QUADRIPARTITIONED NEUTROSOPHIC VAGUE GENERALIZED CLOSED SETS IN QUADRIPARTITIONED NEUTROSOPHIC VAGUE TOPOLOGICAL SPACES

Abstract

In this paper, we introduce the concepts of *Quadripartitioned Neutrosophic Vague Generalized Closed*, *Quadripartitioned Neutrosophic Vague Generalized Pre closed*, *Quadripartitioned Neutrosophic Vague Generalized connected spaces* and *Quadripartitioned Neutrosophic Vague Generalized compact spaces* with some of their properties and we prove some theorems based on *Quadripartitioned Single Valued Neutrosophic Generalized Closed*, *Pre closed*, *connected*, *compact spaces*.

Keywords: Quadripartitioned, Neutrosophic, Vague, Topological.

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I. INTRODUCTION

Nowadays many real life problems includes in the field of engineering, economics deals with the concept of uncertainty, imprecise judgements, ambiguity etc.. . In these situations, we use $Fuzzy\,set$ [11] theory which was founded in 1965 by Zadeh to solve those ambiguity. $Fuzzy\,sets$ which allows the elements to have a $degrees\ of\ membership$ in the set and it lies in the real unit interval of [0, 1]. As an extension of $Fuzzy\,sets$, Atanassov introduced the concept of $Intuitionistic\,Fuzzy\,Set\,(IFS)$ [1] which includes $non-membership\,function$.. IFS theory is utilized in the areas like $logic\ programming$, $decision\,making\,problems$, $medical\,diagnosis$, $engineering\,problems$ etc. Later on, in 1993, Gau & Beuhrer introduced the $Vague\ set$ [4] $theory$.

After some time, in 2005 Smarandache presented the $Neutrosophic\,Set\,theory$ to solve problems contains insufficient, undefined and fickle information. In this theory, the elements in the set are allowed to have $membership$ and $non-membership\,function$. $Neutrosophic\ set\ (NS)\ theory$ deals with uncertainty factor i.e, indeterminacy factor which is independent of truth and falsity values. Since $Neutrosophic\ set(NS)$ is used to solve indeterminate and inconsistent information effectively, we apply NS in many fields like decision support system, semantic web services, new economy's growth, image processing, medical diagnosis etc., . In 2010 Wang et al., [5] developed $Single\ Valued\ Neutrosophic\ set\ (SVNS)$ and he defined some basic operations like $subset$, $equality$, $complement$, $union\ and\ intersection$ on $SVNS$.

In 1977, Belnap [2] introduced a new concept which includes a four valued logic in which any data is denoted by four parameters such as $True(T)$, $False(F)$, $neither\,true\,nor\,false(none)$ and $both\,true\,and\,false(both)$. As an extension of this concept, Smarandache [10] developed four numerical valued $neutrosophic\,logic$ in which $indeterminacy$ is splitted into two terms namely $Unknown(U)$ and $Contradiction(C)$.

Hence a new set $Quadripartitioned\,Single\,Valued\,Neutrosophic\,Set(QSVNS)$ was introduced by Rajashi Chatterjee.,et al [9] in which we have four components T, C, U, F in real unit interval [0,1]. Recently we have fused Vague set and Quadripartitioned Neutrosophic set and found Quadripartitioned Neutrosophic vague set [7]. In this paper, we introduce the concepts of $Quadripartitioned\ Neutrosophic\ Vague\ Generalized\ Closed$, $Quadripartitioned\ Neutrosophic\ Vague\ Generalized\ Pre\ closed$, $Quadripartitioned\ Neutrosophic\ Vague\ Generalized\ connected\ spaces$ and $Quadripartitioned\ Neutrosophic\ Vague\ Generalized\ compact\ spaces$ with some of their properties and we prove some theorems based on $Quadripartitioned\ Single\ Valued\ Neutrosophic\ Generalized\ Closed$, $Pre\ closed$, $connected$, $compact\ spaces$.

II. PRELIMINARIES

Definition 2.1[7]: Let $\mathcal{X}$ be the universe of discourse. A $\boldsymbol{Quadripartitioned\ Neutrosophic\ Vague\ Set\ (QNVS)}$ $\mathfrak{D}_{\mathcal{QNV}}$ on $\mathcal{X}$ written as $\mathfrak{D}_{\mathcal{QNV}} = \left\{\langle \mathfrak{x}; \hat{\mathcal{T}}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}); \hat{\mathcal{C}}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}); \hat{\mathcal{U}}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}); \hat{\mathcal{F}}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\rangle; \mathfrak{x} \in \mathcal{X}\right\}$, whose $truth\quad membership$,

$contradiction\ \ membership$, $ignorance\ \ membership\ \ and\ \ false\ \ membership$ $functions$ is defined as: $\hat{\mathcal{T}}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}) = [\mathcal{T}^-, \mathcal{T}^+]$,$\hat{\mathcal{C}}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}) = [\hat{\mathcal{C}}^-, \hat{\mathcal{C}}^+]$,$\hat{\mathcal{U}}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}) = [\mathcal{U}^-, \mathcal{U}^+]$, $\hat{\mathcal{F}}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}) = [\mathcal{F}^-, \mathcal{F}^+]$ Where, **(1)** $\mathcal{T}^+ = 1 - \mathcal{F}^-$ **(2)** $\mathcal{F}^+ = 1 - \mathcal{T}^-$ **(3)** $\mathcal{C}^+ = 1 - \mathcal{U}^-$ **(4)** $\mathcal{U}^+ = 1 - \mathcal{C}^-$ **(5)** $^-0 \leq \mathcal{T}^- + \mathcal{C}^- + \mathcal{U}^- + \mathcal{F}^- \leq 3^+$

Definition 2.2[7]: A $\boldsymbol{Quadripartitioned\ Neutrosophic\ Vague\ Topology}$ ($\mathcal{QNV}\boldsymbol{T}$) on $\mathcal{X}_{\mathcal{QNV}}$ is a family $\tau_{\mathcal{QNV}}$ of $Quadripartitioned\ Neutrosophic\ Vague\ Sets$ ($\mathcal{QNVS}$) in $\mathcal{X}_{\mathcal{QNV}}$ satisfying the following axioms:

i) $0_{\mathcal{QNV}}, 1_{\mathcal{QNV}} \in \tau_{\mathcal{QNV}}$

ii) $\mathcal{G}_1 \cap \mathcal{G}_2 \in \tau_{\mathcal{QNV}}$, for any $\mathcal{G}_1, \mathcal{G}_2 \in \tau_{\mathcal{QNV}}$

iii) $\cup\, \mathcal{G}_i \in \tau_{\mathcal{QNV}}, \forall\{\mathcal{G}_i : i \in J\} \subseteq \tau_{\mathcal{QNV}}$.

In this case the pair $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is called $Quadripartitioned\ Neutrosophic\ Vague\ Topological\ Space$ ($\mathcal{QNVTS}$) and any $\mathcal{QNVS}$ in $\tau_{\mathcal{QNV}}$ is known as $Quadripartitioned\ Neutrosophic\ Vague\ Open\ Set$ ($\mathcal{QNVOS}$) in $\mathcal{X}_{\mathcal{QNV}}$. The complement $\mathfrak{D}^c_{\mathcal{QNV}}$ of $\mathcal{QNVOS}$ $\mathfrak{D}_{\mathcal{QNV}}$ in $\mathcal{QNVTS}$ $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is called $Quadripartitioned\ Neutrosophic\ Vague\ Closed\ Set$ ($\mathcal{QNVCS}$) in $\mathcal{X}_{\mathcal{QNV}}$.

Definition 2.3 [7]: The $union$ of two $\mathcal{QNVS}$s $\mathcal{D}_{\mathcal{QNV}}$ and $\mathcal{E}_{\mathcal{QNV}}$ is a $\mathcal{QNVS}\ \mathcal{K}_{\mathcal{QNV}}$, written as $\mathcal{K}_{\mathcal{QNV}} = \mathcal{D}_{\mathcal{QNV}} \cup \mathcal{E}_{\mathcal{QNV}}$ whose $truth - membership$, $contradiction - membership$, $ignorance\ membership$ and $false - membership$ functions are related to those of $\mathcal{D}_{\mathcal{QNV}}$ and $\mathcal{E}_{\mathcal{QNV}}$ by

$$\hat{\mathcal{T}}_{\mathcal{K}_{\mathcal{QNV}}}(\mathfrak{x}) = \left[\max\left(\hat{\mathcal{T}}^-_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{T}}^-_{\mathcal{E}_{\mathcal{QNV}}}\right), \max\left(\hat{\mathcal{T}}^+_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{T}}^+_{\mathcal{E}_{\mathcal{QNV}}}\right)\right]$$

$$\hat{\mathcal{C}}_{\mathcal{K}_{\mathcal{QNV}}}(\mathfrak{x}) = \left[\max\left(\hat{\mathcal{C}}^-_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{C}}^-_{\mathcal{E}_{\mathcal{QNV}}}\right), \max\left(\hat{\mathcal{C}}^+_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{C}}^+_{\mathcal{E}_{\mathcal{QNV}}}\right)\right]$$

$$\hat{\mathcal{U}}_{\mathcal{K}_{\mathcal{QNV}}}(\mathfrak{x}) = \left[\min\left(\hat{\mathcal{U}}^-_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{U}}^-_{\mathcal{E}_{\mathcal{QNV}}}\right), \min\left(\hat{\mathcal{U}}^+_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{U}}^+_{\mathcal{E}_{\mathcal{QNV}}}\right)\right]$$

$$\hat{\mathcal{F}}_{\mathcal{K}_{\mathcal{QNV}}}(\mathfrak{x}) = \left[\min\left(\hat{\mathcal{F}}^-_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{F}}^-_{\mathcal{E}_{\mathcal{QNV}}}\right), \min\left(\hat{\mathcal{F}}^+_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{F}}^+_{\mathcal{E}_{\mathcal{QNV}}}\right)\right]$$

Definition 2.4 [7]: The intersection of two $\mathcal{QNVS}$s $\mathcal{D}_{\mathcal{QNV}}$ and $\mathcal{E}_{\mathcal{QNV}}$ is a $\mathcal{QNVS}\ \mathcal{H}_{\mathcal{QNV}}$, written as $\mathcal{H}_{\mathcal{QNV}} = \mathcal{D}_{\mathcal{QNV}} \cap \mathcal{E}_{\mathcal{QNV}}$ whose $truth - membership$, $contradiction - membership$, $ignorance\ membership$ and $false - membership$ functions are related to those of $\mathcal{D}_{\mathcal{QNV}}$ and $\mathcal{E}_{\mathcal{QNV}}$ by

$$\hat{\mathcal{T}}_{\mathcal{H}_{\mathcal{QNV}}}(\mathfrak{x}) = \left[\min\left(\hat{\mathcal{T}}^-_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{T}}^-_{\mathcal{E}_{\mathcal{QNV}}}\right), \min\left(\hat{\mathcal{T}}^+_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{T}}^+_{\mathcal{E}_{\mathcal{QNV}}}\right)\right]$$

$$\hat{\mathcal{C}}_{\mathcal{H}_{\mathcal{QNV}}}(\mathfrak{x}) = \left[\min\left(\hat{\mathcal{C}}^-_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{C}}^-_{\mathcal{E}_{\mathcal{QNV}}}\right), \min\left(\hat{\mathcal{C}}^+_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{C}}^+_{\mathcal{E}_{\mathcal{QNV}}}\right)\right]$$

$\hat{\mathcal{U}}_{\mathcal{H}_{\mathcal{QNV}}}(\mathfrak{x}) = \left[\max\left(\hat{\mathcal{U}}^-_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{U}}^-_{\mathcal{E}_{\mathcal{QNV}}}\right), \max\left(\hat{\mathcal{U}}^+_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{U}}^+_{\mathcal{E}_{\mathcal{QNV}}}\right)\right]$

$\hat{\mathcal{F}}_{\mathcal{H}_{\mathcal{QNV}}}(\mathfrak{x}) = \left[\max\left(\hat{\mathcal{F}}^-_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{F}}^-_{\mathcal{E}_{\mathcal{QNV}}}\right), \max\left(\hat{\mathcal{F}}^+_{\mathcal{D}_{\mathcal{QNV}}}, \hat{\mathcal{F}}^+_{\mathcal{E}_{\mathcal{QNV}}}\right)\right]$

Definition 2.5 [7]: Let $\left\{\mathcal{D}_{i_{\mathcal{QNV}}} : i \in J\right\}$ be an arbitrary family of $\mathcal{QNVS}$s. Then

$$\bigcup \mathcal{D}_{i_{\mathcal{QNV}}} = \left\{ \left\langle \begin{array}{l} \mathfrak{x}; \left(\underset{i \in J}{max}\left(\hat{\mathcal{T}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right), \underset{i \in J}{max}\left(\hat{\mathcal{T}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right)\right), \left(\underset{i \in J}{max}\left(\hat{\mathcal{C}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right), \underset{i \in J}{max}\left(\hat{\mathcal{C}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right)\right), \\ \left(\underset{i \in J}{min}\left(\hat{\mathcal{U}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right), \underset{i \in J}{min}\left(\hat{\mathcal{U}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right)\right), \left(\underset{i \in J}{min}\left(\hat{\mathcal{F}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right), \underset{i \in J}{min}\left(\hat{\mathcal{F}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right)\right) \end{array} \right\rangle; \mathfrak{x} \in \mathcal{X} \right\}$$

$$\bigcap \mathcal{D}_{i_{\mathcal{QNV}}} = \left\{ \left\langle \begin{array}{l} \mathfrak{x}; \left(\underset{i \in J}{min}\left(\hat{\mathcal{T}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right), \underset{i \in J}{min}\left(\hat{\mathcal{T}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right)\right), \left(\underset{i \in J}{min}\left(\hat{\mathcal{C}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right), \underset{i \in J}{min}\left(\hat{\mathcal{C}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right)\right), \\ \left(\underset{i \in J}{max}\left(\hat{\mathcal{U}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right), \underset{i \in J}{max}\left(\hat{\mathcal{U}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right)\right), \left(\underset{i \in J}{max}\left(\hat{\mathcal{F}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right), \underset{i \in J}{max}\left(\hat{\mathcal{F}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}\right)\right) \end{array} \right\rangle; \mathfrak{x} \in \mathcal{X} \right\}$$

Definition 2.6 [7]: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ be $\mathcal{QNVTS}$ and $\mathfrak{D}_{\mathcal{QNV}} = \left\{\left\langle \mathfrak{x}; \left[\hat{\mathcal{T}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{T}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{C}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{C}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{U}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{U}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{F}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{F}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]\right\rangle; \mathfrak{x} \in \mathcal{X}\right\}$ be $\mathcal{QNVS}$ in $\mathcal{X}_{\mathcal{QNV}}$. Then the *Quadripartitioned Neutrosophic vague interior* and *Quadripartitioned Neutrosophic vague closure* are defined by

i) $\mathcal{QNV}\ int(\mathcal{D}_{\mathcal{QNV}}) = \cup \left\{\mathcal{G}_{\mathcal{QNV}} / \mathcal{G}_{\mathcal{QNV}}\ is\ a\ \mathcal{QNVOS}\ in\ \mathcal{X}_{\mathcal{QNV}}\ and\ G_{\mathcal{QNV}} \subseteq \mathcal{D}_{\mathcal{QNV}}\right\}$

ii) $\mathcal{QNV}\ cl(\mathcal{D}_{\mathcal{QNV}}) = \cap \left\{K_{\mathcal{QNV}} / K_{\mathcal{QNV}}\ is\ a\ \mathcal{QNVCS}\ in\ \mathcal{X}_{\mathcal{QNV}}\ and\ \mathcal{D}_{\mathcal{QNV}} \subseteq K_{\mathcal{QNV}}\right\}$

Also for any $\mathcal{QNVS}$ $\mathcal{D}_{\mathcal{QNV}}$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$, we have $\mathcal{QNV}\ cl(\mathcal{D}^c_{\mathcal{QNV}}) = \left(\mathcal{QNV}\ int(\mathcal{D}_{\mathcal{QNV}})\right)^C$ and $\mathcal{QNV}\ int(\mathcal{D}^c_{\mathcal{QNV}}) = \left(\mathcal{QNV}\ cl(\mathcal{D}_{\mathcal{QNV}})\right)^C$.

It can also be shown that $\mathcal{QNV}\ cl(\mathcal{D}_{\mathcal{QNV}})$ is $\mathcal{QNVCS}$ and $\mathcal{QNV}\ int(\mathcal{D}_{\mathcal{QNV}})$ is $\mathcal{QNVOS}$ in $\mathcal{X}_{\mathcal{QNV}}$.

a) $\mathcal{D}_{Q\mathcal{N}\mathcal{V}}$ is $Q\mathcal{N}\mathcal{V}CS$ in $\mathcal{X}_{QNV}$ if and only if $Q\mathcal{N}\mathcal{V}\ cl(\mathcal{D}_{Q\mathcal{N}\mathcal{V}}) = \mathcal{D}_{Q\mathcal{N}\mathcal{V}}$.

b) $\mathcal{D}_{Q\mathcal{N}\mathcal{V}}$ is $Q\mathcal{N}\mathcal{V}OS$ in $\mathcal{X}_{QNV}$ if and only if $Q\mathcal{N}\mathcal{V}\ int(\mathcal{D}_{Q\mathcal{N}\mathcal{V}}) = \mathcal{D}_{Q\mathcal{N}\mathcal{V}}$.

III. QUADRIPARTITIONED NEUTROSOPHIC VAGUE GENERALIZED CLOSED SETS AND QUADRIPARTITIONED NEUTROSOPHIC VAGUE GENERALIZED PRE CLOSED SETS

Definition 3.1: A $Q\mathcal{N}\mathcal{V}S\ \mathcal{D}_{Q\mathcal{N}\mathcal{V}}$ in a $Q\mathcal{N}\mathcal{V}TS\ (\mathcal{X}_{Q\mathcal{N}\mathcal{V}}, \tau_{Q\mathcal{N}\mathcal{V}})$ is called,

i) $Quadripartitioned\ Neutrosophic\ Vague\ regular\ open$ if and only if

$$\mathcal{D}_{Q\mathcal{N}\mathcal{V}} = Q\mathcal{N}\mathcal{V}\ int\left(Q\mathcal{N}\mathcal{V}\ cl(\mathcal{D}_{Q\mathcal{N}\mathcal{V}})\right)$$

ii) $Quadripartitioned\ Neutrosophic\ Vague\ regular\ closed$ if and only if

$$\mathcal{D}_{Q\mathcal{N}\mathcal{V}} = Q\mathcal{N}\mathcal{V}\ cl\left(Q\mathcal{N}\mathcal{V}\ int(\mathcal{D}_{Q\mathcal{N}\mathcal{V}})\right)$$

Definition 3.2: A $Q\mathcal{N}\mathcal{V}S\ \mathcal{D}_{Q\mathcal{N}\mathcal{V}}$ in a $Q\mathcal{N}\mathcal{V}TS\ (X_{Q\mathcal{N}\mathcal{V}}, \tau_{Q\mathcal{N}\mathcal{V}})$ is called,

i) $Quadripartitioned\ Neutrosophic\ Vague\ semi\ open\ set$ ($Q\mathcal{N}\mathcal{V}SOS$) if

$$\mathcal{D}_{Q\mathcal{N}\mathcal{V}} \subseteq Q\mathcal{N}\mathcal{V}\ cl\left(Q\mathcal{N}\mathcal{V}\ int(\mathcal{D}_{Q\mathcal{N}\mathcal{V}})\right)$$

ii) $Quadripartitioned\ Neutrosophic\ Vague\ semi\ closed\ set$ ($Q\mathcal{N}\mathcal{V}SCS$) if

$$Q\mathcal{N}\mathcal{V}\ int\left(Q\mathcal{N}\mathcal{V}\ cl(\mathcal{D}_{Q\mathcal{N}\mathcal{V}})\right) \subseteq \mathcal{D}_{Q\mathcal{N}\mathcal{V}}$$

iii) $Quadripartitioned\ Neutrosophic\ Vague\ pre-open\ set$ ($Q\mathcal{N}\mathcal{V}POS$) if

$$\mathcal{D}_{Q\mathcal{N}\mathcal{V}} \subseteq Q\mathcal{N}\mathcal{V}\ int\left(Q\mathcal{N}\mathcal{V}\ cl(\mathcal{D}_{Q\mathcal{N}\mathcal{V}})\right)$$

iv) $Quadripartitioned\ Neutrosophic\ Vague\ pre-closed\ set$ ($Q\mathcal{N}\mathcal{V}PCS$) if

$$Q\mathcal{N}\mathcal{V}\ cl\left(Q\mathcal{N}\mathcal{V}\ int(\mathcal{D}_{Q\mathcal{N}\mathcal{V}})\right) \subseteq \mathcal{D}_{Q\mathcal{N}\mathcal{V}}$$

v) $Quadripartitioned\ Neutrosophic\ Vague\ \alpha-open\ set$ ($Q\mathcal{N}\mathcal{V}\alpha OS$) if

$$\mathcal{D}_{Q\mathcal{N}\mathcal{V}} \subseteq Q\mathcal{N}\mathcal{V}\ int\left(Q\mathcal{N}\mathcal{V}\ cl\left(Q\mathcal{N}\mathcal{V}\ int(\mathcal{D}_{Q\mathcal{N}\mathcal{V}})\right)\right)$$

vi) $Quadripartitioned\ Neutrosophic\ Vague\ \alpha-closed\ set$ ($Q\mathcal{N}\mathcal{V}\alpha CS$) if

$$Q\mathcal{N}\mathcal{V}\ cl\left(Q\mathcal{N}\mathcal{V}\ int\left(Q\mathcal{N}\mathcal{V}\ cl(\mathcal{D}_{Q\mathcal{N}\mathcal{V}})\right)\right) \subseteq \mathcal{D}_{Q\mathcal{N}\mathcal{V}}$$

vii) $Quadripartitioned\ Neutrosophic\ Vague\ \beta-open\ set$ ($Q\mathcal{N}\mathcal{V}\beta OS$) if

$$\mathcal{D}_{Q\mathcal{N}\mathcal{V}} \subseteq Q\mathcal{N}\mathcal{V}\ cl\left(Q\mathcal{N}\mathcal{V}\ int\left(Q\mathcal{N}\mathcal{V}\ cl(\mathcal{D}_{Q\mathcal{N}\mathcal{V}})\right)\right)$$

viii) $Quadripartitioned\ Neutrosophic\ Vague\ \beta\ -closed\ set(\mathcal{QNV}\beta CS)$ if

$$\mathcal{QNV}\ int\left(\mathcal{QNV}\ cl\left(\mathcal{QNV}\ int(\mathcal{D}_{\mathcal{QNV}})\right)\right) \subseteq \mathcal{D}_{\mathcal{QNV}}$$

Definition 3.3: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ be a $\mathcal{QNV}TS$ and

$\mathfrak{D}_{\mathcal{QNV}} = \left\{\langle \mathfrak{x}; \left[\hat{\mathcal{T}}^-_{\mathcal{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{T}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{C}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{C}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{U}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{U}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{F}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{F}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]\rangle; \mathfrak{x} \in \mathcal{X}\right\}$ be a $\mathcal{QNVS}$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$. Then $Quadripartitioned\ Neutrosophic\ Vague\ semi\ closure$ ($\mathcal{QNVS}\ cl$) and $Quadripartitioned\ Neutrosophic\ Vague\ semi\ interior$ ($\mathcal{QNVS}\ int$) of $\mathcal{D}_{\mathcal{QNV}}$ are defined by,

$$\mathcal{QNVS}\ cl(\mathcal{D}_{\mathcal{QNV}}) = \cap\left\{\mathcal{L}_{\mathcal{QNV}}: \mathcal{L}_{\mathcal{QNV}}\ is\ a\ \mathcal{QNVSCS}\ in\ \mathcal{X}_{\mathcal{QNV}}\ and\ \mathcal{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}\right\}$$

$$\mathcal{QNVS}\ int(\mathcal{D}_{\mathcal{QNV}}) = \cup\left\{\mathcal{M}_{\mathcal{QNV}}: \mathcal{M}_{\mathcal{QNV}}\ is\ a\ \mathcal{QNVSOS}\ in\ \mathcal{X}_{\mathcal{QNV}}\ and\ \mathcal{M}_{\mathcal{QNV}} \subseteq \mathcal{D}_{\mathcal{QNV}}\right\}$$

Result 3.4: Let $\mathcal{D}_{\mathcal{QNV}}$ be a $\mathcal{QNVS}$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$, then

i) $\mathcal{QNVS}\ cl(\mathcal{D}_{\mathcal{QNV}}) = \mathcal{D}_{\mathcal{QNV}} \cup \mathcal{QNV}\ int\left(\mathcal{QNV}\ cl(\mathcal{D}_{\mathcal{QNV}})\right)$

ii) $\mathcal{QNVS}\ int(\mathcal{D}_{\mathcal{QNV}}) = \mathcal{D}_{\mathcal{QNV}} \cap \mathcal{QNV}\ cl\left(\mathcal{QNV}\ int(\mathcal{D}_{\mathcal{QNV}})\right)$

Definition 3.5: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ be a $\mathcal{QNV}TS$ and
$\mathfrak{D}_{\mathcal{QNV}} = \left\{\langle \mathfrak{x}; \left[\hat{\mathcal{T}}^-_{\mathcal{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{T}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{C}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{C}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{U}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{U}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{F}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{F}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]\rangle; \mathfrak{x} \in \mathcal{X}\right\}$ be a $\mathcal{QNVS}$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$. Then $Quadripartitioned\ Neutrosophic\ Vague\ \alpha\ closure$ ($\mathcal{QNV}\alpha\ cl$) and $Quadripartitioned\ Neutrosophic\ Vague\ \alpha\ interior$($\mathcal{QNV}\alpha\ int$) of $\mathfrak{D}_{\mathcal{QNV}}$ are defined by,

i) $\mathcal{QNV}\alpha\ cl(\mathfrak{D}_{\mathcal{QNV}}) = \cap\left\{\mathcal{L}_{\mathcal{QNV}}: \mathcal{L}_{\mathcal{QNV}}\ is\ a\ \mathcal{QNV}\alpha CS\ in\ \mathcal{X}_{\mathcal{QNV}}\ and\ \mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}\right\}$

ii) $\mathcal{QNV}\alpha\ int(\mathfrak{D}_{\mathcal{QNV}}) = \cup\left\{\mathcal{M}_{\mathcal{QNV}}: \mathcal{M}_{\mathcal{QNV}}\ is\ a\ \mathcal{QNV}\alpha OS\ in\ \mathcal{X}_{\mathcal{QNV}}\ and\ \mathcal{M}_{\mathcal{QNV}} \subseteq \mathfrak{D}_{\mathcal{QNV}}\right\}$

Result 3.6: Let $\mathfrak{D}_{\mathcal{QNV}} = \left\{\langle \mathfrak{x}; \left[\hat{\mathcal{T}}^-_{\mathcal{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{T}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{C}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{C}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{U}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{U}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{F}}^-_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{F}}^+_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]\rangle; \mathfrak{x} \in \mathcal{X}\right\}$
be a $\mathcal{QNVS}$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$, then

i) $\mathcal{QNV}\alpha\; cl(\mathfrak{D}_{\mathcal{QNV}}) = \mathfrak{D}_{\mathcal{QNV}} \cup \mathcal{QNV}\; cl\left(\mathcal{QNV}\; int\left(\mathcal{QNV}\; cl(\mathfrak{D}_{\mathcal{QNV}})\right)\right)$

ii) $\mathcal{QNV}\alpha\; int(\mathfrak{D}_{\mathcal{QNV}}) = \mathfrak{D}_{\mathcal{QNV}} \cap \mathcal{QNV}\; int\left(\mathcal{QNV}\; cl\left(\mathcal{QNV}\; int(\mathfrak{D}_{\mathcal{QNV}})\right)\right)$

Definition 3.7: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ be a *Quadripartitioned Neutrosophic Vague Topological space*. A subset $\mathfrak{D}_{\mathcal{QNV}}$ of $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is called *Quadripartitioned Neutrosophic Vague generalized closed set* ($\mathcal{QNV}g - closed$) if $\mathcal{QNV}\; cl(\mathfrak{D}_{\mathcal{QNV}}) \subseteq \mathcal{L}_{\mathcal{QNV}}$ whenever $\mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$ and $\mathcal{L}_{\mathcal{QNV}}$ is a *Quadripartitioned Neutrosophic Vague open set*. Complement of $\mathcal{QNV}g - closed$ set is called $\mathcal{QNV}g - open$ set.

Theorem 3.8: Every *Quadripartitioned Neutrosophic Vague Closed set* is a *Quadripartitioned Neutrosophic Vague generalized closed set* in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$.

Proof:
Let $\mathfrak{D}_{\mathcal{QNV}}$ be a $\mathcal{QNV}CS$ and $\mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$ where $\mathcal{L}_{\mathcal{QNV}}$ be $\mathcal{QNV}OS$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$. Since $\mathfrak{D}_{\mathcal{QNV}}$ is $\mathcal{QNV}CS$, $\mathcal{QNV}\; cl(\mathfrak{D}_{\mathcal{QNV}}) \subseteq \mathfrak{D}_{\mathcal{QNV}}$ [since $\mathfrak{D}_{\mathcal{QNV}} = \mathcal{QNV}\; cl(\mathfrak{D}_{\mathcal{QNV}})$]. Therefore $\mathcal{QNV}\; cl(\mathfrak{D}_{\mathcal{QNV}}) \subseteq \mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$. Hence $\mathfrak{D}_{\mathcal{QNV}}$ is a $\mathcal{QNV}g - closed$ set in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$.

Theorem 3.9: Let $\mathcal{P}_{\mathcal{QNV}}$ and $\mathcal{R}_{\mathcal{QNV}}$ be $\mathcal{QNV}g - closed$ sets in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ then $\mathcal{P}_{\mathcal{QNV}} \cup \mathcal{R}_{\mathcal{QNV}}$ is also $\mathcal{QNV}g - closed$ set in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$.

Proof: Since $\mathcal{P}_{\mathcal{QNV}}$ and $\mathcal{R}_{\mathcal{QNV}}$ are $\mathcal{QNV}g - closed$ sets in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$,we get $\mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}}) \subseteq \mathcal{L}_{\mathcal{QNV}}$ and $\mathcal{QNV}\; cl(\mathcal{R}_{\mathcal{QNV}}) \subseteq \mathcal{L}_{\mathcal{QNV}}$ whenever $\mathcal{P}_{\mathcal{QNV}}, \mathcal{R}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$ where $\mathcal{L}_{\mathcal{QNV}}$ is $\mathcal{QNV}OS$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$. This implies $\mathcal{P}_{\mathcal{QNV}} \cup \mathcal{R}_{\mathcal{QNV}}$ is also a subset of $\mathcal{L}_{\mathcal{QNV}}$ where $\mathcal{L}_{\mathcal{QNV}}$ is $QNVOS$ in $X_{\mathcal{QNV}}$. Then $\mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}} \cup \mathcal{R}_{\mathcal{QNV}}) = \mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}}) \cup \mathcal{QNV}\; cl(\mathcal{R}_{\mathcal{QNV}})$. i.e., $\mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}} \cup \mathcal{R}_{\mathcal{QNV}}) \subseteq \mathcal{L}_{\mathcal{QNV}}$. Therefore $\mathcal{P}_{\mathcal{QNV}} \cup \mathcal{R}_{\mathcal{QNV}}$ is $\mathcal{QNV}g - closed$ set in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$.

Theorem 3.10: Let $\mathcal{P}_{\mathcal{QNV}}$ and $\mathcal{R}_{\mathcal{QNV}}$ be $\mathcal{QNV}g - closed$ sets in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ then $\mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}} \cap \mathcal{R}_{\mathcal{QNV}}) \subseteq \mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}}) \cap \mathcal{QNV}\; cl(\mathcal{R}_{\mathcal{QNV}})$.

Proof: Since $\mathcal{P}_{\mathcal{QNV}}$ and $\mathcal{R}_{\mathcal{QNV}}$ are $\mathcal{QNV}g - closed$ sets in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$, we get $\mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}}) \subseteq \mathcal{L}_{\mathcal{QNV}}$ and $\mathcal{QNV}\; cl(\mathcal{R}_{\mathcal{QNV}}) \subseteq \mathcal{L}_{\mathcal{QNV}}$ whenever $\mathcal{P}_{\mathcal{QNV}}, \mathcal{R}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$ where $\mathcal{L}_{\mathcal{QNV}}$ is $\mathcal{QNV}OS$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$. This implies $\mathcal{P}_{\mathcal{QNV}} \cap \mathcal{R}_{\mathcal{QNV}}$ is also a subset of $\mathcal{L}_{\mathcal{QNV}}$ where $\mathcal{L}_{\mathcal{QNV}}$ is $\mathcal{QNV}OS$. Since $\mathcal{P}_{\mathcal{QNV}} \cap \mathcal{R}_{\mathcal{QNV}} \subseteq \mathcal{P}_{\mathcal{QNV}}$ and $\mathcal{P}_{\mathcal{QNV}} \cap \mathcal{R}_{\mathcal{QNV}} \subseteq \mathcal{R}_{\mathcal{QNV}}$ and also we know that if $\mathcal{P}_{\mathcal{QNV}} \subseteq \mathcal{R}_{\mathcal{QNV}}$ then $\mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}}) \subseteq \mathcal{QNV}\; cl(\mathcal{R}_{\mathcal{QNV}})$. Therefore $\mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}} \cap \mathcal{R}_{\mathcal{QNV}}) \subseteq \mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}})$ and $\mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}} \cap \mathcal{R}_{\mathcal{QNV}}) \subseteq \mathcal{QNV}\; cl(\mathcal{R}_{\mathcal{QNV}})$ which implies that $\mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}} \cap \mathcal{R}_{\mathcal{QNV}}) \subseteq \mathcal{QNV}\; cl(\mathcal{P}_{\mathcal{QNV}}) \cap \mathcal{QNV}\; cl(\mathcal{R}_{\mathcal{QNV}})$. Hence proved.

Remark 3.11: The intersection of two $\mathcal{QNV}g-closed$ sets need not be a $\mathcal{QNV}g-closed$ set.

Theorem 3.12: Let $\mathcal{Y}_{\mathcal{QNV}}$ be $\mathcal{QNV}g-closed$ set in $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}})$ and $\mathcal{Y}_{\mathcal{QNV}}\subseteq\mathcal{Z}_{\mathcal{QNV}}\subseteq\mathcal{QNV}\ cl(\mathcal{Y}_{\mathcal{QNV}})$ then $\mathcal{Z}_{\mathcal{QNV}}$ is $\mathcal{QNV}g-closed$ set in $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}})$.

Proof: Let $\mathcal{Z}_{\mathcal{QNV}}\subseteq\mathcal{L}_{\mathcal{QNV}}$ where $\mathcal{L}_{\mathcal{QNV}}$ is $QNVOS$ in $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}})$. Then $\mathcal{Y}_{\mathcal{QNV}}\subseteq\mathcal{Z}_{\mathcal{QNV}}$ implies $\mathcal{Y}_{\mathcal{QNV}}\subseteq\mathcal{L}_{\mathcal{QNV}}$. Since $\mathcal{Y}_{\mathcal{QNV}}$ is $\mathcal{QNV}g-closed$, we get $\mathcal{QNV}\ cl(\mathcal{Y}_{\mathcal{QNV}})\subseteq\mathcal{L}_{\mathcal{QNV}}$ whenever $\mathcal{Y}_{\mathcal{QNV}}\subseteq\mathcal{L}_{\mathcal{QNV}}$. And also $\mathcal{Y}_{\mathcal{QNV}}\subseteq\mathcal{QNV}\ cl(\mathcal{Z}_{\mathcal{QNV}})$ implies $\mathcal{QNV}\ cl(\mathcal{Z}_{QNV})\subseteq\mathcal{QNV}\ cl(\mathcal{Y}_{QNV})$. Thus $\mathcal{QNV}\ cl(\mathcal{Z}_{\mathcal{QNV}})\subseteq\mathcal{L}_{\mathcal{QNV}}$ and so $\mathcal{Z}_{\mathcal{QNV}}$ is $\mathcal{QNV}g-closed$ set in $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}})$.

Theorem 3.13: A $\mathcal{QNV}g-closed$ set $\mathcal{Y}_{\mathcal{QNV}}$ is $\mathcal{QNV}CS$ if and only if $\mathcal{QNV}\ cl(\mathcal{Y}_{\mathcal{QNV}})-\mathcal{Y}_{\mathcal{QNV}}$ is $\mathcal{QNV}CS$..

Proof: First assume that $\mathcal{Y}_{\mathcal{QNV}}$ is $\mathcal{QNV}CS$ then we get $\mathcal{QNV}\ cl(\mathcal{Y}_{\mathcal{QNV}})=\mathcal{Y}_{\mathcal{QNV}}$ and so $\mathcal{QNV}\ cl(\mathcal{Y}_{\mathcal{QNV}})-\mathcal{Y}_{\mathcal{QNV}}=0_{\mathcal{QNV}}$ which is $\mathcal{QNV}CS$. Conversely assume that $\mathcal{QNV}\ cl(\mathcal{Y}_{\mathcal{QNV}})-\mathcal{Y}_{\mathcal{QNV}}$ is $\mathcal{QNV}CS$. Then $\mathcal{QNV}\ cl(\mathcal{Y}_{\mathcal{QNV}})-\mathcal{Y}_{\mathcal{QNV}}=0_{\mathcal{QNV}}$, i.e., $\mathcal{QNV}\ cl(\mathcal{Y}_{\mathcal{QNV}})=\mathcal{Y}_{\mathcal{QNV}}$ implies that $\mathcal{Y}_{\mathcal{QNV}}$ is $\mathcal{QNV}CS$. Hence proved.

Definition 3.14: Let $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}})$ be a $Quadripartitioned\ Neutrosophic\ Vague\ topological$

$space$. A $\mathcal{QNV}S$ $\mathfrak{D}_{\mathcal{QNV}}$ in $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}})$ is called $Quadripartitioned\ Neutrosophic\ Vague$

$\alpha\ generalized\ closed\ set\ (\mathcal{QNV}\alpha g-closed)$ if $\mathcal{QNV}\ \alpha\ cl(\mathfrak{D}_{\mathcal{QNV}})\subseteq\mathcal{L}_{\mathcal{QNV}}$ whenever $\mathfrak{D}_{\mathcal{QNV}}\subseteq\mathcal{L}_{\mathcal{QNV}}$ and $\mathcal{L}_{\mathcal{QNV}}$ is a $Quadripartitioned\ Neutrosophic\ Vague\ open\ set$ in $\mathcal{X}_{\mathcal{QNV}}$.

Definition 3.15. Let $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}})$ be a $\mathcal{QNV}TS$ and $\mathfrak{D}_{\mathcal{QNV}}$
$=\left\{\langle\mathfrak{x};\left[\hat{\mathcal{T}}^{-}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}),\hat{\mathcal{T}}^{+}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right];\left[\hat{\mathcal{C}}^{-}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}),\hat{\mathcal{C}}^{+}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right];\left[\hat{\mathcal{U}}^{-}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}),\hat{\mathcal{U}}^{+}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right];\left[\hat{\mathcal{F}}^{-}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}),\hat{\mathcal{F}}^{+}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]\rangle;\mathfrak{x}\in\mathcal{X}\right\}$
be a $\mathcal{QNV}S$ in $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}})$. Then $Quadripartitioned\ Neutrosophic\ Vague\ pre\ closure$ ($QNVP\ cl$) and $Quadripartitioned\ Neutrosophic\ Vague\ pre\ interior(\mathcal{QNV}P\ int)$ of $\mathfrak{D}_{\mathcal{QNV}}$ are defined by,

i) $\mathcal{QNV}P\ cl(\mathfrak{D}_{\mathcal{QNV}})=\cap\{\mathcal{L}_{\mathcal{QNV}}:\mathcal{L}_{\mathcal{QNV}}\ is\ a\ \mathcal{QNV}PCS\ in\ \mathcal{X}_{\mathcal{QNV}}\ and\ \mathfrak{D}_{\mathcal{QNV}}\subseteq\mathcal{L}_{\mathcal{QNV}}\}$

ii) $\mathcal{QNV}P\ int(\mathfrak{D}_{\mathcal{QNV}})=\cup\{\mathcal{M}_{\mathcal{QNV}}:\mathcal{M}_{\mathcal{QNV}}\ is\ a\ \mathcal{QNV}POS\ in\ \mathcal{X}_{\mathcal{QNV}}\ and\ \mathcal{M}_{\mathcal{QNV}}\subseteq\mathfrak{D}_{\mathcal{QNV}}\}$.

Result 3.16: Let $\mathfrak{D}_{\mathcal{QNV}}$ be a $QNVS$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$, then

$\mathcal{QNV}P\ cl(\mathfrak{D}_{\mathcal{QNV}}) = \mathfrak{D}_{\mathcal{QNV}} \cup \mathcal{QNV}\ cl\left(\mathcal{QNV}\ int(\mathfrak{D}_{\mathcal{QNV}})\right)$

Definition 3.17: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ be a $Quadripartitioned\ Neutrosophic\ Vague\ topological$

$space$. A $\mathcal{QNVS}$ $\mathfrak{D}_{\mathcal{QNV}}$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is called $Quadripartitioned\ Neutrosophic\ Vague$

$generalized\ pre-closed$ ($\mathcal{QNV}gP-closed$) set if $\mathcal{QNV}P\ cl(\mathfrak{D}_{\mathcal{QNV}}) \subseteq \mathcal{L}_{\mathcal{QNV}}$ whenever $\mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$ and $\mathcal{L}_{\mathcal{QNV}}$ is a $Quadripartitioned\ Neutrosophic\ Vague\ open\ set\ in\ \mathcal{X}_{\mathcal{QNV}}$. The family of all

$\mathcal{QNV}gP-closed$ set of a $\mathcal{QNV}TS$ $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is denoted by $\mathcal{QNV}gPC(X_{QNV})$.

Theorem 3.18: Every $Quadripartitioned\ Neutrosophic\ Vague\ closed\ set$($\mathcal{QNV}CS$) is a $Quadripartitioned\ Neutrosophic\ Vague\ generalized\ pre-closed$ ($\mathcal{QNV}gP-closed$) but not conversely.

Proof. Let $\mathfrak{D}_{\mathcal{QNV}}$ be a $\mathcal{QNV}CS$ in $\mathcal{X}_{\mathcal{QNV}}$ and $\mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$ where $\mathcal{L}_{\mathcal{QNV}}$ be $\mathcal{QNV}OS$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$. Since $\mathcal{QNV}Pcl(\mathfrak{D}_{\mathcal{QNV}}) \subseteq \mathcal{QNV}\ cl(\mathfrak{D}_{\mathcal{QNV}})$ and $\mathfrak{D}_{\mathcal{QNV}}$ is a $\mathcal{QNV}CS$ in $\mathcal{X}_{\mathcal{QNV}}$, $\mathcal{QNV}Pcl(\mathfrak{D}_{\mathcal{QNV}}) \subseteq \mathcal{QNV}\ cl(\mathfrak{D}_{\mathcal{QNV}}) = \mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$. Hence $\mathfrak{D}_{\mathcal{QNV}}$ is a $\mathcal{QNV}gP-closed$ set in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$.

Theorem 3.19: Every $Quadripartitioned$ Neutrosophic $Vague\ \alpha\ closed\ set$($\mathcal{QNV}\alpha CS$) is a $Quadripartitioned\ Neutrosophic\ Vague\ generalized\ pre-closed$ ($\mathcal{QNV}gP-closed$) set but not conversely.

Proof: Let $\mathfrak{D}_{\mathcal{QNV}}$ be a $\mathcal{QNV}\alpha CS$ in $\mathcal{X}_{\mathcal{QNV}}$ and $\mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$ where $\mathcal{L}_{\mathcal{QNV}}$ be $\mathcal{QNV}OS$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$. By hypothesis, $\mathcal{QNV}\ cl\left(\mathcal{QNV}\ int\left(\mathcal{QNV}\ cl(\mathfrak{D}_{\mathcal{QNV}})\right)\right) \subseteq \mathfrak{D}_{\mathcal{QNV}}$ and since $\mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{QNV}\ cl(\mathfrak{D}_{\mathcal{QNV}})$,
$\mathcal{QNV}\ cl\left(\mathcal{QNV}\ int(\mathfrak{D}_{\mathcal{QNV}})\right) \subseteq \mathcal{QNV}\ cl\left(\mathcal{QNV}\ int\left(\mathcal{QNV}\ cl(\mathfrak{D}_{\mathcal{QNV}})\right)\right) \subseteq \mathfrak{D}_{\mathcal{QNV}}$. Here $\mathcal{QNV}\ cl(\mathfrak{D}_{\mathcal{QNV}}) \subseteq \mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$. Therefore $\mathfrak{D}_{\mathcal{QNV}}$ is a $\mathcal{QNV}gP-closed$ set in $\mathcal{X}_{\mathcal{QNV}}$.

Theorem 3.20: Every $Quadripartitioned\ Neutrosophic\ Vague\ generalized\ closed$ ($\mathcal{QNV}g-closed$) set is a $Quadripartitioned\ Neutrosophic\ Vague\ generalized\ pre-closed$ ($\mathcal{QNV}gP-closed$) set but not conversely.

Proof: Let $\mathfrak{D}_{\mathcal{QNV}}$ be a $\mathcal{QNV}g-closed$ set in $\mathcal{X}_{\mathcal{QNV}}$ and $\mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$ where $\mathcal{L}_{\mathcal{QNV}}$ be $\mathcal{QNV}OS$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$. Since $\mathcal{QNV}Pcl(\mathfrak{D}_{\mathcal{QNV}}) \subseteq \mathcal{QNV}\ cl(\mathfrak{D}_{\mathcal{QNV}})$ and by hypothesis, $\mathcal{QNV}Pcl(\mathfrak{D}_{\mathcal{QNV}}) \subseteq F_{QNV}$. Therefore $\mathfrak{D}_{\mathcal{QNV}}$ is a $\mathcal{QNV}gP-closed$ set in $\mathcal{X}_{\mathcal{QNV}}$.

Theorem 3.21: Every $Quadripartitioned\ Neutrosophic\ Vague\ pre-closed$ ($\mathcal{QNV}PCS$) set is a $Quadripartitioned\ Neutrosophic\ Vague\ generalized\ pre-closed$ ($\mathcal{QNV}gP-closed$) set but not conversely.

Proof: Let $\mathfrak{D}_{\mathcal{QNV}}$ be a $\mathcal{QNV}PCS$ in $\mathcal{X}_{\mathcal{QNV}}$ and $\mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$ where $\mathcal{L}_{\mathcal{QNV}}$ be a $\mathcal{QNV}OS$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$. Since $\mathcal{QNV}\ cl\left(\mathcal{QNV}\ int(\mathfrak{D}_{\mathcal{QNV}})\right) \subseteq \mathfrak{D}_{\mathcal{QNV}}$ which implies $\mathcal{QNV}Pcl(\mathfrak{D}_{\mathcal{QNV}}) = \mathfrak{D}_{\mathcal{QNV}} \cup \mathcal{QNV}\ cl\left(\mathcal{QNV}\ int(\mathfrak{D}_{\mathcal{QNV}})\right) \subseteq \mathcal{L}_{\mathcal{QNV}}$. Therefore $\mathcal{QNV}Pcl(\mathfrak{D}_{\mathcal{QNV}}) \subseteq \mathcal{L}_{\mathcal{QNV}}$. Hence $\mathfrak{D}_{\mathcal{QNV}}$ is a $\mathcal{QNV}gP-closed$ set in $\mathcal{X}_{\mathcal{QNV}}$.

IV. QUADRIPARTITIONED NEUTROSOPHIC VAGUE GENERALIZED CONNECTED SPACE AND QUADRIPARTITIONED NEUTROSOPHIC VAGUE GENERALIZED COMPACT SPACE.

Definition 4.1: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ be a $Quadripartitioned\ Neutrosophic\ Vague\ topological\ space$. A $\mathcal{QNV}S\ \mathfrak{D}_{\mathcal{QNV}}$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is called $Quadripartitioned\ Neutrosophic\ Vague$

$generalized\ semi\ closed(\mathcal{QNV}gS-closed)$ set if $\mathcal{QNV}S\ cl(\mathfrak{D}_{\mathcal{QNV}}) \subseteq \mathcal{L}_{\mathcal{QNV}}$ whenever $\mathfrak{D}_{\mathcal{QNV}} \subseteq \mathcal{L}_{\mathcal{QNV}}$ and $\mathcal{L}_{\mathcal{QNV}}$ is a $Quadripartitioned\ Neutrosophic\ Vague\ open\ set$ in $\mathcal{X}_{\mathcal{QNV}}$. The family of all $\mathcal{QNV}gS-closed$ sets of a $\mathcal{QNV}TS$ $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is denoted by $\mathcal{QNV}gSC(\mathcal{X}_{\mathcal{QNV}})$.

Definition.4.2: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$, $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ be any two $\mathcal{QNV}TS$s. Then

1. A function $\mu: (\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1}) \longrightarrow (Y_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ is known as $Quadripartitioned\ Neutrosophic\ Vague\ generalized\ continuous$ ($\mathcal{QNV}g-continuous$) if μ^{-1} of every $Quadripartitioned\ Neutrosophic\ Vague\ closed\ set$ (respectively $open\ set$) in $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ is $\mathcal{QNV}g-closed$ set (respectively $\mathcal{QNV}g-open$) in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$.

2. A function $\mu: (\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1}) \longrightarrow (\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ is known as $Quadripartitioned\ Neutrosophic\ Vague\ generalized\ irresolute$ if μ^{-1} of every $\mathcal{QNV}g-closed$ set (respectively $\mathcal{QNV}g-open$) in $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ is $\mathcal{QNV}g-closed$ set (respectively $\mathcal{QNV}g-open$) in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$.

3. A function $\mu: (\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1}) \longrightarrow (\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ is known as $Quadripartitioned\ Neutrosophic\ Vague\ strongly\ continuous$ if $\mu^{-1}(\mathcal{V}_{\mathcal{QNV}})$ is both $Quadripartitioned\ Neutrosophic\ Vague\ open$ and $Quadripartitioned\ Neutrosophic\ Vague\ closed$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$ for each $Quadripartitioned\ Neutrosophic\ Vague\ set$ $\mathcal{V}_{\mathcal{QNV}}$ in $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$.

4. A function $\mu: (\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1}) \longrightarrow (\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ is known as $Quadripartitioned\ Neutrosophic\ Vague\ strongly\ generalized\ continuous$ if $\mu^{-1}(\mathcal{V}_{\mathcal{QNV}})$ is both $\mathcal{QNV}g-$

$closed$ and $\mathcal{QNV}g - open$ set in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$ for each $Quadripartitioned\ Neutrosophic\ Vague\ set\ \mathcal{V}_{\mathcal{QNV}}$ in $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$.

Definition 4.3: A $\mathcal{QNV}TS\ (\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is known as $Quadripartitioned\ Neutrosophic\ Vague\ connected$ if no non empty $Quadripartitioned\ Neutrosophic\ Vague\ set$ is both $Quadripartitioned\ Neutrosophic\ Vague\ open$ and $Quadripartitioned\ Neutrosophic\ Vague\ closed\ set$.

Definition 4.4: A $\mathcal{QNV}TS\ (\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is said to be $Quadripartitioned\ Neutrosophic\ Vague$ ($\mathcal{QNV}T_{1/2}$) space if every $\mathcal{QNV}g - closed$ set is a $Quadripartitioned\ Neutrosophic\ Vague\ closed$ in $\mathcal{X}_{\mathcal{QNV}}$.

Definition.4.5: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ be any $\mathcal{QNV}TS$. Then $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is known as $Quadripartitioned\ Neutrosophic\ Vague\ generalized\ disconnected$ ($\mathcal{QNV}g - disconnected$) if there exists a $\mathcal{QNV}g - open$ and $\mathcal{QNV}g - closed$ set $A_{\mathcal{QNV}}$ such that $A_{\mathcal{QNV}} \neq 0_{\mathcal{QNV}}$ and $A_{\mathcal{QNV}} \neq 1_{\mathcal{QNV}}$. $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is known as $\mathcal{QNV}g - connected$ if it is not $\mathcal{QNV}g - disconnected$.

Proposition 4.6: Every $\mathcal{QNV}g - connected$ space is $Quadripartitioned\ Neutrosophic\ Vague\ connected$. But the converse is not true.

Proof: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ be a $\mathcal{QNV}T$ space and assume that it is not $Quadripartitioned\ Neutrosophic\ Vague\ connected$. Hence there exist a $Quadripartitioned\ Neutrosophic\ Vague\ set$

$$\mathfrak{D}_{\mathcal{QNV}} = \left\{\langle \mathfrak{x}; \left[\hat{\mathcal{T}}^{-}_{\mathcal{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{T}}^{+}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{C}}^{-}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{C}}^{+}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{U}}^{-}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{U}}^{+}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{F}}^{-}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x}), \hat{\mathcal{F}}^{+}_{\mathfrak{D}_{\mathcal{QNV}}}(\mathfrak{x})\right]\rangle; \mathfrak{x} \in \mathcal{X}\right\}$$

such that $\mathfrak{D}_{\mathcal{QNV}}$ is both $\mathcal{QNV}OS$ and $\mathcal{QNV}CS$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$. Since every $Quadripartitioned\ Neutrosophic\ Vague\ open$ and $Quadripartitioned\ Neutrosophic\ Vague\ closed\ set$ is $\mathcal{QNV}g - open$, $\mathcal{QNV}g - closed$ respectively. It shows that $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is $\mathcal{QNV}g - connected$. Hence the proof.

Theorem 4.7: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ be a $\mathcal{QNV}T_{1/2}$ space. Then $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is $Quadripartitioned\ Neutrosophic\ Vague\ connected$ if and only if $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is $\mathcal{QNV}g - connected$.

Proof: First assume that $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is $\mathcal{QNV}g - disconnected$. Then there exist a $\mathcal{QNV}g - open$ and $\mathcal{QNV}g - closed$ set $\mathfrak{D}_{\mathcal{QNV}}$ such that $\mathfrak{D}_{\mathcal{QNV}} \neq 0_{QNV}$ and $\mathfrak{D}_{\mathcal{QNV}} \neq 1_{QNV}$. Since $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is $\mathcal{QNV}T_{1/2}$ space $\mathfrak{D}_{\mathcal{QNV}}$ is both $Quadripartitioned\ Neutrosophic\ Vague\ open$ and $Quadripartitioned\ Neutrosophic\ Vague\ closed$. Hence $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is not $Quadripartitioned\ Neutrosophic\ Vague\ connected$. Conversely assume that

$(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is not $Quadripartitioned\ Neutrosophic\ Vague\ connected$. Then there exist a $Quadripartitioned\ Neutrosophic\ Vague\ open$ and $Quadripartitioned\ Neutrosophic\ Vague\ closed\ set$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$. Since every $Quadripartitioned\ Neutrosophic\ Vague\ open$ and $closed\ set$ is $\mathcal{QNV}g-open$ and $\mathcal{QNV}g-closed$, $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is not $\mathcal{QNV}g-connected$. Hence the proof.

Proposition 4.8: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$, $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ are two $QNVTS$ s. If $\mu: (\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1}) \longrightarrow (\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ is $\mathcal{QNV}g-continuous$ surjection and $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$ is $\mathcal{QNV}g-connected$ then $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ is $\mathcal{QNV}g-connected$.

Proof: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$ be not $\mathcal{QNV}g-connected$. Then there exists a $\mathcal{QNV}g-open$ and $\mathcal{QNV}g-closed$ set $\mathfrak{D}_{\mathcal{QNV}}$ in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$ such that $\mathfrak{D}_{\mathcal{QNV}} \neq 0_{\mathcal{QNV}}$ and $\mathfrak{D}_{\mathcal{QNV}} \neq 1_{\mathcal{QNV}}$. Since μ is $\mathcal{QNV}g-continuous$, $\mu^{-1}(\mathfrak{D}_{\mathcal{QNV}})$ is $\mathcal{QNV}g-open$ and $\mathcal{QNV}g-closed$ set in $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$. Thus $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ is not $\mathcal{QNV}$ g-connected. Hence the proof.

Definition 4.9: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ be a $Quadripartitioned\ Neutrosophic\ Vague\ topological\ space$. If a family

$\left\{\langle \mathfrak{x}; \left[\hat{\mathcal{T}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{T}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{C}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{C}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{U}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{U}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{F}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{F}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]\rangle; i \in J\right\}$ of $\mathcal{QNV}g-open\ sets\ in$ $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ satisfies the condition, $\left\{\langle \mathfrak{x}; \left[\hat{\mathcal{T}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{T}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{C}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{C}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{U}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{U}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{F}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{F}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]\rangle; i \in J\right\} = 1_{\mathcal{QNV}}$ then it is known as $\mathcal{QNV}$g $-$ open cover of $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$.

A finite subfamily of a $\mathcal{QNV}$g $-$ open cover $\left\{\langle \mathfrak{x}; \left[\hat{\mathcal{T}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{T}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{C}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{C}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{U}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{U}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{F}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{F}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]\rangle; i \in J\right\}$ of $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ which is also a $\mathcal{QNV}$g $-$ open cover of $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is known as finite sub cover of $\left\{\langle \mathfrak{x}; \left[\hat{\mathcal{T}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{T}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{C}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{C}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{U}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{U}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{F}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{F}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]\rangle; i \in J\right\}$.

Definition 4.10: A $\mathcal{QNV}$TS $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ is called Quadripartitioned Neutrosophic Vague generalized compact ($\mathcal{QNV}$g $-$ compact) if and only if every $\mathcal{QNV}$g $-$ open cover of $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ has a te sub cover.

Theorem 4.11: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$, $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ be two $QNVTS$ s and $\mu: (\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1}) \longrightarrow (\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ be $\mathcal{QNV}g-continuous\ surjection$. If $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$ is $\mathcal{QNV}g-compact$ then $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ is also $\mathcal{QNV}g-compact$.

Proof: Let $\mathcal{D}_{i_{\mathcal{QNV}}} = \left\{\langle \mathfrak{x}; \left[\hat{\mathcal{T}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{T}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{C}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{C}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{U}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{U}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{F}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{F}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]\rangle; i \in J\right\}$ be a $\mathcal{QNV}g-open\ cover$ in $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ with

$$\cup \left\{\langle \mathfrak{x}; \left[\hat{\mathcal{T}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{T}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{C}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{C}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{U}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{U}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]; \left[\hat{\mathcal{F}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}), \hat{\mathcal{F}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]\rangle; i \in J\right\} = \bigcup_{i\in j} \mathcal{D}_{i_{\mathcal{QNV}}} = 1_{\mathcal{QNV}}$$

Since μ is $\mathcal{QNV}g-continuous$,

$$\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right) = \left\{\langle \begin{matrix} y; \left[\hat{\mathcal{T}}^-_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y), \hat{\mathcal{T}}^+_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y)\right]; \\ \left[\hat{\mathcal{C}}^-_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y), \hat{\mathcal{C}}^+_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y)\right]; \\ \left[\hat{\mathcal{U}}^-_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y), \hat{\mathcal{U}}^+_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y)\right]; \\ \left[\hat{\mathcal{F}}^-_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y), \hat{\mathcal{F}}^+_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y)\right] \end{matrix} \rangle; i \in J\right\}$$

is $\mathcal{QNV}g-open\ cover$ of $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$.

Now, $\cup_{i\in j} \mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right) = \mu^{-1}\left(\cup_{i\in j} \mathcal{D}_{i_{\mathcal{QNV}}}\right) = 1_{\mathcal{QNV}}$...................... (1)

Since $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^1})$ is $\mathcal{QNV}g-compact$, there exists a finite sub cover $J_0 \subseteq J$ such that,

$$\bigcup_{i\in J_0} \mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right) = 1_{\mathcal{QNV}}$$

Hence, $\mu\left(\cup_{i\in J_0} \mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)\right) = 1_{\mathcal{QNV}}$

$\mu\left(\mu^{-1}\left(\cup_{i\in j} \mathcal{D}_{i_{\mathcal{QNV}}}\right)\right) = 1_{\mathcal{QNV}}$ [by (1)]

$$\bigcup_{i\in j} \mathcal{D}_{i_{\mathcal{QNV}}} = 1_{\mathcal{QNV}}$$

Therefore $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$ is $\mathcal{QNV}g-compact$.

Definition 4.12: Let $(\mathcal{X}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}})$ be a $\mathcal{QNVTS}$ and $\mathcal{E}_{\mathcal{QNV}}$ be a $\mathcal{QNVS}$ in $\mathcal{X}_{\mathcal{QNV}}$. If a family

$$\left\{\langle e; \left[\hat{\mathcal{T}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e), \hat{\mathcal{T}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right]; \left[\hat{\mathcal{C}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e), \hat{\mathcal{C}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right]; \left[\hat{\mathcal{U}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e), \hat{\mathcal{U}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right]; \left[\hat{\mathcal{F}}^-_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e), \hat{\mathcal{F}}^+_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right]\rangle; i \in J\right\}$$

of $\mathcal{QNV}g-open$ sets in $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}})$ satisfies the condition $\mathcal{E}_{\mathcal{QNV}}\subseteq$ $\left\{\langle e;\left[\hat{\mathcal{T}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{T}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right];\left[\hat{\mathcal{C}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{C}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right];\left[\hat{\mathcal{U}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{U}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right];\left[\hat{\mathcal{F}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{F}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right]\rangle;i\in J\right\}$ $=1_{\mathcal{QNV}}$ then it is known as $\mathcal{QNV}g-open$ cover of $\mathcal{E}_{\mathcal{QNV}}$.

A finite subfamily of a $\mathcal{QNV}g-open$ cover $\left\{\langle e;\left[\hat{\mathcal{T}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{T}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right];\left[\hat{\mathcal{C}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{C}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right];\left[\hat{\mathcal{U}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{U}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right];\left[\hat{\mathcal{F}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{F}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right]\rangle;i\in J\right\}$ of $\mathcal{E}_{\mathcal{QNV}}$ which is also a $\mathcal{QNV}g-open$ cover of $\mathcal{E}_{\mathcal{QNV}}$ is known as $finite\ sub\ cover$ of $\left\{\langle e;\left[\hat{\mathcal{T}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{T}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right];\left[\hat{\mathcal{C}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{C}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right];\left[\hat{\mathcal{U}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{U}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right];\left[\hat{\mathcal{F}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e),\hat{\mathcal{F}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(e)\right]\rangle;i\in J\right\}$.

Definition 4.13: A $Quadripartitioned\ Neutrosophic\ Vague\ set$ $\mathcal{E}_{\mathcal{QNV}}$ in $\mathcal{QNVTS}$ $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}})$ is known as $\mathcal{QNV}g-compact$ if and only if every $\mathcal{QNV}g-open\ cover$ of $\mathcal{E}_{\mathcal{QNV}}$ has a $finite\ sub\ cover$.

Theorem 4.14: Let $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}^1})$, $(\mathcal{Y}_{\mathcal{QNV}},\tau_{\mathcal{QNV}^2})$ be any two $\mathcal{QNVTS}$ s and $\mu:(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}^1})\longrightarrow(\mathcal{Y}_{\mathcal{QNV}},\tau_{\mathcal{QNV}^2})$ be an $\mathcal{QNV}g-continuous$ function. If $\mathcal{D}_{\mathcal{QNV}}$ is $\mathcal{QNV}g-compact$ in $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}^1})$ then $\mu(\mathcal{D}_{\mathcal{QNV}})$ is $\mathcal{QNV}g-compact$ in $(\mathcal{Y}_{\mathcal{QNV}},\tau_{\mathcal{QNV}^2})$.

Proof: Let $\mathcal{D}_{i_{\mathcal{QNV}}}=$ $\left\{\langle\mathfrak{x};\left[\hat{\mathcal{T}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}),\hat{\mathcal{T}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right];\left[\hat{\mathcal{C}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}),\hat{\mathcal{C}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right];\left[\hat{\mathcal{U}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}),\hat{\mathcal{U}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right];\left[\hat{\mathcal{F}}^{-}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x}),\hat{\mathcal{F}}^{+}_{\mathcal{D}_{i_{\mathcal{QNV}}}}(\mathfrak{x})\right]\rangle;i\in J\right\}$ be a $\mathcal{QNV}g-open$ cover of $\mu(\mathcal{D}_{\mathcal{QNV}})$ in $(\mathcal{Y}_{\mathcal{QNV}},\tau_{\mathcal{QNV}^2})$ i.e, $\mu(\mathcal{D}_{\mathcal{QNV}})\subseteq\bigcup_{i\in j}\mathcal{D}_{i_{\mathcal{QNV}}}$

Since μ is $\mathcal{QNV}g-continuous$,

$$\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)=\left\{\left\langle\begin{array}{l}y;\left[\hat{\mathcal{T}}^{-}_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y),\hat{\mathcal{T}}^{+}_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y)\right];\\ \left[\hat{\mathcal{C}}^{-}_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y),\hat{\mathcal{C}}^{+}_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y)\right];\\ \left[\hat{\mathcal{U}}^{-}_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y),\hat{\mathcal{U}}^{+}_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y)\right];\\ \left[\hat{\mathcal{F}}^{-}_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y),\hat{\mathcal{F}}^{+}_{\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)}(y)\right]\end{array}\right\rangle;i\in J\right\}$$

is $\mathcal{QNV}\mathrm{g}-$ open cover of $\mathcal{D}_{\mathcal{QNV}}$ in $(\mathcal{X}_{\mathcal{QNV}},\tau_{\mathcal{QNV}^1})$.

Now, $\mathcal{D}_{\mathcal{QNV}}\subseteq\mu^{-1}\left(\bigcup_{i\in j}\mathcal{D}_{i_{\mathcal{QNV}}}\right)\subseteq\bigcup_{i\in J}\mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)$

Since $\mathcal{D}_{\mathcal{QNV}}$ is $\mathcal{QNV}\mathrm{g}-$ compact, then there exist a finite sub cover $J_0\subseteq J$ such that,

$$\mathcal{D}_{\mathcal{QNV}} \subseteq \bigcup_{i\in J_0} \mathcal{D}_{i_{\mathcal{QNV}}} = 1_{\mathcal{QNV}}$$

Hence, $\mu(\mathcal{D}_{\mathcal{QNV}}) \subseteq \mu\left(\bigcup_{i\in J_0} \mu^{-1}\left(\mathcal{D}_{i_{\mathcal{QNV}}}\right)\right) = \bigcup_{i\in J_0} \mathcal{D}_{i_{\mathcal{QNV}}}$

$\mu(\mathcal{D}_{\mathcal{QNV}})$ is $\mathcal{QNV}$g – compact in $(\mathcal{Y}_{\mathcal{QNV}}, \tau_{\mathcal{QNV}^2})$.

V. CONCLUSION

We have introduced the concepts of Quadripartitioned Neutrosophic Vague Generalized Closed , Quadripartitioned Neutrosophic Vague Generalized Pre closed , Quadripartitioned Neutrosophic Vague Generalized connected spaces and Quadripartitioned Neutrosophic Vague Generalized compact spaces with some of their properties and we prove some theorems based on Quadripartitioned Single Valued Neutrosophic Generalized Closed, Pre closed, connected, compact spaces.

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