COMPARATIVE ANALYSIS OF MATHEMATICAL MODELS FOR NON-NEWTONIAN STRESS RELAXATION FLOW OF BLOOD IN AN ARTERY WITH MULTIPLE STENOSIS

Abstract

In this research, a mathematical model is developed to investigate the stress relaxation flow of blood through a stenosed artery with multiple constrictions. The study derives analytical expressions for flux and velocity, considering suitable boundary conditions. The research conducts a quantitative analysis of flux, flow velocity, resistive impedances, and temporal variations in wall shear stress. The axial velocity is graphically presented for various values of the Jeffrey parameter within the narrowed region of the artery.

Keywords: Stress relaxation, tapered artery, multiple constrictions, flux, wall shear stress, axial velocity, Jeffrey parameter.

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I. INTRODUCTION

Stenosis refers to the abnormal and unnatural growth inside an artery's lumen, which disrupts the flow pattern. Such constricted arteries can block 50%-90% of the area. The mathematical modeling and analysis of arteries with multiple stenoses are highly valuable for Bio-mathematicians and medical scientists, as they explore various characteristics of blood flow. The main objective of this theoretical research is to investigate the mechanics of non-Newtonian blood flow in an artery with multiple stenoses. This investigation is crucial within the realm of medical sciences, where precise information about various blood flow parameters is lacking.

There is compelling evidence supporting the significant role of fluid dynamics components such as resistance of flow, wall shear stress and apparent viscosity, in the evolution and advancement of arterial stenosis. Recently, numerous theoretical and analytical studies have been conducted on the basis of blood flow within narrow arteries with stenosis, with a predominant focus on non-Newtonian blood behavior. Mathur and Jain (2013) investigated the mathematical model for non-Newtonian flow of blood through arteries with stenosis and blood is considering as a power-law fluid. The results they obtained demonstrated that within the selected non-Newtonian model, both pressure drop and shear stress escalate as the size of the stenosis increases. Ramesh Babu and Savita (2019) studied the flow of Jeffrey fluid through narrow arteries with multiple stenoses, exploring variations in flow velocities and volumetric flow across distinct flow regions under various boundary conditions. Halder et al. (2017) conducted studies on both Newtonian and non-Newtonian pulsatile flows of blood through arterial stenosis. They presented three-dimensional modeling and analyzed the blood flow in stenosed arteries to simulate atherosclerosis artery disease under various pulsatile flow scenarios. Sriyab (2020) analyzed the non-Newtonian behavior of blood and effects of stenotic geometry in arterial stenosis. Their mathematical model considered several stenosis shapes, such as shape of a bell and cosine shape and identified stenosed artery geometry, length of stenosis, stenosis thickness and power law index (in non-Newtonian behavior) as primary factors influencing the flow of blood through the stenosed artery. Shit et al. (2012) conducted a mathematical model for the circulation of blood in constricted, overlapping arteries with changing viscosity. They observed that hematocrit as well as magnetic field and artery shape significantly influenced pressure gradient, velocity profiles and wall shear stress. Blood's variable viscosity was treated as a porous medium, and they analytically solved the problem using the Frobenius method. Nanda and Bose (2012) investigated a mathematical model for flow of blood through the narrow arteries with multiple stenoses. They studied rheological parameters, length of stenosis and fluid yield stress, which strongly influenced flow characteristics both qualitatively and quantitatively. Nanda and Mallik (2012) analyzed a two-phase non-Newtonian fluid model for blood flow through narrow arteries under stenotic conditions. They performed large-scale numerical simulations and developed computer codes to study measurable flow variables with physiological significance. Mandal et. al. (2007) formulated a mathematical framework that accounts for blood's non-Newtonian behavior, employing the comprehensive power-law model which encompasses both shear-thickening and shear-thinning characteristics. Their proposed model considered the pulsating pressure gradients generated by the heart's rhythmic activity, addressing the fluctuating flow within narrowed arteries. The elastic cylindrical tube representing the arterial wall included a stenosis within its inner passage.

The primary aim of this study is to theoretically explore the mathematical modeling of blood flow with non-Newtonian stress relaxation through an artery afflicted by multiple stenosis. The study focuses on deriving analytical results for flow velocity and flux. These analytical expressions are then utilized to examine the variations in flow velocities and volumetric flow across distinct flow regions.

II. MATHEMATICAL MODEL

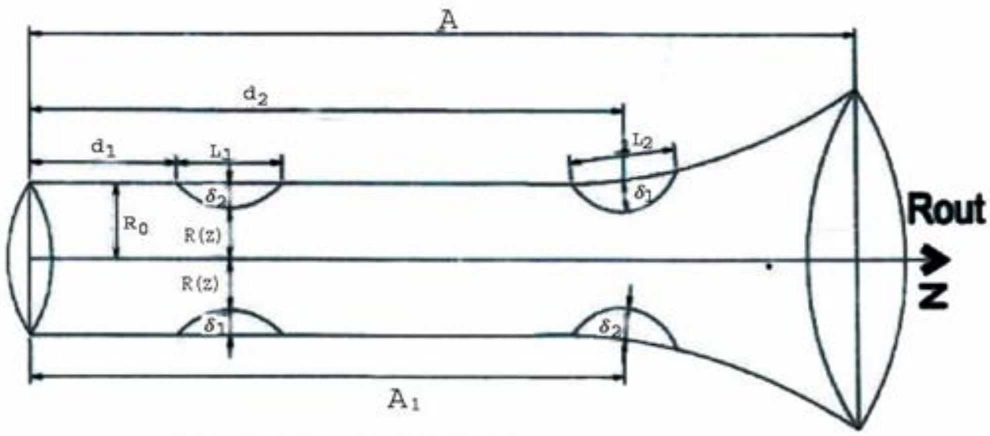

Figure 1: Physical Model of Multiple Stenosis

In this scenario, we are examining the steady movement of a Jeffrey fluid within a tube of varying cross-sectional shape, encompassing two gentle and axially symmetric constrictions. In order to depict the tube's shape, we employ a cylindrical polar coordinate system denoted as (r, z). Here, z signifies the dimension along the axis of the tube, while r is oriented perpendicular to the tube's axis. The tube's radius is considered to be..

$$h(z) = R(z) \text{ and } R(z) = R_0 \text{ where } 0 \le z \le d_1$$

$$= R_0 - \frac{\delta_1}{2}\left\{1+\cos\frac{2\pi}{L_1}\left(z-d_1-\frac{L_1}{2}\right)\right\} \text{where } d_1 \le z \le d_1 + L_1$$

$$= R_0 \qquad \text{where } d_1 + L_1 \le z \le A - \frac{L_1}{2}$$

$$= R_0 - \frac{\delta_2}{2}\left\{1+\cos\frac{2\pi}{L_2}\left(z-A_1-\frac{L_2}{2}\right)\right\} \text{where } A_1 - L_2 \le z \le A$$

$$= R^* - \frac{\delta_2}{2}\left\{1+\cos\frac{2\pi}{L_2}\left(z-A_1-\frac{L_2}{2}\right)\right\} \text{where } A_1 \le z \le A_1 + \frac{L_2}{2}$$

$$= R^*(z)$$

Where $A_1 + \frac{L_2}{2} \le z \le A$ (1)

Here, lengths of the two stenoses are L_i (where i=1,2) and the optimal width of two stenoses are δ_i (where i=1,2) and the criteria for the mild stenosis are fulfilled.

$$\delta_i << \min\left(R_0, R_{out}\right)$$

$$\delta_i << L_i \left(i = 1,2\right)$$

Where $R_{out} = R(z) \ at \ z = A$

The fundamental equation governing the flow is...

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{1+\mu_1}\frac{\partial w}{\partial r}\right) = \frac{1}{\rho}\frac{\partial p}{\partial z} \tag{2}$$

Where μ_1 is Jeffrey parameter, p is the pressure gradient, ρ is the viscosity of the fluid, R_0 is the radius of the tube Here, the Jeffrey parameter is represented by μ_1 , while p signifies the pressure gradient. Additionally, ρ stands for the fluid's viscosity and R_0 corresponds to the tube's radius.

The conditions at the boundaries are

$$\frac{\partial w}{\partial r} = 0 \quad \text{where} \quad r = 0 \tag{3}$$

$$w = 0 \quad \text{where} \quad r = R(z) \tag{4}$$

Introducing the subsequent dimensionless variable

$$\begin{gathered} \bar{r} = \frac{r}{R_0},\ \bar{p} = \frac{pR_0^{\ 2}}{\rho UA},\ \bar{d} = \frac{d_1}{A},\ \overline{L_2} = \frac{L_2}{A},\ \bar{w} = \frac{w}{\rho},\ \bar{z} = \frac{z}{A}, \\ \overline{L_1} = \frac{L_1}{A},\ \overline{A_1} = \frac{A_1}{A}, \bar{R}(z) = \frac{R(z)}{R_0},\ \bar{Q} = \frac{Q}{\pi U R_0^{\ 2}},\ \bar{\delta_t} = \frac{R(z)}{R_0} \end{gathered} \tag{5}$$

Making the governing equations dimensionless by eliminating the bars:

$$\frac{\partial}{\partial r}\left(\frac{r}{1+\mu_1}\frac{\partial w}{\partial r}\right) = r.\frac{\partial p}{\partial z} \tag{6}$$

The dimensionless boundary conditions are:

$$\frac{\partial w}{\partial r}=0 \qquad \text{Where } r=0 \tag{7}$$

$$w=0 \qquad \text{Where } r=R(z) \tag{8}$$

III. SOLUTION OF THE PROBLEM

1. **Velocity Distribution:** By integrating equation (1) while enforcing boundary conditions (7) and (8), the axial velocity can be derived as follows:

$$w=\frac{1}{4(1+\mu_1)}\frac{\partial p}{\partial z}\left(R^2-r^2\right) \tag{9}$$

The volumetric flow rate is determined as:

$$Q=\frac{1}{16(1+\mu_1)}\frac{\partial p}{\partial z}R^4(z) \tag{10}$$

2. **Pressure Difference:** The pressure gradient Δp across the entire length of the tube is expressed as follows:

$$\Delta p=\int_0^1 \frac{Q.16(1+\mu_1)}{R^4(z)} \tag{11}$$

IV. RESULT AND DISCUSSION

We have examined the axial velocity variation as a function of r for different values of the Jeffrey parameter μ_1, using equation (9) within the constricted regions $d_1 \le z \le d_1+L_1$ and $A_1-\frac{L_2}{2}\le z\le A_1$.

These results are depicted in figures (2) and (3). Importantly, it was observed that the velocity diminishes with an increase in the Jeffery parameter μ_1 for both constriction areas. Furthermore, we derived the volumetric flux in the constricted region $d_1 \le z \le d_1+L_1$ using equation (10) for various Jeffery parameter μ_1 and its depiction is visualized in figure 4. The curve adopts an inverted parabolic configuration, and significantly, it is worth noting that the lowest flux rate transpires precisely at the midpoint ($z=0.3$) of the stenosis.

Similarly, for the constricted region $A_1 - \frac{L_2}{2} \le z \le A_1$, we computed the volumetric flux across various Jeffery parameters and this is depicted in figure (5). Under these circumstances, the flux rate diminishes as the Jeffery parameter μ_1 increases within the constricted region.

Volumetric flux is computed across a range of k values, as depicted in figure (6). For numerical computation, it is assumed within the third constricted region that:

$$\frac{R^*(z)}{R_0} = e^{k(z-A_1)^2} = e^{\bar{k}(\bar{z}-\bar{A}_1)^2} \qquad \text{Where } \bar{k} = k.A^2$$

$$\text{And} \quad \overline{d_1} = 0.2,\ \overline{L_1} = 0.2,\ \overline{A_1} = 0.2,\ \overline{L_2} = 0.8 \ .$$

In this observation, we have noted an escalation in flux as the value of k increases. The alteration of Q flow rate concerning various values of Jeffery parameter μ_1 within the third constricted $A_1 \le z \le A_1 + \frac{L_2}{2}$, as depicted in figure (7). In this instance, it is evident that the flux declines with an escalation in the Jeffery parameter μ_1.

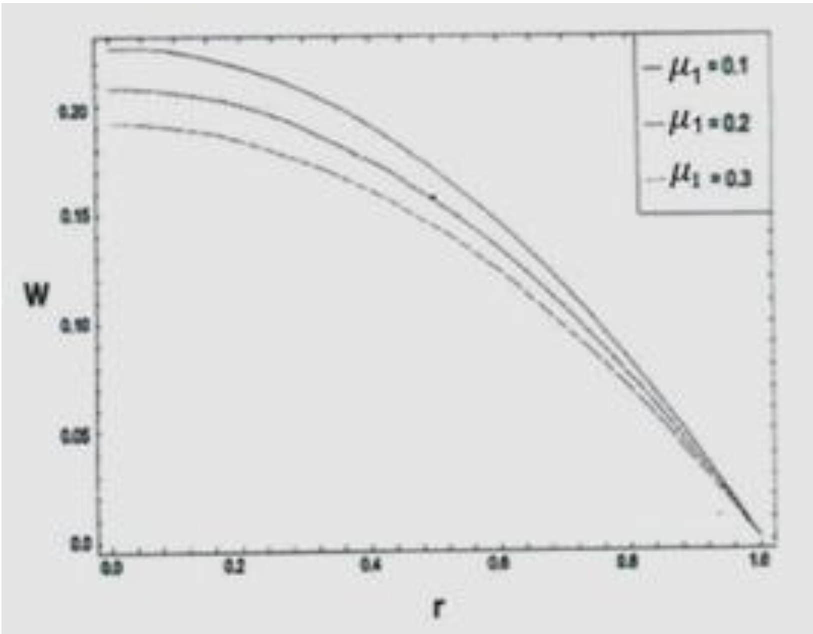

Figure 2: Variations of Flow Velocities for Various Values of Jeffery Parameter μ_1 in First Constricted Region $d_1 \le z \le d_1 + L_1$

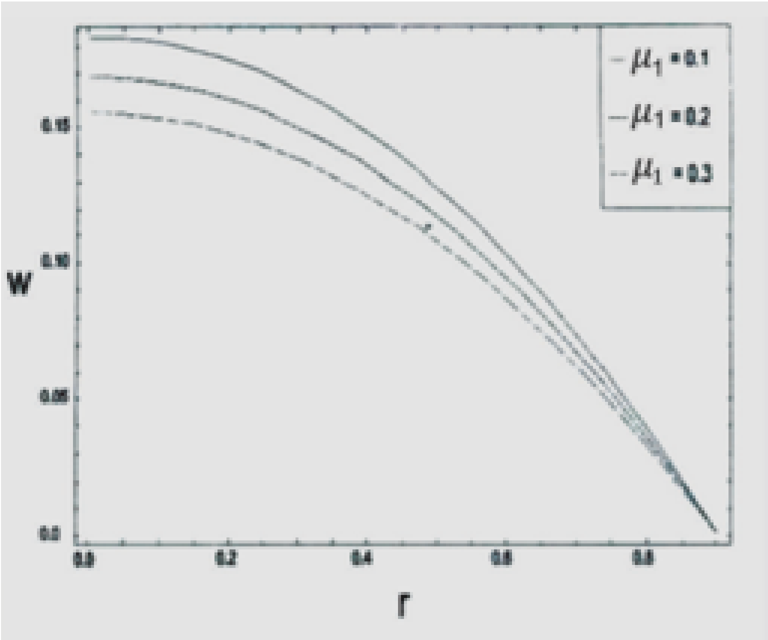

Figure 3: Variations of Flow Velocities for Various Values of Jeffery Parameter μ_1 in Second Constricted Region $A_1 - \frac{L_2}{2} \le z \le A_1$

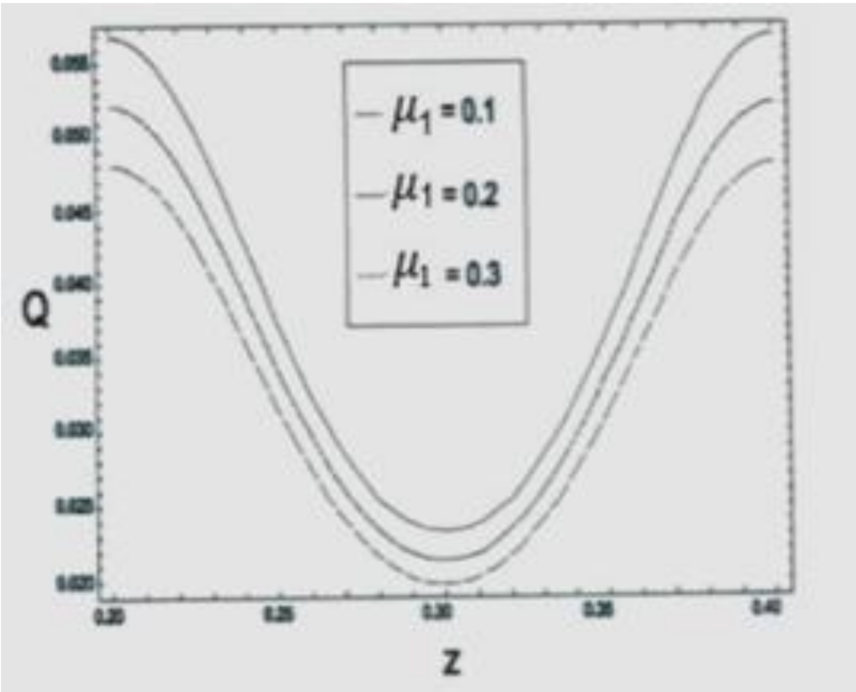

Figure 4: Volumetric Flux Q for Various Values of Jeffery Parameter μ_1 in First Constricted Region $d_1 \le z \le d_1 + L_1$

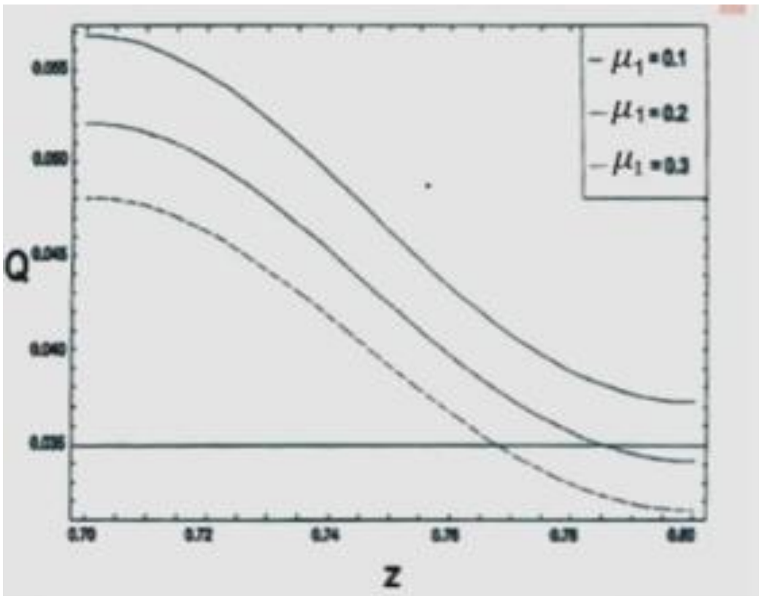

Figure 5: Volumetric Flux Q for Various Values of Jeffery Parameter μ_1 in Second Constricted Region $A_1 - \frac{L_2}{2} \leq z \leq A_1$

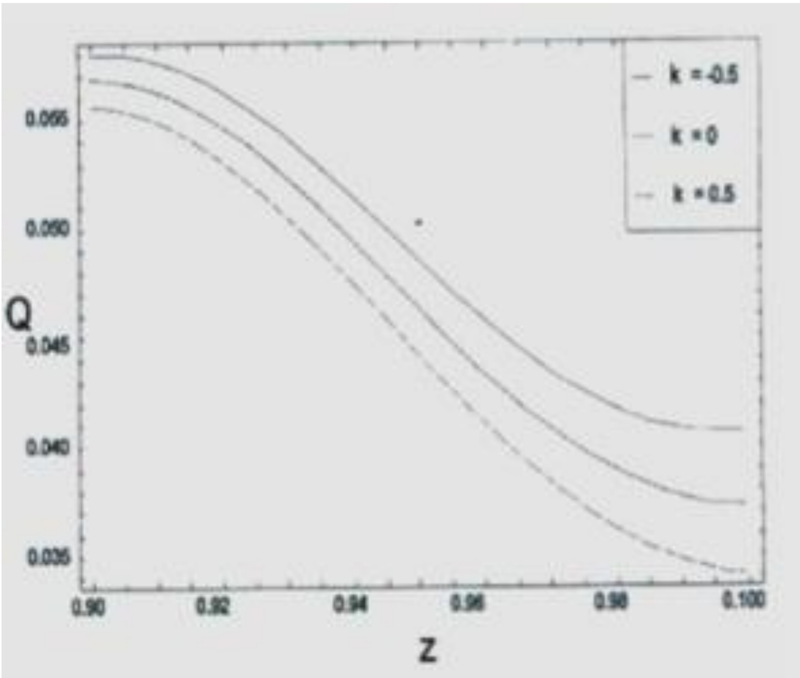

Figure 6: Volumetric Flux Q for Various Values of k in the Third Constricted Region $A_1 \leq z \leq A_1 + \frac{L_2}{2}$

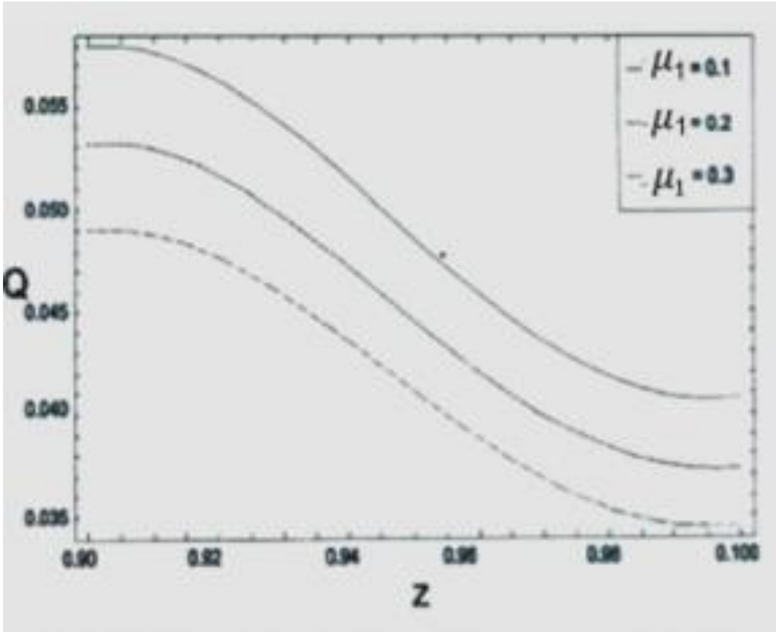

Figure 7: Volumetric Flux Q for Various Values of Jeffery Parameter μ_1 in the Third Constricted Region $A_1 \le z \le A_1 + \frac{L_2}{2}$

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