LOCAL ISOLATE DOMINATION IN GRAPHS

Abstract

In this Communication a new parameter called Local Isolate Domination in graphs is defined and Studied. A Dominating Set S is a Local Dominating set iff for each u in S, < N(u) > has an isolate.

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Author

S. V. Padmavathi
P.G and Research
Department of Mathematics
Saraswathi Narayanan College
Madurai, Tamil Nadu, India
svsripadhma@gmail.com

I. INTRODUCTION

Throughout this paper , Simple Finite Graphs without loops and multiple edges are considered. For terminologies and notations refer Chartrand And Lesniak[3]. Domination and related topics are dealt in [1 , 4, 5, 6].

A subset S of the vertex set V(G) is a Dominating set if each vertex in the set V \ S is adjacent to a vertex in S. Minimum cardinality of a minimal dominating set is the Domination number of a Graph denoted by$\gamma(G)$.

It is an isolate dominating set if the induced graph<S> contains an isolate and is introduced and studied in [7].

A Dominating set S is called a Doubly Isolate Dominating set if both the induced graphs < S > and < V \ S > have isolates. Doubly Isolate Dominating set is introduced and studied in [2].

Also when the concept of Isolate Dominating set is localized to the Neighbour set we arrive at a new variant called Local Isolate Dominationin Graphs. This motivated us to define a new parameter that is introduced and studied in this communication.

II. PRELIMINARY RESULTS

Theorem 2.1: **"**For a Graph G with order atleast 3, $\Delta(G)= n-1$ and minimum degree atleast 2, G has no Local Isolate Dominating Set."

Proof: From the hypothesis we observe that G is a graph without isolates and with Domination number as one and hence this dominating set is not a Local Isolate Dominating set of G. "Suppose S is any dominating set of G and $S \setminus \{v\} \neq \phi$ where {v} is a full degree vertex of G.Now for each u in $\{S \setminus \{v\}\}$ the induced graph < N(u) > has no isolated vertex. Hence S is not a Local Isolate Dominating Set of G.

Corollary 2.2: **"**The Local Isolate Dominating Set Does Not Exist For The Following Graphs:

- Complete Graph K_n.
- Wheel Graph W_n.
- Fan Graph F_n."

Observation 2.3: **"**The Local Isolate Dominating Set Does Not Exist For Complete r-Partite $K_{n1,n2,\ldots nr}$, $r \geq 3$ Graph."

III. MAIN RESULTS

Proposition 3.1

1. For the Paths P_n and the Cycles C_n we have $\gamma_{lo}(P_n)=\gamma_{lo}(C_n)=\left\lceil \frac{n}{3} \right\rceil$, $\Gamma_{lo}(P_n)=\left\lceil \frac{n}{2} \right\rceil$ and $\Gamma_{lo}(C_n)=\left\lfloor \frac{n}{2} \right\rfloor$
2. If G is a Graph of Order n, Then $(G^+)=\Gamma_{lo}(G^+)=n$, Where G+ is the Graph that was produced from G by attaching at each of G's vertex's e Edges."

Proof

1. "Obviously $\gamma_{lo}(P_4)=2$ and when n≠4, any γ-set of P_n is a local isolate dominating set as well, so that $\gamma_{lo}(P_n)\le\gamma(P_n)$." Every Local Isolate D dominating set is a dominating set so $\gamma(P_n)\le\gamma_{lo}(P_n)$ thus $\gamma_{lo}(P_n)=\gamma(P_n)$ and so $\gamma_{lo}(P_n)=\left\lceil \frac{n}{3} \right\rceil$ as $\gamma(P_n)\left\lceil \frac{n}{3} \right\rceil$. Now if P_n = $\{v_1,v_2,v_3,\ldots v_n\}$ then the set s = $\left\{ v_{2i-1} \setminus 1\le i\le \left\lceil \frac{n}{2} \right\rceil \right\}$ is a minimal isolate dominating set so that $\Gamma_{lo}(P_n)\ge\left\lceil \frac{n}{2} \right\rceil$. Additionally, since any set that contains more than vertices of Pn is no longer able to be a minimal isolation dominating, we have $\Gamma_{lo}(P_n)=\left\lceil \frac{n}{2} \right\rceil$. Similar to this, one may prove $\gamma_{lo}(C_n)=\left\lceil \frac{n}{3} \right\rceil$ and $\Gamma_{lo}(C_n)=\left\lfloor \frac{n}{2} \right\rfloor$
2. Each pendant vertex is required to be present in any minimal isolate-dominating set S of G+ or one of its neighbours, in order to have at least n vertices. "Further, if $|S|>n$, S must consequently include a pendant vertex along with its support and so $S-\{v\}$, where v is the support, is an isolate dominating set of G^+, a contradiction to the minimality of S." Hence $|S|=n$.

Theorem 3.2: "For a Graph G of order at least 2, $\gamma_{lo}(G)=1$ iff there exists a pair u,vin V(G), $deg_G(u) = 1$ and $deg_G(v) = n-1$."

Proof: Let G be a graph with n≥2. Suppose $\gamma_{lo}(G)=1$. Let S={v} be a Local Isolate Dominating set of G. "Since S is a dominating set and |V(G)\ S|=n−1, $deg_G(v)=n-1$." Also since S is a γ_{lo}-set of G, <n(v) > has an isolate vertex, say u. Therefore u is a pendent vertex of G. Hence $deg_G(u)=1$. Conversely, {V} is a dominant set of G since there is a vertex v with $deg_G(v)=n$. Since "$deg_G(u)=1$, u is a isolate vertex in <n(v)>", thus $\gamma_{lo}(G)=1$.

Corollary 3.3: "For a Star Graph S_n with n≥2, $\gamma_{lo}(S_n) = 1$."

Theorem 3.4: "If G is a Tree with n ≥ 2 then G has a Local Isolate Dominating Set."

Proof: Let G be a Tree of Order n $\geq$2 and S be any Dominating Set of G. "Suppose G has no Local Isolate Dominating Set, there exist a vertex v $\in$S, and < n(v)>has no isolate vertex." Thus <n(v)> is a connected Graph. This implies < n[v] > contains a cycle, which contradicts that G is a Tree. Therefore G has a Local Isolate Dominating Set.

Corollary 3.5: "For any Tree T, $\gamma(T)=\gamma_o(T)=\gamma_{lo}(T)$".

Theorem 3.6: "Let S be any Local Isolate Dominating Set of a Graph G and U $\in$ S. Then there exist a vertex v $\in$ V (G) such that uv $\in$ E (G) and N (U) $\cap$ N (V) = ϕ."

Theorem 3.7: "For a Complete bipartite Graph $K_{m,n}$, $\gamma_{lo}(K_{m,n})=2$, m $\geq$2, n $\geq$2."

Theorem 3.8: "A Local isolate dominating set S of a graph G is minimal iff it is 1-minimal."

Proof: "Let S be a 1-minimal Local Isolate Dominating Set of a graph G. Suppose there exists a S'$\subset$ S that is also a Local Isolated Dominating Set of G, then for all v in S', <n(v) >has an isolate vertex". Since S' is a Dominating Set, for all vertex in u in S \ S' is adjacent to at least one vertex in S'and either u is an isolate vertex in <n(v)>,v $\in$ S' or <n(v) >has an isolated vertex in V \ S.

Case (i): u is an isolate vertex in <n(v)>, v$\in$ S' then S\{v} is Local Isolate Dominating Set of G which contradicts the "1-minimality of S".

Case (ii): <n(v)>has an isolated vertex in V \ S. Let w$\in$<n(v)>be isolate vertex in V \ S then S \{u} is Local Isolate Dominating Set of G which contradicts the 1-minimality of S. Hence S is minimal. Converse is obvious.

Theorem 3.9 "A Local Isolate Dominating Set S of a Graph G is Minimal iff every vertex in S has a Private Neighbor with respect to S."

Corollary 3.10 "A minimal Local Isolate Dominating Set S of a Graph G is also a minimal Dominating Set of a Graph G."

IV. JOIN OF GRAPHS

Observation 4.1 "Let G and H be any two Graphs of order m , n $\geq$3 with isolate vertex and S be a Local Isolate Dominating Set of G+H. Then S$\cap$V(G) $\neq \phi$ and S$\cap$V(H) $\neq \phi$."

Theorem 4.2 "Let G and H be any two Graphs. Then S a subset of V(G+H) is a Local Isolate Dominating Set of G+H iff G and H have isolated vertices."

Proof "Let G and H be any two Graphs and S $\subseteq$V(G+H) be a Local Isolate Dominating Set of G+H." Suppose G and H have no isolated vertex then for each u $\in$ S, < n(u)>is connected, which is a contradiction. Therefore there are isolated vertex in both G and H.

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Conversely, U and V be isolated vertices of G and H respectively, “Then S={u,v} is a Dominating Set of G + H and also n(u) $\geq$V(H) and n(v)$\geq$V(G).” Thus <n(u)> and < n(v)> have isolated vertex.
Therefore “S is a Local Isolate Dominating Set of G+H.”

Corollary 4.3 “Let G and H be any Graphs with isolated vertex, Then $\gamma_{lo}(G+H)\leq 2$.”

Proof: “Let G and H be Graphs with isolated vertex.” Suppose either G=K_1 or H= K_1or G= H= K_1 Then Clearly, $\gamma_{lo}(G+H) = 1$. Suppose G$\neq K_1$ and H $\neq K_{1,}$ by the theorem 4.2, $\gamma_{lo}(G+H) = 2$. Thus $\gamma_{lo}(G+H)\leq 2$.

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