nu, B\omega, \kappa\tau) \geq$
$a_1 \Phi(S\nu, T\omega, \tau ) + a_2 \Phi(A\nu, T\omega, \tau) + a_3 \Phi(B\omega, S\nu, \tau)$
$= a_1\Phi(A\nu, B\omega, \tau) + a_2 \Phi(A\nu, B\omega, \tau) + a_3\Phi(B\omega, A\nu, \tau)$
$= (a_1 + a_2 + a_3) \Phi ( A\nu, B\omega, \tau)$
this gives contradiction since $a_1 + a_2 + a_3 > 1$.Similarly, by inequality (ii)

$\Psi(A\nu, B\omega, \kappa\tau) \leq b_1 \Psi(S\nu, T\omega, \tau) + b_2 \Psi(A\nu, T\omega, \tau) + b_3 \Psi(B\omega, S\nu, \tau)$
$= b_1 \Psi(A\nu, B\omega, \tau) + b_2\Psi(A\nu, B\omega, \tau) + b_3\Psi(B\omega, A\nu, \tau)$
$= (b_1 + b_2 + b_3) \Psi(A\nu, B\omega, \tau)$

We get contradiction, because $(b_1 + b_2 + b_3) < 1$. And by Lemma 1.3 $A\nu = B\omega$, i.e. $A\nu = S\nu = B\omega = T\omega$. Let us assume that there is another point $\lambda$ such that $A\lambda = S\lambda$, then by (i) and (ii), we have $A\lambda = S\lambda = B\omega = T\omega$. This gives, $A\nu = A\lambda$ and $\mu = A\nu = S\nu$ ,we have conclude that the unique point of coincidence of A and S is $\mu$ . Also by Lemma 1.4 common fixed point of A and S is only $\mu$ , i. e. $\mu = A\mu = S\mu$ In the similar way a unique point $\lambda \in X$ s.t. $\lambda = B\lambda = T\lambda$.

Let us assume that $\mu \neq \lambda$. We have, $\Phi(\mu, \lambda, \kappa\tau) = \Phi(A\mu, B\lambda, \kappa\tau)$
$\geq a_1\Phi(S\mu, T\lambda, \tau) + a_2 \Phi(A\mu, T\lambda, \tau) + a_3\Phi(B\lambda, S\mu, \tau)$
$= a_1\Phi(\mu, \lambda, \tau) + a_2\Phi(\mu, \lambda, \tau) + a_3\Phi(\lambda, \mu, \tau)$
$= (a_1 + a_2 + a_3) \Phi(\mu, \lambda, \tau)$
this gives contradiction since $(a_1 + a_2 + a_3) > 1$.

Similarly,
$\Psi(\mu, \lambda, \kappa\tau) = \Psi(A\mu, B\lambda, \kappa\tau)$
$\leq b_1 \Psi(S\mu, T\lambda, \tau) + b_2\Psi(A\mu, T\lambda, \tau) + b_3 \Psi(B\lambda, S\mu, \tau)$
$= b_1 \Psi(\mu, \lambda, \tau) + b_2\Psi(\mu, \lambda, \tau) + b_3 \Psi(\lambda, \mu, \tau)$
$= (b_1 + b_2 + b_3) \Psi(\mu, \lambda, \tau)$
Again contradiction, because $(b_1 + b_2 + b_3) < 1$. And by Lemma 1.3, $\lambda = \mu$. Also by Lemma 1.4, the common fixed point of A, B, S and T is $\lambda$. From (i) and (ii) the uniqueness of the fixed point holds ♦

**Theorem 2.2** Let the pairs (A, S) and (B, T) are OWC self mappings on complete IFMS $(X, \Phi, \Psi, ^{o}, \triangle)$ for any $\nu, \omega \in X$ and $\tau > 0$ with positive number $\kappa \in (0, 1)$ such that

$$\Phi(A\nu, B\omega, \kappa\tau) \geq a_1 \min\{\Phi(S\nu, T\omega, \tau), \Phi(S\nu, A\nu, \tau)\} + b_1 \min\{\Phi(B\omega, T\omega, \tau), \Phi(A\nu, T\omega,\tau)\} + c_1\Phi(B\omega, S\nu, \tau) \quad \ldots(iii)$$

and

$$\Psi(A\nu, B\omega, \kappa\tau) \leq a_2 \min\{\Psi(S\nu, T\omega, \tau), \Psi(S\nu, A\nu, \tau)\} + b_2 \min\{\Psi(B\omega, T\omega, \tau), \Psi(A\nu,T\omega, \tau)\} + c_2\Psi(B\omega, S\nu, \tau) \quad \ldots(iv)$$

Inequalities (iii) and (iv) are true for all $\nu, \omega \in X$, where $a_1, a_2, b_1, b_2, c_1, c_2 > 0$, $a_1 + b_1 + c_1 > 1$ & $a_2 + b_2 + c_2 < 1$ then there exist a unique point $\mu \in X$ such that $A\mu = S\mu = \mu$ and a unique point $\lambda \in X$ such that $B\lambda = T\lambda = \lambda$. Moreover, $\lambda = \mu$, is a unique common fixed point of all four self mappings on X.

**Proof : -** It is given that the pairs {A, S} and {B, T} are OWC, so there are points $\nu, \omega$ inIFM Such that $A\nu = S\nu$ and $B\omega = T\omega$ . We claim that, $A\nu = B\omega$. If not, by inequality (iii) $\Phi(A\nu, B\omega, \kappa\tau) \geq a_1 \min\{\Phi(S\nu, T\omega, \tau), \Phi(S\nu, A\nu, \tau)\} + b_1\min\{\Phi(B\omega, T\omega, \tau),$

$\Phi(A\nu, T\omega, \tau)\} + c_1\Phi(B\omega, S\nu, \tau)$

$= a_1 \min\{\Phi(A\nu, B\omega, \tau), \Phi(A\nu, A\nu, \tau)\} + b_1\min\{\Phi(B\omega, B\omega, \tau), \Phi(A\nu, B\omega,\tau)\} + c_1\Phi(B\omega, A\nu, \tau)$

$= a_1 \min\{\Phi(A\nu, B\omega, \tau), 1\} + b_1 \min\{1, \Phi(A\nu, B\omega, \tau)\} + c_1\Phi(B\omega, A\nu, \tau)$

$= a_1 \Phi(A\nu, B\omega, \tau) + b_1 \Phi(A\nu, B\omega, \tau) + c_1\Phi(A\nu, B\omega, \tau)$

$= (a_1 + b_1 + c_1) \Phi(A\nu, B\omega, \tau)$

Above inequality gives us contradiction because $a_1 + b_1 + c_1 > 1$ Similarly, by inequality (iv)

$\Psi(A\nu, B\omega, \kappa\tau) \le a_2 \min\{\Psi(S\nu, T\omega, \tau), \Psi(S\nu, A\nu, \tau)\} + b_2 \min\{\Psi(B\omega, T\omega, \tau), \Psi(A\nu, T\omega, \tau)\} + c_2\Psi(B\omega, S\nu, \tau)$

$= a_2 \min\{\Psi(A\nu, B\omega, \tau), \Psi(A\nu, A\nu, \tau)\} + b_2 \min\{\Psi(B\omega, B\omega, \tau), \Psi(A\nu, B\omega,\tau)\} + c_2\Psi(B\omega, A\nu, \tau)$

$= a_2 \min\{\Psi(A\nu, B\omega, \tau), 1\} + b_2 \min\{1, \Psi(A\nu, B\omega, \tau)\} + c_2\Psi(B\omega, A\nu, \tau)$

$= a_2 \Psi(A\nu, B\omega, \tau) + b_2 \Psi(A\nu, B\omega, \tau) + c_2 \Psi(A\nu, B\omega, \tau)$

$= (a_2 + b_2 + c_2) \Psi(A\nu, B\omega, \tau)$

We get again contradiction, since $a_2 + b_2 + c_2 < 1$. And by Lemma 1.3, $A\nu = B\omega$, i.e. $A\nu = S\nu = B\omega = T\omega$. Let us assume that another point $\lambda$ s.t. $A\lambda = S\lambda$, then by (iii) and (iv),we have $A\lambda = S\lambda = B\omega = T\omega$. Hence we have, $A\nu = A\lambda$ and the unique point of coincidence of A and S is $\mu = A\nu = S\nu$. By Lemma 1.4, the only common fixed point ofA and S, i. e. $\mu = A\mu = S\mu$. In the similar way a unique point $\lambda \in X$ s.t. $\lambda = B\lambda = T\lambda$.

Let(Hyp.) $\mu \ne \lambda$. We have,

$\Phi(\mu, \lambda, \kappa\tau) = \Phi(A\mu, B\lambda, \kappa\tau)$

$\ge a_1 \min\{\Phi(S\mu, T\lambda, \tau), \Phi(S\mu, A\mu, \tau)\} + b_1 \min\{\Phi(B\lambda, T\lambda, \tau), \Phi(A\mu, T\lambda, \tau)\} + c_1\Phi(B\lambda, S\mu, \tau) = a_1\min\{\Phi(\mu, \lambda, \tau), \Phi(\mu, \mu, \tau)\} + b_1\min\{\Phi(\lambda, \lambda,\tau), \Phi(\mu, \lambda,\tau)\} + c_1\Phi(\lambda, \mu, \tau)$

$= a_1\min\{\Phi(\mu, \lambda, \tau), 1\} + b_1 \min\{1, \Phi(\mu, \lambda, \tau)\} + c_1 \Phi(\mu, \lambda, \tau)$

$= a_1\Phi(\mu, \lambda, \tau) + b_1 \Phi(\mu, \lambda, \tau) + c_1\Phi(\mu, \lambda, \tau)$

$= (a_1 + b_1 + c_1) \Phi(\mu, \lambda, \tau)$

We again get contradiction since a1+ b1 + c1 > 1 Similarly,

$\Psi(\mu, \lambda, \kappa\tau) = \Psi(A\mu, B\lambda, \kappa\tau)$

$\le a2 \min\{\Psi(S\mu, T\lambda, \tau),\Psi(S\mu, A\mu, \tau)\} + b2 \min\{\Psi(B\lambda, T\lambda, \tau),\Psi(A\mu, T\lambda, \tau)\} + c2\Psi(B\lambda, S\mu, \tau)$

$= a2 \min\{\Psi(\mu, \lambda, \tau), \Psi(\mu, \mu, \tau)\} + b2 \min\{\Psi(\lambda, \lambda, \tau), \Psi(\mu, \lambda, \tau)\} + c2\Psi(\lambda, \mu, \tau)$

$= a2 \min\{\Psi(\mu, \lambda, \tau), 1\} + b2 \min\{1, \Psi(\mu, \lambda, \tau)\} + c2\Psi(\mu, \lambda, \tau)$

$= a2 \Psi(\mu, \lambda, \tau) + b2 \Psi(\mu, \lambda, \tau) + c2 \Psi(\mu, \lambda, \tau)$

$= (a2 + b2 + c2) \Psi(\mu, \lambda, \tau)$

Again gives contradiction, because a2 + b2 + c2 < 1. And by Lemma 1.3 $\lambda = \mu$. Also by Lemma 1.4, the common fixed point of A, B, S and T is $\lambda$. Also from (iii) and (iv) the uniqueness of the fixed point holds •

## II. CONCLUSION

We establish common fixed point solutions for occassionally weakly compatible in intuitive fuzzy metric spaces, which enhances and generalizes the work of several writers who have previously presented their findings in the fixed point theory of fuzzy metric spaces.

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