SYSTEM OF FIRST ORDER ORDINARY DIFFERENTIAL EQUATIONS BY USING COMPLEX EFG TRANSFORM

Abstract

Recently Kuffi, Karaaslan and sadkhan developed EFG integral transform. We apply EFG transform for solving first order differential equation’s system.

Keywords: Integral transform, System of differential equation, Ordinary differential equation.

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I. INTRODUCTION

Integral transforms are very much useful in differential equations and hence play important role. Now a day's lot of researchers are interested and engaged in developing new integral transform and using those in different types of differential integral as well as Integro-differential equations and their systems.

Recently Kushare transform [2] and Soham transform [3] are introduced by Kushare, Khakale and Patil. Kuffietal introduced Complex EFG transformation [1] (2022). Patil [4, 5, 6, 7, 8, 9] used various integral transforms for solving various systems of differential equations.

In this chapter we use EFG transform for solving system of first order differential equations.

1. **Preliminary:** In this section we state some definitions, properties and formulae of complex EFG transform which are required to solve the system of first order ordinary differential equations.

Definition [1]: For the function of exponential order in set B defined as

B={f(t : there exist m, L_1, L_2 > 0} ,f(t) $<$ m $e^{-iLj(t)}, if\ j = 1,2, \ldots..$ (1)

where, $i^2 = -1$, m is finite for a particular function in the set B while $L_1$and L_2may be finite or infinite; we can define Complex EFG transform.

We denote the complex EFG transform by$G^c\{\}$ and define it as

G^c(f(t))= $\lim_{s\to\infty} \int_{t=0}^{s} f\ (t\ )e^{-is(u)t}$dt = f (iu) t≥0, L_1≤s(u)≤L_2

Table 1: Some basic functions and their Complex EFG integral transform.

Sr. No.	Function	EFG Transform
1	1	$\frac{-i}{s(u)}, s(u) \neq 0$
2	k	$\frac{-ik}{s(u)}, \quad s(u) \neq 0$
3	t	$\frac{-1}{s(u)^2}, \quad s(u) \neq 0, Re(s(u)) > 0$
4	t^2	$\frac{2!\,i}{s(u)^3}, \quad s(u) \neq 0, Re(s(u)) > 0$
5	t^n	$\frac{(-1)^n n!\,(i)^{(n-1)}}{[s(u)]^{n+1}}, \quad s(u) \neq 0, Re(s(u)) > 0$
6	e^{at}	$-\left[\frac{a}{a^2+(s(u))^2} + i\frac{s(u)}{a^2+(s(u))^2}\right], \quad a - is(u) \neq 0, Re(a - is(u) > 0)$

7	sin(at)	$\frac{-a}{(s(u))^2 - a^2}$, $s(u) > \lvert a \rvert$
8	cos(at)	$\frac{-is(u)}{(s(u))^2 - a^2}$, $s(u) > \lvert a \rvert$
9	sinh(at)	$\frac{-a}{(s(u))^2 + a^2}$, $Re(s(u)) > 0 , s(u) \neq \pm ai$
10	cosh(at)	$\frac{-is(u)}{(s(u))^2 + a^2}$, $Re(s(u)) > 0 , s(u) \neq \pm ai$

Properties of EFG transform: [1]

Property 1: If$G^c\{t\} = F(iu)$ then $G^{c^{-1}}\{F(iu-a)\} = e^{at}G^C\{f(t)\}$

$$G^c\{t\} = F(iu) = \lim_{p\to\infty}\int_0^p f(t)e^{is(u)t}\,dt$$

$$\mathrm{F(iu-a)} = \lim_{p\to\infty}\int_0^p f(t)e^{-[(is(u)t-a)]}\,dt$$

$$=\lim_{p\to\infty}\int_0^p f(t)e^{-is(u)t}e^{at}\,dt$$

$$=\lim_{p\to\infty}\int_0^p G^C(e^{at}f(t))\,dt$$

$$G^{c^{-1}}\{F(iu-a)\} = e^{at}G^C\{f(t)\}$$

Property: 2 shifting property for the EFG transform

If $F(t) = F(is(u))$ then; $G^c\{e^{at}F(is(u))\} = F(is(u) - a)$, where a is a realconstant number .

Theorem: Transform of derivatives [1]
Let F(u) be the Complex EFG transform of the f(t) then $G^c(f^{'}(t))$= - f(0) + is(u). F(iu)

Linearity Property: If f(t) and g(t) are two functions then,

$$G^c\{\alpha\, f(t) + \beta\, g(t)\} = \alpha G^c\{f(t)\} + \beta\, G^c\{g(t)\}$$

where α and β are arbitrary constant

2. **Applications for system of equation:** In this section we use Complex EFG transform to solve following first order differential equation's system.

Example:1 Consider the system of differential equations.

$$\frac{dy}{dt} + x = 0 \quad (1)$$

$$\frac{dx}{dt} + y = 0 \tag{2}$$

With the given initial conditionx$(0) = 1$ and y$(0) = 1$
By using EFG transform to equation (1) and equation (2),

$$G^c\{\frac{dy}{dt}\} + G^c\{x\} = 0$$

$$G^c\{\frac{dx}{dt}\} + G^c\{y\} = 0$$

$$-y(0) + is(u)G^c\{y\} + G^c\{x\} = 0$$
$$-x(0) + is(u)G^c\{x\} + G^C\{y\} = 0$$

Using initial conditions,

$$-1 + is(u)G^c\{y\} + G^c\{x\} = 0$$
$$-1 + is(u)G^c(x) + G^C(y) = 0$$

$$is(u)G^c\{y\} + G^c\{x\} = 1 \tag{3}$$
$$is(u)G^c\{x\} + G^C\{y\} = 1 \tag{4}$$

Multiplying equation (3) by is(u) and equation (4) by 1and subtracting we get,

$$[(is(u))^2 - 1]G^c(y) = is(u) - 1$$

$$[(is(u))^2 - (1)^2]G^c\{y\} = is(u) - 1$$

$$(is(u) - 1)(is(u) + 1)G^c\{y\} = is(u) - 1$$
$$\therefore G^c(y) = \frac{1}{is(u) + 1}$$
$$\therefore G^c(y) = \frac{1}{is(u) + 1} \times \frac{1 - is(u))}{1 - is(u)}$$
$$G^c\{\text{y}\} = \frac{1 - is(u))}{1 + (is(u))^2} (\text{since } i^2 = -1)$$
$$G^c\{y\} = \frac{1}{(1 + (s(u))^2)} - \frac{is(u)}{(1 + (s(u))^2)}$$
$$G^c\{y\} = -\left[\frac{-1}{(1 + (s(u)^2)} + \frac{is(u)}{(1 + (s(u))^2)}\right]$$

Now applying inverse complex EFG transform we obtain

$$y = e^{-t}$$

From equation (3)

$$is(u)G^c(y) + G^c(x) = 1$$
$$is(u)\left[\frac{1}{(1 + (s(u))^2)} - \frac{is(u)}{(1 + (s(u))^2)}\right] + G^c\{x\} = 1$$
$$\left[\frac{is(u)}{(1 + (s(u))^2)} - \frac{i^2(s(u))^2}{(1 + (s(u))^2)}\right] + G^c\{x\} = 1$$

Solving these two equations simultaneously,

$$G^c\{x\} = [1 - \left[\frac{is(u) + (s(u))^2}{(1 + (s(u))^2)}\right]]$$

$$G^c\{x\} = \frac{1 - is(u)}{1 - (is(u))^2} \qquad \text{(since} i^2 = -1)$$

$$G^c\{x\} = \frac{1}{(1 + (s(u)^2)} - \frac{is(u)}{(1 + (s(u))^2)}$$

$$G^c\{x\} = -[\frac{-1}{(1 + (s(u)^2)} + \frac{is(u)}{(1 + (s(u))^2)}]$$

By using inverse complex EFG transform, we obtain

$$x = e^{-t}$$

Required solution is $x = e^{-t}$ and $y = e^{-t}$.

Example:2 Consider system of differential equation

$$\frac{dy}{dt} + 2y = x \qquad (1)$$

$$\frac{dx}{dt} - 2x = y \qquad (2)$$

With the given initial condition x(0)=0 & y(0)=1

By using EFG transform

$$G^c\left\{\frac{dy}{dt}\right\} + 2G^c\{y\} = G^c\{x\}$$

$$G^c\left\{\frac{dx}{dt}\right\} - 2G^c\{x\} = G^c\{y\}$$

$$-(y(0)) + (is(u))G^c\{y\} + 2G^c\{y\} = G^c\{x\}$$

$$-(x(0)) + (is(u))G^c\{x\} - 2G^c\{x\} = G^c\{y\}$$

$$(is(u) + 2)G^c\{y\} = G^c\{x\}$$

$$(is(u) - 2)G^c\{x\} - G^c\{y\} = 0$$

$$G^c\{x\} - (is(u) + 2)G^c\{y\} = -1 \qquad (3)$$

$$(is(u) - 2)\, G^c\{x\} - G^c\{y\} = 0 \qquad (4)$$

We solve equations (3) and (4),

$$G^c\{x\} - (is(u) + 2)G^c\{y\} = (-1)$$

$$(is(u) + 2)(is(u) - 2)G^c\{x\} - (is(u) + 2)G^c\{y\} = 0$$

Solving these two equations simultaneously,

$$[1 - (is(u) + 2)G^c\{x\}] = (-1)$$

$$(1 + ((s(u))^2 + 4)G^C\{x\} = (-1)$$

$$G^c\{x\} = \frac{(-1)}{((s(u))^2 + 5)}$$

$$G^c\{x\} = [\left(\frac{1}{\sqrt{5}}\right)\frac{-\sqrt{5}}{((s(u))^2 + 5)}]$$

By using inverse complex EFG transform we get,

$$x = \frac{1}{\sqrt{5}} sinh\sqrt{5}y$$

From equation 4

$$G^c\{y\} = [(is(u) - 2)\left(\frac{-1}{((s(u))^2 + 5)}\right)]$$
$$= [\frac{-is(u)}{((s(u))^2+5)} + \frac{2}{((s(u))^2+5)}]$$

By using inverse EFG transform,

$$y = cosh\sqrt{5}t + \frac{2}{\sqrt{5}} sinh\sqrt{5}t$$

It is the required solution.

II. CONCLUSION

We used Complex EFG integral transform to solve the first order system of ordinary differential equations successfully. Answers obtained by using Complex EFG integral transform are same as obtained byanother integral transform methods.

REFERENCES

[1] Emad A. Kuffi, Faruk Karaaslan, GhaithS. Sadkhan, The complex EFG integral transformand its applications, International Journal of Health Sciences, 6(S3), pp. 237-547.
[2] S. R. Kushare, D. P. Patil and A. M. Takate,The new integral transform, “Kushare transform”, International Journal of Advances in Engineering and Management, Vol.3, Issue 9, Sept.2021, PP. 1589-1592
[3] Dinkar P. Patil and Savita S. Khakale, The new integral transforms “Soham transform, International Journal of Advances in Engineering and Management, Vol.3, issue 10, Oct. 2021.
[4] Dinkar P. Patil, Yashashri S. Suryawanshi and Mohini D Nehete, Application of Soham transform for solving mathematical models occuring in health science and biotechnology, International Journal of Mathematics, Statistics and Operations Research, Vol. 2, Number 2, 2022, pp. 273-288.
[5] Dinkar Patil, Shweta Rathi and Shrutika Rathi, The new integral transform Soham thransform for system of differential equations, International Journal of Advances in Engineering and Management, Vol. 4, Issue 5, May 2022, PP. 1675- 1678.
[6] Dinkar P. Patil, Prerana D. Thakare and Prajakta R. Patil, General Integral Transform for the Solution of Models in Health Sciences, International Journal of Innovative Science and Research Technology, Vol. 7, Issue 12, December 2022, pp. 1177- 1183.
[7] Dinkar P. Patil, Sonal R. Borse, Darshana P. Kapadi, Kushare Transform for the Solution of Models in Health Sciences, International Journal of Novel Research and Development, Vol. 8, Issue 1, January 2023, pp c617- c 623.
[8] D. P. Patil, Aboodh and Mahgoub transform in boundary Value problems of System of ordinary differential equations, International Journal of Advanced Research in Science, communication and Technology, Vol.6, Issue 1, June 2021, pp. 67-75.
[9] Dinkar P. Patil, Shweta L. Kandalkar and Nikita D. Gatkal, Applications of Kushare transform in the system of differential equations, International Advanced Research in Science, Engineering and Technology, Vol. 9, Issue 7, July 2022, pp. 192-195.