COMPLEX FERMATEAN PENTAPATITIONED NEUTROSOPHIC SETS AND ITS APPLICATIONS

Abstract

Complex fuzzy sets and complex intuitionistic fuzzy sets are unable to dealing with inexact, undetermined, discrepant, and inadequate periodic fact. To deal this issue, propose a complex fermatean pentapartitioned neutrosophic set whose complex valued truth membership function, complex valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, and complex valued false membership function are the combination of real valued truth amplitude term in related with phase term, real-valued contradiction amplitude term in related with phase term, real-valued unknown membership function, and complex valued false membership function. Complex fuzzy sets and complex intuitionistic fuzzy sets are incapable of dealing with inexact, undetermined, discrepant, and inadequate periodic fact. To deal this issue, propose a complex fermatean pentapartitioned neutrosophic set whose complex valued truth membership function, complex valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, and complex valued false membership function are the combination of real valued truth amplitude term in related with phase term, real-valued contradiction amplitude term in related with phase term, real-valued unknown membership function, and complex valued false membership function.

Keywords: Complex fermatean pentapartitioned neutrosophic set, complex fermatean pentapartitioned neutrosophic relation, complex fermatean pentapartitioned neutrosophic applications.

Authors

R. Subha
Research Scholar
Department of Mathematics
Nirmala College for Women
Coimbatore, Tamil Nadu, India.
Subhamurali6@Gmail.Com

K. Mohana
Assistant Professor
Department of Mathematics
Nirmala College for Women
Coimbatore, Tamil Nadu, India.
Riyaraju1116@Gmail.Com

I. INTRODUCTION

Zedeh's significant paper was the first to propose fuzzy sets. This unique notion is successfully employed in modeling uncertainty in many real-world applications. The membership function of [0, 1] characterizes a fuzzy set. Fuzzy sets and its applications have been significantly researched in several areas. In 1986, Atanassov created intuitionistic fuzzy sets, which include the uncertain degree known as the uncertain margin. The uncertain margin is equal to one less the sum of membership and non-membership. As a result, the intuitionistic fuzzy set is defined by a membership function and a non-membership function with values between [0, 1].

Ramot et al. [6] developed a new manner for fuzzy set extension by introducing complex fuzzy sets in which the degree of membership is swapped by a complex value of the type $r_s(x).e^{jw_s(x)}, j = \sqrt{-1}$ Where $r_s(x)$ and $w_s(x)$ are both in the range [0, 1]. The range in complex unit disk is $r_s(x).e^{jw_s(x)}$. In 1995, Florentin Smarandache greeted the concept of Neutrosphic set, enables the insight of unbiased thought by greeting a new factor in the set termed indeterminacy. As a result, the neutrosophic set was created, which contain the components of the truth membership function (T), indeterminacy membership function (I), and falsity membership function (F). Neutrosophic sets are engaged with the irregular interval of [0, 1].

Wang [4] (2010) developed the idea of single-valued nuetrosophic sets (SVNS), commonly stretching out of intuitionistic fuzzy sets, which has since been a hot study area. Fermatean Pentapartitioned Single Valued Neutronosophic Sets, proposed by Rajashi Chatterjee et al., be based on Belnap's four logics and Smarandache's four numerical logics. In the FPSVNS, indeterminacy is divided into two functions known as 'contradiction' (both true and false) and 'unknown' (neither true nor false), resulting in five components: T_A,C_A, K_A, U_A and F_A, which lie in the irregular interval [0, 1].

The research applies Ramot et al. [6], Alkouri and Saleh [1], and Cai and Zhang et al. [8]'s work to neutrosophic sets. To introduce a sophisticated neutrosophic set, we basically follow the theory of Ramot et al. [6]. A complex valued truth membership function, a complex valued indeterminate membership function, and a complex valued falsity membership function define the complex neutrosophic. In addition, the complex neutrosophic set is often used because it not only generalizes existing structures, but also defines information accordingly.

A relation, like a set, is crucial in all engineering, science and mathematics-oriented fields.

Relations explain the logic, approximation analysis, rule-based systems, nonlinear simulation, synthetic evaluation, classification, pattern recognition, and control. The relationship between fuzzy sets and intuitionistic fuzzy sets has been broadly researched in the neutrosophic surroundings. Yang et al. suggested and investigated single valued neutrosophic relations (SVRS).

The remaining article is structured as follows: Section 2 recaps some of the core principles associated with complicated neutrosophic sets and CNSs. In Section 3, we have to define the

Complex Fermatean Pentapartitioned Neutrosophic Sets (CFPNs) as a precursor to the idea of CNR. Following that, the notion of CNR will be defined. This component also generates a decision-making algorithm. Using the CFPNR features, this algorithm investigates the efficacy of a variety of training strategies. In Section 4, we describe various fundamental operations on CFPNRs, such as complement, inverse and composition. This section also includes the definition of projection for CFPNRs. Section 5 compares CFPNR and other current approaches to the supremacy of our suggested strategy in detail. Section 6 summarizes the paper's conclusion and proposes future study topics.

II. PRELIMINARIES

A complex neutrosophic set is described as follows:

Definition 2.1:

A complex neutrosophic set A is described on a universe X, for any x ϵ X, $T_A(x)$ a truth membership function, an indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$ assign a complex-valued grade of $T_A(x)$, $I_A(x)$, and $F_A(x)$ in S. The values of $T_A(x)$, $I_A(x)$, and $F_A(x)$, as well as their sum, may be within the complex plane's unit circle and also have the following form:

$T_A(x) = p_s(x).\, e^{ju_s(x)}$
$I_A(x) = q_s(x).\, e^{jv_s(x)}$
$F_A(x) = r_s(x).\, e^{jw_s(x)}$ where $p_s(x)$, $q_s(x)$, $r_s(x)$ and $u_s(x)$, $v_s(x)$, $w_s(x)$ are respectively, real valued and $p_s(x), q_s(x), r_s(x)\ \epsilon\ [0,1]$ such that $0 \le p_s(x) + q_s(x) + r_s(x) \le 3$.
The complex neutrosophic set S can be expressed as

$A = \{(x, T_A(x) = a_T, I_A(x) = a_I, F_A(x) = a_F : x\,\epsilon\, X)\}$,
where $T_A : \{X \to a_T : a_T \epsilon C, |a_T| \le 1\}$, $I_A : \{X \to a_I : a_I \epsilon C, |a_I| \le 1\}$ and
$F_A : \{X \to a_F : a_F \epsilon C, |a_F| \le 1\}$, and $|T_A(x) + I_A(x) + F_A(x)| \le 3$.

2. 2. Complex Neutrosophic Relations

Here, we define the Cartesian product of two CNSs, followed by the above definition of the CNR.

Definition 2.2 .1

Let X and Y represent two complex neutrosophic sets over U and V, respectively. The Cartesian product of X and Y, represented by X × Y, is a CNS described as

$X \times Y = \{\langle (u,v), T_{X\times Y}(u,v), I_{X\times Y}(u,v), F_{X\times Y}(u,v) \rangle : (u,v) \epsilon\, U \times V\}$, where $T_{X\times Y}(u,v)$ is a complex valued truth membership function $I_{X\times Y}(u,v)$ is a complex valued indeterminacy membership function and $F_{X\times Y}(u,v)$ is a complex- valued falsity membership function and $\forall (u,v) \epsilon\, U \times V$,

$T_{X\times Y}(u,v) = \min\left(p_X(u).\, p_Y(v).\, e^{jmin\,(\mu_X(u)\,(\mu_X(v)}\right)$,

$I_{X \times Y}(u,v) = \max\left(q_X(u).\, q_Y(v).\, e^{jmax\,(\vartheta_X(u)\,(\vartheta_X(v)}\right),$

$T_{X \times Y}(u,v) = \max\left(r_X(u).\, r_Y(v).\, e^{jmax\,(\omega_X(u)\,(\omega_X(v)}\right).$

Definition 2.3: [7]

Consider X as a universal set. An object of the form A Fermatean pentapartitioned neutrosophic set (FPN) A on X is written as A = {< x, $T_{A,}$ C_A , K_A , U_A , $F_{A,}$) >: x ϵ X }

$(T_A)^3 + (C_A)^3 + (K_A)^3 + (U_A)^3 + (F_A)^3 \leq 3$

Here, T_A(x) is the truth membership.
C_A(x) is contradiction membership,
K_A(x) is ignorance membership
U_A (x) is unknown membership,
F_A(x) is the false membership.

III. COMPLEX FERMATEAN PENTAPARTITIONED NEUTROSOPHIC SETS

The definition of a complex fermatean pentapartitioned neutrosophic set is:

Definition 3.1

A complex fermatean pentapartitioned neutrosophic set A described on a universal set of X, It is distinguished by a truth membership function T_A(x), C_A(x) is contradiction membership function, K_A(x) is ignorance membership function, U_A (x) is unknown membership function and F_A(x) that fix a complex-valued grade of T_A(x), C_A(x), K_A(x) , U_A (x) ,and F_A(x) in S for any x ϵ X. The values T_A(x), C_A(x), K_A(x), U_A (x) and F_A(x) is the false membership function. The entire can take the following form and be contained entirely within the complex plane.

$T_A(x) = p_s(x).\, e^{ju_s(x)}$

$C_A(x) = q_s(x).\, e^{jv_s(x)}$

$K_A(x) = r_s(x).\, e^{jw_s(x)}$

$U_A(x) = s_s(x).\, e^{jx_s(x)}$

$F_A(x) = t_s(x).\, e^{jy_s(x)}$ where $p_s(x)$, $q_s(x)$, $r_s(x)$, $s_s(x)$, $t_s(x)$ and $u_s(x), v_s(x), w_s(x), x_s(x), y_s(x)$

are respectively, real valued and $p_s(x), q_s(x), r_s(x), s_s(x), t_s(x)$ ϵ [0,1] such that

$0 \leq p_s(x), q_s(x), r_s(x), s_s(x), t_s(x) \leq 3.$

The complex fermatean pentapartitioned neutrosophic set S can be expressed as

$A = \{(x, T_A(x) = a_T,\ C_A(x) = a_C, K_A(x) = a_K, U_A(x) = a_U, F_A(x) = a_F : x \epsilon X)\}$

where $T_A : \{X \to a_T : a_T \epsilon C, |a_T| \leq 1\}$, $C_A : \{X \to a_C : a_{IC} \epsilon C, |a_C| \leq 1\}$, $K_A : \{X \to a_K : a_K \epsilon C, |a_K| \leq 1\}$, $U_A : \{X \to a_U : a_U \epsilon C, |a_U| \leq 1\}$, and $F_A : \{X \to a_F : a_F \epsilon C, |a_F| \leq 1\}$, and $|T_A(x) + C_A(x) + K_A(x) + U_A(x) + F_A(x)| \leq 3$.

Definition 3.2.

Given X to be a universal set, a complex fermatean pentapartitioned neutrosophic set A is described as:

A = {< x, $T_A(x)$, $C_A(x)$, $K_A(x)$, $U_A(x)$, $F_A(x)$) >: x ϵ X } where $T_A(x)$ is the degree of truth membership, $C_A(x)$ is called the degree of contradiction membership, $K_A(x)$ is called the degree of ignorance membership ,

$U_A(x)$ is called the degree of unknown membership and $F_A(x)$ is the degree of false membership that has a mapping $T_A: X \to \{z_1: z_1 \in C: |z_1| \leq 1\}$

$C_A : X \to \{ z_2 : z_2 \in C : |z_2| \leq 1\}$, $K_A : X \to \{ z_3 : z_3 \in C : |z_3| \leq 1\}$, $U_A : X \to \{ z_4 : z_4 \in C : |z_4| \leq 1\}$

and $F_A: X \to \{z_5: z_5 \in C: |z_5| \leq 1\}$. For all x ∈ X, the degree of truth membership is

$T_A(x) = A_A(x).e^{ia_A(x)}$, the degree of contradiction membership is $C_A(x) = B_A(x).e^{ib_A(x)}$, the degree of ignorance membership $K_A(x) = C_A(x).e^{ic_A(x)}$, the degree of unknown membership

$U_A(x) = D_A(x).e^{id_A(x)}$ and the degree of false membership is $F_A(x) = E_A(x).e^{ie_A(x)}$ respectively.

Where $T_A, C_A, K_A, U_A, F_A \in [0, 1]$, $a_A, b_A, c_A, d_A, e_A \in [0, 2\pi]$

$0 \leq T_A^3(x) + C_A^3(x) + K_A^3(x) + U_A^3(x) + F_A^3(x) \leq 3$, $i = \sqrt{-1}$.

$0 \leq a_A^3(x) + b_A^3(x) + c_A^3(x) + d_A^3(x) + e_A^3(x) \leq 3$.

3.3. Concept of complex fermatean pentapartitioned neutrosophic sets

The concept of complex valued truth membership function, complex-valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, complex valued false membership function and complex valued is not a simple task in understanding. Real valued truth membership function, real-valued contradiction membership function, real-valued ignorance membership function, real valued unknown membership function, real-valued false membership function in [0, 1] can be easily perceptive.

Understanding the concept of complex fermatean pentapartitioned neutrosophic set is straightforward when looking at its membership functions, which are represented by the following: ignorance, truth, contradiction, unknown, and false.

Definition 3.3.1.

$T_A(x) = p_s(x). e^{ju_s(x)}$

$C_A(x) = q_s(x). e^{jv_s(x)}$

$K_A(x) = r_s(x). e^{jw_s(x)}$

$U_A(x) = s_s(x). e^{jx_s(x)}$

$F_A(x) = t_s(x). e^{jy_s(x)}$

It is clear that a truth amplitude term $p_s(x)$ and a truth phase term $u_s(x)$ define the complex grade of truth membership function. Similarly, the complex grade of contradiction membership function is defined as a contradiction amplitude term $q_s(x)$ and a contradiction phase term $v_s(x)$; the complex grade of ignorance membership function is defined as an ignorance amplitude term $r_s(x)$ and a contradiction phase term $w_s(x)$; the complex grade of unknown membership function is defined as an unknown amplitude term $s_s(x)$ and a contradiction phase term $x_s(x)$; and the complex grade of false membership function is defined as a false amplitude term $t_s(x)$ and a false phase term $y_s(x)$, respectively. The truth amplitude term $p_s(x)$ equal to $|T_A(x)|$, the amplitude term $T_A(x)$. The contradiction amplitude $q_s(x)$ is equal to $|C_A(x)|$, ignorance amplitude $r_s(x)$ is equal to $|K_A(x)|$, unknown amplitude $s_s(x)$ is equal $|U_A(x)|$, and false amplitude terms $t_s(x)$ is equal to $|F_A(x)|$.

Neutronosophic sets are generalized into complex fermatean pentapartitioned sets. Representing a neutrosophic set as a complex fermatean pentapartitioned neutrosophic set is a simple task. As an illustration, the real-valued truth membership function, contradiction membership function, ignorance membership function, unknown membership function, and false membership function all describe the neutrosophic set S. Complex fermatean pentapartitioned neutrosophic sets are generalizations of neutrosophic sets. A neutrosophic set in the form of a complex fermatean pentapartitioned neutrosophic set is an easy task. For example, the neutrosophic set S is characterized by a real-valued truth membership function $\alpha_{s_1}(x)$, a contradiction membership function $\alpha_{s_2}(x)$, an ignorance membership function $\alpha_{s_3}(x)$, an unknown membership function $\alpha_{s_4}(x)$, and a false membership function $\alpha_{s_5}(x)$. By setting the truth amplitude term $p_s(x)$ is equal to $\alpha_{s_1}(x)$ and the truth phase term $u_s(x)$ equal to zero for all x. Similarly, we can set the contradiction amplitude term $q_s(x)$ equal to $\alpha_{s_2}(x)$ and a contradiction phase term $v_s(x)$ equal to zero for all x. The ignorance membership amplitude term $r_s(x)$ equal to $\alpha_{s_3}(x)$ and a ignorance phase term $w_s(x)$ equal to zero for all x. The unknown amplitude term $s_s(x)$ equal to $\alpha_{s_4}(x)$ and an unknown phase term $x_s(x)$ equal to zero for all x. The false amplitude term $t_s(x)$ equal to $\alpha_{s_5}(x)$ and a false phase term $y_s(x)$ equal to zero for all x. Consequently, it is observed that a fermatean pentapartitioned neutrosophic set can be generated from a complex fermatean pentapartitioned set. The explanation that follows identifies the following: the truth amplitude term corresponds to the real-valued grade of the truth membership function; the contradiction amplitude term, to the real-valued grade of the contradiction membership function; the ignorance amplitude term, to the real-valued grade of the ignorance membership function; the unknown amplitude term, to the real-valued grade of the unknown membership function; and the false amplitude term, to the real-valued grade of the false membership function.

The truth phase term, contradiction phase term, ignorance phase term, unknown phase term, and false phase term are the sole variable that separate them. This sets apart the regular neutrosophic set from the complex fermatean neutrosophic set. Put another way, the complicated fermatean pentapartitioned neutrosophic set will effectively transform into the fermatean pentapartitioned neutrosophic set if all five phase terms are eliminated. The fact that $p_s(x)$, $q_s(x)$, $r_s(x)$, $s_s(x)$ and $t_s(x)$ and have [0, 1], the real-valued grade of membership in the truth, the real-valued grade of membership in contradiction, the real-valued grade of membership in ignorance, the real-valued grade of membership in the unknown and the real-valued grade of false membership supports every discussion in this article.

Accordingly, complex fermatean pentapartitioned neutrosophic sets are the most sophisticated establishment of all existence techniques; as a result, complex fermatean neutrosophic sets represent a unique and noteworthy idea.

3.4 Illustration of a complex fermatean pentapartitioned neutrosophic set

Problem 3.4.1.

Consider a universal set X. Then S is a complex fermatean pentapartitioned neutrosophic set in X, as given below:

$$S = \frac{o.6e^{j.0.8}, o.3e^{j.\frac{3\pi}{4}}, o.5e^{j.0.3}, o.65e^{j.0.5} o.2e^{j.0.4}}{x_1} + \frac{o.7e^{j.0.1}, o.1e^{j.\frac{2\pi}{4}}, o.2e^{j.0.9}, o.5e^{j.0.3}, o.8e^{j.0.4}}{x_2} + \frac{o.9e^{j.0.1}, o.4e^{j.\pi}, o.7e^{j.0.7}, o.6e^{j.0.5} o.3e^{j.0.4}}{x_3}$$

Set theoretic operations on a complex fermatean pentapartitioned neutrosophic set Ramot et al. [6] determine the complement of the membership phase term $\mu_s(x)$ by several possible methods, such as $\mu_s^c(x) = \mu_s(x), 2\pi - \mu_s(x)$. Zhang [8] described the complement of the membership phase term by the rotation of $\mu_s(x)$ by π radian as $\mu_s(x) = \mu_s(x) + \pi$.

To define the complement of a complex fermatean pentapartitioned neutrosophic set, we define the neutrosophic complement for the truth amplitude term $p_s(x)$, the contradiction amplitude term $q_s(x)$, the ignorance amplitude term $r_s(x)$, the unknown amplitude term $s_s(x)$, and the false amplitude term $t_s(x)$.

Definition 3.5. Complement of complex fermatean pentapartitioned neutrosophic set.

Consider A = {< x, $T_A(x)$, $C_A(x)$, $K_A(x)$, $U_A(x)$, $F_A(x)$) >: x ϵ X} being a complex fermatean pentapartitioned neutrosophic set in X. Then the complement of a complex fermatean pentapartitioned neutrosophic set A is defined as c (A) and is defined by

$c(A) = \{<$ x, T_A^c(x) , C_A^c(x) , K_A^c(x) , U_A^c(x) , F_A^c(x) $>$: x ϵ X},
Where T_A^c(x)= c $(p_s(x).e^{ju_s(x)})$, C_A^c(x) $= c(q_s(x).e^{jv_s(x)})$, K_A^c(x) = c$\left(r_s(x).e^{jw_s(x)}\right)$, U_A^c(x) = c$(s_s(x).e^{jx_s(x)})$, F_A^c(x) $= c(t_s(x).e^{jy_s(x)})$ in which c ($p_s(x).e^{ju_s(x)}$) = c $(p_s(x).e^{ju_s^c(x)})$ is such that $c\ (p_s(x)) =\ t_s(x)$ and $u_s^c(x) = u_s(x), 2\pi - u_s(x)$ or $u_s(x) + \pi$.

Similarly, $c\left(q_s(x)\right) = c\left(s_s(x)\right)$and $v_s^c(x) = v_s(x), 2\pi - v_s(x)$ or $v_s(x) + \pi$.
$c\left(r_s(x)\right) = 1 - r_s(x)$ and $r_s^c(x) = r_s(x), 2\pi - r_s(x)$ or $r_s(x) + \pi$.

$c\left(s_s(x)\right) = c\left(q_s(x)\right)$ and $s_s^c(x) = s(x), 2\pi - s_s(x)$ or $s_s(x) + \pi$.

Finally, c$\left(t_s(x).e^{jy_s(x)}\right) =$ c$(t_s(x).e^{jy_s^c(x)})$ where c$\left(t_s(x)\right) = \left(p_s(x)\right)$ and $w_s^c(x) = w_s(x), 2\pi - w_s(x)$ Or $w_s(x) + \pi$.

Proposition 3.6

Let us consider a complex fermatean pentapartitioned neutrososphic set A on X, then, c(c(A)) = A.

Proof By definition 3.1. , it is clearly demonstrated.

Proposition 3.7.

Let us consider two complex fermatean pentapartitioned neutrososphic sets A and B on X.

Next, $c(A \cap B = c(A) \cap c(B)$.

Definition 3.8.

Union of complex fermatean pentapartitioned neutrososphic sets.

The union of two complex fermatean pentapartitioned neutrososphic sets A and B were defined as follows by Ramot et al. [6]: Let $\mu_A(x) = r_A(x).e^{jw_A(x)}$ and $\mu_B(x) = r_B(x).e^{jw_B(x)}$ are the complex-valued membership functions of A and B respectively. Then the membership of $A \cup B$ is given by $\mu_{A\cup B}(x) = \left([r_A(x) \oplus r_B(x)].e^{jw_{A\cup B}(x)}\right)$. Since $r_A(x)$ and $r_B(x)$ are real valued in [0,1], the operators max and min may be used on them to determine the membership union of several methods are provided for computing the phase term $w_{A\cup B}(x)$. Now we define the following as the union of two complex fermatean pentapartitioned neutrososphic sets.

Given two complex fermatean pentapartitioned neutrososphic sets X, A and B are characterized as follows:

A = {< x, T_A(x), C_A(x) , K_A(x) , U_A (x) , F_A(x)) >: x ϵ X } and

B = {< x, T_B(x), C_B(x) , K_B(x) , U_B (x) , F_B(x)) >: x ϵ X }

Then the union of A and B is represented as A∪B and is provided as

$A \cup B = \mathrm{A} = \{< \mathrm{x}, \mathrm{T}_{A\cup B}(\mathrm{x}), \mathrm{C}_{A\cup B}(\mathrm{x}), \mathrm{K}_{A\cup B}(\mathrm{x}), \mathrm{U}_{A\cup B}(\mathrm{x}), \mathrm{F}_{A\cup B}(\mathrm{x})) >: \mathrm{x} \epsilon \mathrm{X}\}$

Here, the truth membership function $T_{A\cup B}(x)$, the contradiction membership function $C_{A\cup B}(x)$, the ignorance membership function $K_{A\cup B}(x)$, the unknown membership function $U_{A\cup B}(x)$ and false membership function $F_{A\cup B}(x)$ are described by

$$T_{A\cup B}(x) = p_A(x) \vee p_B(x). e^{ju_{A\cup B}(x)}$$
$$C_{A\cup B}(x) = q_A(x) \vee q_B(x). e^{jv_{A\cup B}(x)}$$
$$K_{A\cup B}(x) = r_A(x) \wedge r_B(x). e^{jw_{A\cup B}(x)}$$
$$U_{A\cup B}(x) = s_A(x) \wedge s_B(x). e^{jx_{A\cup B}(x)}$$
$$F_{A\cup B}(x) = t_A(x) \wedge t_B(x). e^{jy_{A\cup B}(x)}$$

Where ∨ and ∧ indicate the max and min operators respectively. To calculate phase terms $e^{ju_{A\cup B}(x)}, e^{jv_{A\cup B}(x)}, e^{jw_{A\cup B}(x)}, e^{jx_{A\cup B}(x)}$ and $e^{jy_{A\cup B}(x)}$ we clarify the following:

Definitions 3.9

Given two complex fermatean pentapartitioned neutrososphic sets in X, A and B have the following membership functions: complex-valued truth $T_A(x)$ and $T_B(x)$; complex-valued contradiction $C_A(x)$ and $C_B(x)$; complex-valued ignorance $K_A(x)$ and $K_B(x)$; complex-valued unknown $U_A(x)$ and $U_B(x)$; and complex-valued false $F_A(x)$ and $F_B(x)$, respectively. The function associated with the union of the complex fermatean pentapartitioned neutrososphic sets A and B are represented by:

$$\theta: \{(a_T, a_C, a_K, a_K, a_F): a_T, a_C, a_K, a_K, a_F \epsilon C, |a_T + a_C + a_K + a_K + a_F| \leq 5, |a_T|, |a_C|, |a_K|, |a_K|, |a_F| \leq 1\}$$

X $\{(b_T, b_C, b_K, b_K, b_F): b_T, b_C, b_K, b_K, b_F \epsilon C, |b_T + b_C + b_K + b_K + b_F| \leq 5, |b_T|, |b_C|, |b_K|, |b_K|, |b_F| \leq 1\}$
$\rightarrow \{(d_T, d_C, d_K, d_K, d_F): d_T, d_C, d_K, d_K, d_F \epsilon C, |d_T + d_C + d_K + d_K + d_F| \leq 5, |d_T|, |d_C|, |d_K|, |d_K|, |d_F| \leq 1\}$.

A complex value is imposed by θ, for all x ϵ X,

$\theta(T_A(x), T_B(x)) = T_{A\cup B}(x) = d_T$, $\theta(C_A(x), C_B(x)) = C_{A\cup B}(x) = d_C$, $\theta(K_A(x), K_B(x)) = K_{A\cup B}(x) = d_T$, $\theta(U_A(x), U_B(x)) = U_{A\cup B}(x) = d_U$ and $\theta(F_A(x), F_B(x)) = F_{A\cup B}(x) = d_F$.

The following axiomatic conditions must be met, at the very least, by this function θ: Regarding any

a, b, c, d, e, $x, y \epsilon \{z: \mathrm{z} \epsilon \mathrm{C}, |z| \leq 1\}$:

Axiom 1: $|\theta_T(a,0)| = |a|$, $|\theta_C(b,0)| = |b|$, $|\theta_K(c,1)| = |c|$,

$|\theta_U(d,1)| = |d|$ and $|\theta_F(e,1)| = |e|$ (boundary conditions).

Axiom 2 : $\theta_T(a,x) = \theta_T(x,a)$, $\theta_C(b,x) = \theta_C(x,b)$, $\theta_K(c,x) = \theta_K(x,c)$
$\theta_U(d,x) = \theta_U(x,d)$ and $\theta_F(e,x) = \theta_F(x,e)$ (commutative condition).

Axiom 3 : If $|x| \le |y|$, then $|\theta_T(a,x)| \le |\theta_T(a,y)|$, $|\theta_C(b,x)| \le |\theta_C(b,y)|$
$|\theta_K(c,x)| \le |\theta_K(c,y)|$, $|\theta_U(d,x)| \le |\theta_U(d,y)|$ and $|\theta_F(e,x)| \le |\theta_F(e,y)|$
(Monotonic condition).

Axiom 4:$\theta_T(\theta_T(a,x),y) = \theta_T(a,\theta_T(x,y))$,$\theta_C(\theta_C(b,x),y) = \theta_C(b,\theta_C(x,y))$,
$\theta_K(\theta_K(c,x),y) = \theta_K(c,\theta_K(x,y))$, $\theta_U(\theta_U(d,x),y) = \theta_U(d,\theta_U(x,y))$,

and $\theta_F(\theta_F(e,x),y) = \theta_F(e,\theta_F(x,y))$ (associative condition).

It may be conceivable in a few cases that the taking after are too held:
Axiom 5: Here θ is continuous function (continuity).

Axiom 6: $|\theta_T(a,a)| > |a|$, $|\theta_C(b,b)| > |b|$,
$|\theta_K(c,c)| < |c|$, $|\theta_U(d,c)| < |c|$ and $|\theta_F(e,e)| < |e|$ (super idempotency).
Axiom 7: $|a| \le |c|$ and $|b| \le |d|$, then $|\theta_T(a,b)| \le |\theta_T(c,d)|$,

$|a_1| \le |c_1|$ and $|b_1| \le |d_1|$, then $|\theta_C(a_1,b_1)| \le |\theta_C(c_1,d_1)|$,

$|a_2| \ge |c_2|$ and $|b_2| \ge |d_2|$, then $|\theta_K(a_2,b_2)| \ge |\theta_K(c_2,d_2)|$

$|a_3| \ge |c_3|$ and $|b_3| \ge |d_3|$, then $|\theta_U(a_3,b_3)| \ge |\theta_U(c_3,d_3)|$

and $|a_4| \ge |c_4|$ and $|b_4| \ge |d_4|$, then $|\theta_F(a_4,b_4)| \ge |\theta_F(c_4,d_4)|$
(Strict monotonicity).

The phase term of complex truth membership function, complex contradiction membership function, complex ignorance membership function, complex unknown membership function, and complex false membership function pertains to $(0,2\pi)$.Now we assume that

$p_{T_{A\cup B}}(x) = p_{A\cup B}(x)$, $q_{C_{A\cup B}}(x) = q_{A\cup B}(x)$,

$r_{K_{A\cup B}}(x) = r_{A\cup B}(x)$, $s_{U_{A\cup B}}(x) = s_{A\cup B}(x)$, $t_{F_{A\cup B}}(x) = t_{A\cup B}(x)$.

We now use those forms to define the phase terms that Ramot et al [6] gave.
$\mathrm{T}_{A\cup B}(\mathrm{x})$, $\mathrm{C}_{A\cup B}(\mathrm{x})$, $\mathrm{K}_{A\cup B}(\mathrm{x})$, $\mathrm{U}_{A\cup B}$ (x) and $\mathrm{F}_{A\cup B}(\mathrm{x})$) respectively.

Sum: $p_{A\cup B}(x) = p_A(x) + p_B(x)$ $q_{A\cup B}(x) = q_A(x) + q_B(x)$
$r_{A\cup B}(x) = r_A(x) + r_B(x)$ $s_{A\cup B}(x) = s_A(x) + s_B(x)$ and
$t_{A\cup B}(x) = t_A(x) + t_B(x)$.

Maximum: $p_{A\cup B}(x) = max\ (p_A(x).\ p_B(x))$ $q_{A\cup B}(x) = \max(q_A(x).q_B(x))$

$r_{A\cup B}(x) = \max(r_A(x).\ r_B(x))$ $s_{A\cup B}(x) = \max(s_A(x).s_B(x))$ *and*
$t_{A\cup B}(x) = \ max\ (t_A(x).\ t_B(x).$

Minimum: $p_{A\cup B}(x) = \ \ min\ (p_A(x).\ p_B(x))$ $q_{A\cup B}(x) = \min(q_A(x).q_B(x))$
$r_{A\cup B}(x) = \min(r_A(x).\ r_B(x))$ $s_{A\cup B}(x) = \min(s_A(x).s_B(x))$ *and*
$t_{A\cup B}(x) = \ min\ (t_A(x).\ t_B(x).$

"A contest between the victor, the contradictor, the ignorant, the unknown, and the loser":

$$p_{A\cup B}(x) = \begin{cases} p_A\ (x)\ if & p_A > \ p_B \\ p_B\ (x)\ if & p_B > \ p_A \end{cases}$$

$$q_{A\cup B}(x) = \begin{cases} q_A\ (x)\ if & q_A > \ q_B \\ q_B\ (x)\ if & q_B > \ q_A \end{cases}$$

$$r_{A\cup B}(x) = \begin{cases} r_A\ (x)\ if & r_A < \ r_B \\ r_B\ (x)\ if & r_B < \ r_A \end{cases}$$

$$s_{A\cup B}(x) = \begin{cases} s_A\ (x)\ if & s_A < \ s_B \\ s_B\ (x)\ if & s_B < \ s_A \end{cases}$$

$$t_{A\cup B}(x) = \begin{cases} t_A\ (x)\ if & t_A < \ t_B \\ t_B\ (x)\ if & t_B < \ t_A \end{cases}$$

A novel proposal was put up by Ramot et al. [6]: the "winner take all" concept for the union of phase words is generalized to the game winner, contradictor, ignorant person, unknown person, and loser.

Example 3.9.1

Consider $x = \{x_1, x_2, x_3\}$ as a universal set, and let A and B be complex fermatean pentapartitioned neutrosophic sets. It can be expressed as

$$\mathrm{A} = \left(\frac{\mathrm{o.6e^{j.0.8}, o.3e^{j.\frac{3\pi}{4}}, o.5e^{j.0.3}, o.65e^{j.0.5} o.2e^{j.0.4}}}{\mathrm{x_1}}\right) + \left(\frac{\mathrm{o.7e^{j.0.1}, o.1e^{j.\frac{2\pi}{4}}, o.2e^{j.0.9}, o.5e^{j.0.3}, o.8e^{j.0.4}}}{\mathrm{x_2}}\right) + \left(\frac{\mathrm{o.9e^{j.0.1}, o.4e^{j.\pi}, o.7e^{j.0.7}, o.6e^{j.0.5} o.3e^{j.0.4}}}{\mathrm{x_3}}\right),$$

and

$$\mathrm{B} = \left(\frac{\mathrm{o.8e^{j.0.9}, o.2e^{j.\frac{\pi}{4}}, o.5e^{j.0.3}, o.1e^{j.0.5}, o.4e^{j.0.5}}}{\mathrm{x_1}}\right) + \left(\frac{\mathrm{o.6e^{j.0.1}, o.1e^{0.6}, o.4e^{j.0.6}, o.3e^{j.0.4}, o.01e^{j.\frac{4\pi}{3}}}}{\mathrm{x_2}}\right)$$

$$+\left(\frac{0.2e^{j.0.3}, 0.e^{j.2\pi}, 0.6e^{j.0.4}, 0.7e^{j.0.5}, 0.5e^{j.0.5}}{x_3}\right)$$

Then $A \cup B = \left(\frac{0.8e^{j.0.9}, 0.3e^{j\frac{3\pi}{4}}, 0.5e^{j.0.3}, 0.1e^{j.0.5}, 0.01e^{j.0.4}}{x_1}\right), \left(\frac{0.7e^{j.0.1}, 0.1e^{j\pi}, 0.2e^{j.0.6}, 0.3e^{j.0.3}, 0.01e^{j.0.4}}{x_2}\right),$

$$\left(\frac{0.9e^{j.0.3}, 0.4e^{j2\pi}, 0.6e^{j.0.4}, 0.6e^{j.0.5}, 0.3e^{j.0.4}}{x_3}\right)$$

Definition 3.10: Intersection of a complex fermatean pentapartitioned neutrosophic set.

Considering two complex fermatean pentapartitioned neutrosophic sets A and B in X, it can be written as

A = {< x, $T_A(x)$, $C_A(x)$, $K_A(x)$, U_A (x) , $F_A(x)$) >: x ϵ X } and

B = {< x, $T_B(x)$,$C_B(x)$, $K_B(x)$, U_B (x) , $F_B(x)$) >: x ϵ X }

Next, A and B is represented as A∩B and can be written as

$A \cap B = \mathrm{A} = \{< \mathrm{x}, \mathrm{T}_{A\cap B}(\mathrm{x}), \mathrm{C}_{A\cap B}(\mathrm{x}), \mathrm{K}_{A\cap B}(\mathrm{x}), \mathrm{U}_{A\cap B}(\mathrm{x}), \mathrm{F}_{A\cap B}(\mathrm{x})) >: \mathrm{x} \epsilon \mathrm{X}\}$

Here the truth membership function is denoted by $T_{A\cap B}(x)$, the contradiction membership function is denoted by $C_{A\cap B}(x)$, the ignorance membership function is denoted by $K_{A\cap B}(x)$, the unknown membership function is denoted by $U_{A\cap B}(x)$ and false membership function is denoted by $F_{A\cap B}(x)$. It can be expressed as

$$T_{A\cap B}(x) = p_A(x) \wedge p_B(x).e^{ju_{A\cap B}(x)}$$

$$C_{A\cap B}(x) = q_A(x) \wedge q_B(x).e^{jv_{A\cap B}(x)}$$

$$K_{A\cap B}(x) = r_A(x) \vee r_B(x).e^{jw_{A\cap B}(x)}$$

$$U_{A\cap B}(x) = s_A(x) \vee s_B(x).e^{jx_{A\cap B}(x)}$$

$$F_{A\cap B}(x) = t_A(x) \vee t_B(x).e^{jy_{A\cap B}(x)}$$

Here ∨ and ∧ operators are indicated the max and min respectively. To compute the phase terms $e^{ju_{A\cap B}(x)}$, $e^{jv_{A\cap B}(x)}$, $e^{jw_{A\cap B}(x)}$, $e^{jx_{A\cap B}(x)}$ and $e^{jy_{A\cap B}(x)}$.

Definitions 3.11

Let us consider A and B be two complex fermatean pentapartitioned neutrososphic sets on X, the complex-valued truth membership functions are $T_A(x)$ and $T_B(x)$, the complex-valued contradiction membership functions are $C_A(x)$ and $C_B(x)$, the complex-valued ignorance membership functions are $K_A(x)$ and $K_B(x)$, the complex-valued unknown membership functions are $U_A(x)$ and $U_B(x)$ and the complex-valued false membership functions are $F_A(x)$ and $F_B(x)$ respectively. The intersection of the complex fermatean pentapartitioned neutrososphic sets

$A \cap B$ can be written as

$$\theta: \{(a_T, a_C, a_K, a_K, a_F): a_T, a_C, a_K, a_K, a_F \epsilon C, |a_T + a_C + a_K + a_K + a_F| \leq 5, |a_T|, |a_C|, |a_K|, |a_K|, |a_F| \leq 1\}$$

$$\mathrm{X}\ \{(b_T, b_C, b_K, b_K, b_F): b_T, b_C, b_K, b_K, b_F \epsilon C, |b_T + b_C + b_K + b_K + b_F| \leq 5, |b_T|, |b_C|, |b_K|, |b_K|, |b_F| \leq 1\}$$

$$\rightarrow \{(d_T, d_C, d_K, d_K, d_F): d_T, d_C, d_K, d_K, d_F \epsilon C, |d_T + d_C + d_K + d_K + d_F| \leq 5, |d_T|, |d_C|, |d_K|, |d_K|, |d_F| \leq 1\}.$$

A complex value θ, for all x ϵ X,

$\theta\big(T_A(x), T_B(x)\big) = T_{A\cap B}(x) = d_T$, $\theta\big(C_A(x), C_B(x)\big) = C_{A\cap B}(x) = d_C$, $\theta\big(K_A(x), K_B(x)\big) = K_{A\cap B}(x) = d_T$, $\theta\big(U_A(x), U_B(x)\big) = U_{A\cap B}(x) = d_U$ and $\theta\big(F_A(x), F_B(x)\big) = F_{A\cap B}(x) = d_F$.

Here, the function θ needs the following axioms:

Let a, b, c, d, e, $x, y \ \epsilon \ \{z$: z ϵ C, $|z| \leq 1\}$:

Axiom 1: $|\theta_T(a,1)| = |a|$,$|\theta_C(b,1)| = |b|$, $|\theta_K(c,0)| = |c|$, $|\theta_U(d,0)| = |d|$ and $|\theta_F(e,0)| = |e|$ (boundary conditions).

Axiom 2: $\theta_T(a,x) = \theta_T(x,a)$, $\theta_C(b,x) = \theta_C(x,b)$, $\theta_K(c,x) = \theta_K(x,c)$ $\theta_U(d,x) = \theta_U(x,d)$ and $\theta_F(e,x) = \theta_F(x,e)$ (commutative condition).

Axiom 3: If $|x| \leq |y|$, then $|\theta_T(a,x)| \leq |\theta_T(a,y)|$, $|\theta_C(b,x)| \leq |\theta_C(b,y)|$ $|\theta_K(c,x)| \leq |\theta_K(c,y)|$, $|\theta_U(d,x)| \leq |\theta_U(d,y)|$ and $|\theta_F(e,x)| \leq |\theta_F(e,y)|$ (Monotonic condition).

Axiom 4: $\theta_T(\theta_T(a,x),y) = \theta_T(a,\theta_T(x,y))$, $\theta_C(\theta_C(b,x),y) = \theta_C(b,\theta_C(x,y))$, $\theta_K(\theta_K(c,x),y) = \theta_K(c,\theta_K(x,y))$, $\theta_U(\theta_U(d,x),y) = \theta_U(d,\theta_U(x,y))$, and $\theta_F(\theta_F(e,x),y) = \theta_F(e,\theta_F(x,y))$ (associative condition).

In certain circumstances, it's feasible that the following also holds true:

Axiom 5: θ is continuous function (continuity).

Axiom 6: $|\theta_T(a,a)| > |a|$,$|\theta_C(b,b)| > |b|$, $|\theta_K(c,c)| < |c|$, $|\theta_U(d,c)| < |c|$ and $|\theta_F(e,e)| < |e|$ (super idempotency).
Axiom 7: $|a| \leq |c|$ and$|b| \leq |d|$, then$|\theta_T(a,b)| \leq |\theta_T(c,d)|$,

$|a_1| \leq |c_1|$ & $|b_1| \leq |d_1|$, then $|\theta_C(a_1, b_1)| \leq |\theta_C(c_1, d_1)|$,

$|a_2| \geq |c_2|$& $|b_2| \geq |d_2|$, then $|\theta_K(a_2, b_2)| \geq |\theta_K(c_2, d_2)|$

$|a_3| \geq |c_3|$& $|b_3| \geq |d_3|$, then $|\theta_U(a_3, b_3)| \geq |\theta_U(c_3, d_3)|$

and $|a_4| \geq |c_4|$ and $|b_4| \geq |d_4|$, then $|\theta_F(a_4, b_4)| \geq |\theta_F(c_4, d_4)|$
(Strict monotonicity).

On the same lines by winner, contradiction person, ignorance person, unknown person, and loser game, calculate the phase terms $e^{ju_{A\cap B}(x)}$, $e^{jv_{A\cap B}(x)}$, $e^{jw_{A\cap B}(x)}$, $e^{jx_{A\cap B}(x)}$ and $e^{jy_{A\cap B}(x)}$.

Preposistion 3.12

Let A, B & C be three complex fermatean pentapartitioned neutrososphic sets on X, then
$(A \cup B) \cap C = (A \cap B) \cup (A \cap B)$

$(A \cap B) \cup C = (A \cup B) \cap (A \cup B)$

Proof: We merely demonstrate that portion 1 here.

Assuming that A, B and C are three complex fermatean pentapartitioned neutrososphic sets on X, then we have

$A = \{\langle \mathrm{x}, \mathrm{T}_A(\mathrm{x}), \mathrm{C}_A(\mathrm{x}), \mathrm{K}_A(\mathrm{x}), \mathrm{U}_A\ (\mathrm{x}), \mathrm{F}_A(\mathrm{x})\rangle : \mathrm{x} \in \mathrm{X}\}$,

$B = \{\langle \mathrm{x}, \mathrm{T}_B(\mathrm{x}), \mathrm{C}_B(\mathrm{x}), \mathrm{K}_B(\mathrm{x}), \mathrm{U}_B(\mathrm{x}), \mathrm{F}_B(\mathrm{x})\rangle : \mathrm{x} \in \mathrm{X}\}$ and

$C = \{\langle \mathrm{x}, \mathrm{T}_C(\mathrm{x}), \mathrm{C}_C(\mathrm{x}), \mathrm{K}_C(\mathrm{x}), \mathrm{U}_C(\mathrm{x}), \mathrm{F}_C(\mathrm{x})\rangle : \mathrm{x} \in \mathrm{X}\}$

are the complex valued truth membership function, complex-valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, complex valued false membership function.

Next, we have

$T_{(A\cup B)\cap C}(x) = p_{(A\cup B)\cap C}(x). e^{ju_{(A\cup B)\cap C}(x)}$=min($p_{(A\cup B)}(x). p_C(x) e^{j\,\min(u_{(A\cup B)}(x).u_C(x)}$
= min (max

$(p_A(x), p_B(x)), p_C(x)). e^{j.\min(\max(u_A(x), u_B(x)), u_C(x))}$
= max (min

$(p_A(x), p_C(x)), \min(p_B(x), p_C(x))). e^{j.\max(\min(u_A(x), u_B(x)), min(u_B(x), u_C(x)))}$

=max $((p_{(A\cap C)}(x), p_{(B\cap C)}(x)),. e^{j.\max(u_{(A\cap C)}(x), u_{(B\cap C)}(x))}$

= $(p_{(A\cap C)\cup(B\cap C)}(x). e^{j.(u_{(A\cap C)\cup(B\cap C)}(x)}$= $T_{(A\cap C)\cup(B\cap C)}(x)$

Likewise, we may demonstrate it for $C_{(A\cup B)\cap C}(x) = \ C_{(A\cap C)\cup(B\cap C)}(x)$,

$K_{(A\cup B)\cap C}(x) = K_{(A\cap C)\cup(B\cap C)}(x), U_{(A\cup B)\cap C}(x) = U_{(A\cap C)\cup(B\cap C)}(x)$ & $F_{(A\cup B)\cap C}(x) = F_{(A\cap C)\cup(B\cap C)}(x)$ in the same way.

Preposition 3.13

Let A and B are two complex fermatean pentapartitioned neutrososphic sets on X, then

$(A \cup B) \cap A = A$

$(A \cap B) \cup A = A$

Proof: We establish it in part 1. Given A and B are two complex fermatean pentapartitioned neutrosophic sets on X,

Let $A = \{\langle x, T_A(x), C_A(x), K_A(x), U_A(x), F_A(x)\rangle : x \epsilon X\}$,

$B = \{\langle x, T_B(x), C_B(x), K_B(x), U_B(x), F_B(x)\rangle : x \epsilon X\}$ are the complex valued truth, complex-valued contradiction, complex valued ignorance, complex valued unknown, and complex valued false membership functions.

Next,

$$T_{(A\cup B)\cap A}(x) = p_{(A\cup B)\cap A}(x).e^{ju_{(A\cup B)\cap A}(x)} = \min\left(p_{(A\cup B)}(x).p_A(x)\right)e^{j\min(u_{(A\cup B)}(x).u_A(x)}$$

$$= \min\left(\max(p_A(x), p_B(x)), p_A(x)\right).e^{j.\min(\max(u_A(x),u_B(x)),u_A(x))}$$

$$= T_A(x).$$

Likewise, we may demonstrate it for

$C_{(A\cup B)\cap A}(x) = C_A(x)$,

$K_{(A\cup B)\cap A}(x) = K_A(x), U_{(A\cup B)\cap A}(x) = U_A(x)$& $F_{(A\cup B)\cap A}(x) = F_A(x)$ respectively.

Definition 3.14

Consider two complex fermatean pentapartitioned neutrososphic sets on X, denoted by A and B. Thus, $T_A(x) = p_A(x).e^{ju_A(x)}, C_A(x) = q_A(x).e^{jv_A(x)}, K_A(x) = r_A(x).e^{jw_A(x)}$,

$U_A(x) = s_A(x).e^{jx_A(x)}$, $F_A(x) = t_A(x).e^{jy_A(x)}$ and $T_B(x) = p_B(x).e^{ju_B(x)}$
$C_B(x) = q_B(x).e^{jv_B(x)}$, $K_B(x) = r_B(x).e^{jw_B(x)}$, $U_B(x) = s_B(x).e^{jx_B(x)}$

$F_B(x) = t_B(x).e^{jy_B(x)}$,are the corresponding complex-valued truth membership function, complex-valued contradiction membership function, complex-valued ignorance membership function, complex-valued unknown membership function, complex-valued false membership function. The product of two complex fermatean pentapartitioned neutrososphic of A and B is denoted by AoB and it can be expressed

as$T_{A\,o\,B}(x) = p_{A\,o\,B}(x)e^{ju_{A\,o\,B}(x)} = (p_A(x).p_B(x)).e^{j.2\pi\left(\frac{u_A(x)}{2\pi}.\frac{u_B(x)}{2\pi}\right)}$,

$$C_{\mathrm{A\,o\,B}}(x) = q_{\mathrm{A\,o\,B}}(x)e^{jv_{\mathrm{A\,o\,B}}(x)} = (q_A(x).q_B(x)).e^{\mathrm{j}.2\pi\left(\frac{v_A(x)}{2\pi}.\frac{v_B(x)}{2\pi}\right)},$$

$$K_{\mathrm{A\,o\,B}}(x) = r_{\mathrm{A\,o\,B}}(x)e^{jw_{\mathrm{A\,o\,B}}(x)} = (r_A(x).r_B(x)).e^{\mathrm{j}.2\pi\left(\frac{w(x)}{2\pi}.\frac{w_B(x)}{2\pi}\right)},$$

$$U_{\mathrm{A\,o\,B}}(x) = s_{\mathrm{A\,o\,B}}(x)e^{jx_{\mathrm{A\,o\,B}}(x)} = (s_A(x).s_B(x)).e^{\mathrm{j}.2\pi\left(\frac{x_A(x)}{2\pi}.\frac{x_B(x)}{2\pi}\right)},$$

$$F_{\mathrm{A\,o\,B}}(x) = t_{\mathrm{A\,o\,B}}(x)e^{jy_{\mathrm{A\,o\,B}}(x)} = (t_A(x).t_B(x)).e^{\mathrm{j}.2\pi\left(\frac{y_A(x)}{2\pi}.\frac{y_B(x)}{2\pi}\right)}.$$

Example 3.14.1. Let $X = \{x_1, x_2, x_3\}$ be a universal set. Assume that A and B are two complex fermatean pentapartitioned neutrosophic sets with the following characteristics:

$$\mathrm{A} = \frac{0.6\mathrm{e}^{\mathrm{j}.0.8\pi}, 0.3\mathrm{e}^{\mathrm{j}.0.6\pi}, 0.5\mathrm{e}^{\mathrm{j}.0.3\pi}, 0.65\mathrm{e}^{\mathrm{j}.0.5\pi}, 0.2\mathrm{e}^{\mathrm{j}.0.4\pi}}{\mathrm{x}_1} + \frac{0.7\mathrm{e}^{\mathrm{j}.0.1\pi}, 0.1\mathrm{e}^{\mathrm{j}.0.2\pi}, 0.2\mathrm{e}^{\mathrm{j}.0.9\pi}, 0.5\mathrm{e}^{\mathrm{j}.0.3\pi}, 0.8\mathrm{e}^{\mathrm{j}.0.4\pi}}{\mathrm{x}_2} + \frac{0.9\mathrm{e}^{\mathrm{j}.0.1\pi}, 0.4\mathrm{e}^{\mathrm{j}.\pi}, 0.7\mathrm{e}^{\mathrm{j}.0.7\pi}, 0.6\mathrm{e}^{\mathrm{j}.0.5\pi}, 0.3\mathrm{e}^{\mathrm{j}.0.4\pi}}{\mathrm{x}_3}$$

And

$$\mathrm{B} = \frac{0.8\mathrm{e}^{\mathrm{j}.0.9\pi}, 0.2\mathrm{e}^{\mathrm{j}.0.2\pi}, 0.5\mathrm{e}^{\mathrm{j}.0.3\pi}, 0.1\mathrm{e}^{\mathrm{j}.0.5\pi}, 0.4\mathrm{e}^{\mathrm{j}.0.5\pi}}{\mathrm{x}_1} + \frac{0.6\mathrm{e}^{\mathrm{j}.0.1\pi}, 0.1\mathrm{e}^{0.6\pi}, 0.4\mathrm{e}^{\mathrm{j}.0.6\pi}, 0.3\mathrm{e}^{\mathrm{j}.0.4\pi}, 0.01\mathrm{e}^{\mathrm{j}.0.4\pi}}{\mathrm{x}_2} + \frac{0.2\mathrm{e}^{\mathrm{j}.0.3\pi}, 0.1\mathrm{e}^{\mathrm{j}.2\pi}, 0.6\mathrm{e}^{\mathrm{j}.0.4\pi}, 0.7\mathrm{e}^{\mathrm{j}.0.5\pi}, 0.5\mathrm{e}^{\mathrm{j}.0.5\pi}}{\mathrm{x}_3}$$

Then $AoB =$

$$\frac{0.48\mathrm{e}^{\mathrm{j}.0.36\pi}, 0.06\mathrm{e}^{\mathrm{j}.0.06\pi}, 0.25\mathrm{e}^{\mathrm{j}.0.045\pi}, 0.065\mathrm{e}^{\mathrm{j}.0.125\pi}, 0.08\mathrm{e}^{\mathrm{j}.0.1\pi}}{\mathrm{x}_1}, \frac{0.42\mathrm{e}^{\mathrm{j}.0.005\pi}, 0.01\mathrm{e}^{0.06\pi}, 0.08\mathrm{e}^{\mathrm{j}.0.27\pi}, 0.15\mathrm{e}^{\mathrm{j}.0.06\pi}, 0.008\mathrm{e}^{\mathrm{j}.0.08\pi}}{\mathrm{x}_2},$$

$$\frac{0.18\mathrm{e}^{\mathrm{j}.0.015\pi}, 0.04\mathrm{e}^{\mathrm{j}.\pi}, 0.42\mathrm{e}^{\mathrm{j}.0.14\pi}, 0.42\mathrm{e}^{\mathrm{j}.0.125\pi}, 0.15\mathrm{e}^{\mathrm{j}.0.1\pi}}{\mathrm{x}_3}.$$

Definition 3.15

Let A_n $(\mathrm{n} = 1, 2, 3 \ldots.\mathrm{N})$ be a N complex fermatean pentapartitioned neutrosophic sets on X, such that

$T_{A_n}(x) = p_{A_n}(x).e^{ju_{A_n}(x)}$, $C_{A_n}(x) = q_{A_n}(x).e^{jv_{A_n}(x)}$, $K_{A_n}(x) = r_{A_n}(x).e^{jw_{A_n}(x)}$, $U_{A_n}(x) = s_{A_n}(x).e^{jx_{A_n}(x)}$, $F_{A_n}(x) = t_{A_n}(x).e^{jy_{A_n}(x)}$ are the complex-valued truth membership function, complex-valued contradiction membership function, complex-valued ignorance membership function, complex-valued unknown membership function, complex-valued false membership function respectively.

Now the Cartesian product of A_n is denoted by $A_1 \times A_2 \times \ldots . \times A_N$ and it can be written as

$T_{A_1 \times A_2 \times \ldots . \times A_N}(x) = p_{A_1 \times A_2 \times \ldots . \times A_N}(x). e^{ju_{A_1 \times A_2 \times \ldots . \times A_N}(x)}$= min

$\left(\left((p_{A_1}(x), p_{A_2}(x) \ldots . p_{A_N}(x)\right). e^{j.\min(u_{A_1}(x), u_{A_2}(x) \ldots \ldots . u_{A_N}(x))}\right)$

$C_{A_1 \times A_2 \times \ldots . \times A_N}(x) = q_{A_1 \times A_2 \times \ldots . \times A_N}(x). e^{jv_{A_1 \times A_2 \times \ldots . \times A_N}(x)}$

= min $\left(\left((q_{A_1}(x), q_{A_2}(x) \ldots . q_{A_N}(x)\right). e^{j.\min(v_{A_1}(x), v_{A_2}(x) \ldots \ldots . v_{A_N}(x)}\right)$

$K_{A_1 \times A_2 \times \ldots . \times A_N}(x) = r_{A_1 \times A_2 \times \ldots . \times A_N}(x). e^{jw_{A_1 \times A_2 \times \ldots . \times A_N}(x)}$ = max

$\left(\left((r_{A_1}(x), r_{A_2}(x) \ldots . r_{A_N}(x)\right). e^{j.\max\left(w_{A_1}(x), w_{A_2}(x) \ldots \ldots . w_{A_N}(x)\right)}\right)$

$U_{A_1 \times A_2 \times \ldots . \times A_N}(x) = s_{A_1 \times A_2 \times \ldots . \times A_N}(x). e^{jx_{A_1 \times A_2 \times \ldots . \times A_N}(x)}$ = max

$\left(\left((s_{A_1}(x), s_{A_2}(x) \ldots . s_{A_N}(x)\right). e^{j.\max\left(x_{A_1}(x), x_{A_2}(x) \ldots \ldots . x_{A_N}(x)\right)}\right)$

and $F_{A_1 \times A_2 \times \ldots . \times A_N}(x) = t_{A_1 \times A_2 \times \ldots . \times A_N}(x). e^{jy_{A_1 \times A_2 \times \ldots . \times A_N}(x)}$

= max $\left(\left((t_{A_1}(x), t_{A_2}(x) \ldots . t_{A_N}(x)\right). e^{j.\max(y_{A_1}(x), y_{A_2}(x) \ldots \ldots . y_{A_N}(x)}\right)$

Where $x = \{x_1, x_2, x_3 \ldots \ldots x_N\} \in X \times X \times \ldots \ldots \times X$.

IV. δ-EQUALITIES OF COMPLEX FERMATEAN PENTAPARTITIONED NEUTROSOPHIC SETS

In this section, we discussed the distance measure and additionally the operational characteristics of complex feramtean pentapartitioned neutrosophic sets.

Definition 4.1

The collection of all complex feramtean pentapartitioned neutrosophic sets on X is denoted by CN(X).

Let A and B are members of CN(x). If $T_A(x) \leq T_B(x)$ is such that the amplitude terms $p_A(x) \leq p_B(x)$ and a truth phase term $u_A(x) \leq u_B(x)$ and $C_A(x) \leq C_B(x)$ such that the amplitude terms

$q_A(x) \leq q_B(x)$ and a contradiction phase term $v_A(x) \leq v_B(x)$, $K_A(x) \leq K_B(x)$ amplitude terms $r_A(x) \leq r_B(x)$ and the phase terms $w_A(x) \leq w_B(x)$, $U_A(x) \leq U_B(x)$ amplitude terms $s_A(x) \leq s_B(x)$ and a contradiction phase terms $x_A(x) \leq x_B(x)$, $F_A(x) \leq F_B(x)$ amplitude terms $t_A(x) \leq t_B(x)$ and a falsity phase terms $y_A(x) \leq y_B(x)$ respectively. Then A is contained in or equal to B that is $A \subseteq$ B .

Definition 4.2

The two complex fermatean pentapartitioned neutrosophic sets A and B are said to be equal if and only if the amplitude terms are denoted by $p_A(x) \leq p_B(x)$, $q_A(x) \leq q_B(x)$,

$r_A(x) \leq r_B(x), s_A(x) \leq s_B(x)$ & $t_A(x) \leq t_B(x)$ and the phase terms are denoted by

$u_A(x) \leq u_B(x)$, $v_A(x) \leq v_B(x)$,$w_A(x) \leq w_B(x)$, $x_A(x) \leq x_B(x)$ & $y_A(x) \leq y_B(x)$.

Definitions: 4.3

The distance of complex fermatean pentapartitioned neutrosophic sets is defined as

$d_{CFPNS}: CN(X) \times CN(X) \rightarrow [0,1]$, such that for any A, B, C$\epsilon CN(X)$
$0 \leq d_{CFPNS}(A,B) \leq 1$, $d_{CFPNS}(A,B) = 0 \; if \; and \; only \; if \; A = B$
$d_{CFPNS}(A,B) = d_{CFPNS}(B,A)$,

$d_{CFPNS}(A,B) \leq d_{CFPNS}(A,C) + d_{CFPNS}(C,B)$.

Let $d_{CFPNS}: CN(X) \times CN(X) \rightarrow [0,1]$ is defined as

$$d_{CFPNS}(A,B) = max\begin{pmatrix} \max(sup_{x\epsilon X}|p_A(x) - p_B(x)|, sup_{x\epsilon X}|q_A(x) - q_B(x)|, sup_{x\epsilon X}|r_A(x) - r_B(x)|, sup_{x\epsilon X}|s_A(x) - s_B(x)|, sup_{x\epsilon X}|t_A(x) - t_B(x)|) \\ \max(\frac{1}{2\pi} sup_{x\epsilon X}|u_A(x) - u_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|v_A(x) - v_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|w_A(x) - w_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|x_A(x) - x_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|y_A(x) - y_B(x)| \end{pmatrix}$$

Theorem 4.4

Consider A and B are two complex fermatean pentapartitined neutrosophic set on X, then $d_{CFPNS}(A,B)$ is also a complex fermatean pentapartitined neutrosophic set on X.

Proof: The proof is simple to understand.

Definition 4.5

Let A and B are two complex fermatean pentapartitioned neutrosophic sets on,
Then

$T_A(x) = p_A(x).\, e^{ju_A(x)}$, $C_A(x) = q_A(x).\, e^{jv_A(x)}$, $K_A(x) = r_A(x).\, e^{jw_A(x)}$,

$U_A(x) = s_A(x).\, e^{jx_A(x)}$, $F_A(x) = t_A(x).\, e^{jy_A(x)}$ and $T_B(x) = p_B(x).\, e^{ju_B(x)}$

$C_B(x) = q_B(x).\, e^{jv_B(x)}$, $K_B(x) = r_B(x).\, e^{jw_B(x)}$, $U_B(x) = s_B(x).\, e^{jx_B(x)}$

$F_B(x) = t_B(x).e^{jy_B(x)}$ are complex-valued truth membership function, complex-valued contradiction membership function, complex-valued ignorance membership function, complex-valued unknown membership function, complex-valued false membership function respectively.

Then A and B are said to be δ-equal, if and only if $d_{CNS}(A, B) \leq 1 - \delta$ where $0 \leq \delta \leq 1$ it is indicated by $A = (\delta)B$.

Preposition 4.6

Let A, B, and C are three complex fermatean pentapartitioned neutrosophic sets on X, such that the following conditions are satisfied:

i) A = (0) B.
ii) A = (1) B if and only if A = B.
iii) If $A = (\delta)B$ if and only if $B = (\delta)A$
iv) $A = (\delta_1)B$ and $\delta_2 \leq \delta_1$,then $A = (\delta_2)B$
v) If $A = (\delta_\alpha)B$, then $A = (sup_{\alpha\epsilon j}\delta_\alpha)B$ for all $\alpha\epsilon j$, where J is an index set.
vi) If $A = (\delta')B$ andthere exist a unique δ such that $A = (\delta)B$, then $\delta' \leq \delta$ for all A,B
vii) If $A = (\delta_1)B$ and$B = (\delta_2)C$, then $A = (\delta)C$ where $\delta = \delta_1 * \delta_2$.

Proof: The conditions 1,2,3,4 & 6 are easily demonstrable. Here only 5 & 7 are explained.

Now $A = (\delta_\alpha)B$, for all $\alpha\epsilon j$, we have

$$d_{CFPNS}(A, B) = max\begin{pmatrix} \max(sup_{x\epsilon X}|p_A(x) - p_B(x)|, sup_{x\epsilon X}|q_A(x) - q_B(x)|, sup_{x\epsilon X}|r_A(x) - r_B(x)|, sup_{x\epsilon X}|s_A(x) - s_B(x)|, sup_{x\epsilon X}|t_A(x) - t_B(x)|) \\ \max(\frac{1}{2\pi}sup_{x\epsilon X}|u_A(x) - u_B(x)|, \frac{1}{2\pi}sup_{x\epsilon X}|v_A(x) - v_B(x)|, \frac{1}{2\pi}sup_{x\epsilon X}|w_A(x) - w_B(x)|, \frac{1}{2\pi}sup_{x\epsilon X}|x_A(x) - x_B(x)|, \frac{1}{2\pi}sup_{x\epsilon X}|y_A(x) - y_B(x)| \end{pmatrix} \leq 1 - \delta_\alpha$$

Accordingly, $sup_{x\epsilon X}|p_A(x) - p_B(x)| \leq 1 - sup_{\alpha\epsilon j}\delta_\alpha$

$sup_{x\epsilon X}|q_A(x) - q_B(x)| \leq 1 - sup_{\alpha\epsilon j}\delta_\alpha$,

$sup_{x\epsilon X}|r_A(x) - r_B(x)| \leq 1 - sup_{\alpha\epsilon j}\delta_\alpha$,

$sup_{x\epsilon X}|s_A(x) - s_B(x)| \leq 1 - sup_{\alpha\epsilon j}\delta_\alpha$,

$sup_{x\epsilon X}|t_A(x) - t_B(x)| \leq 1 - sup_{\alpha\epsilon j}\delta_\alpha$, and

$\frac{1}{2\pi}sup_{x\epsilon X}|u_A(x) - u_B(x)| \leq 1 - sup_{\alpha\epsilon j}\delta_\alpha$,

$\frac{1}{2\pi}sup_{x\epsilon X}|v_A(x) - u_B(x)| \leq 1 - sup_{\alpha\epsilon j}\delta_\alpha$,

$\frac{1}{2\pi}sup_{x\epsilon X}|w_A(x) - w_B(x)| \leq 1 - sup_{\alpha\epsilon j}\delta_\alpha$,

$\frac{1}{2\pi} sup_{x\epsilon X}|x_A(x) - x_B(x)| \leq 1 - sup_{\alpha\epsilon j}\delta_\alpha$,

$\frac{1}{2\pi} sup_{x\epsilon X}|y_A(x) - y_B(x)| \leq 1 - sup_{\alpha\epsilon j}\delta_\alpha$.

Thus,

$$d_{CFPNS}(A,B) = max\begin{pmatrix} \max(sup_{x\epsilon X}|p_A(x) - p_B(x)|, sup_{x\epsilon X}|q_A(x) - q_B(x)|, sup_{x\epsilon X}|r_A(x) - r_B(x)|, sup_{x\epsilon X}|s_A(x) - s_B(x)|, sup_{x\epsilon X}|t_A(x) - t_B(x)|) \\ \max(\frac{1}{2\pi} sup_{x\epsilon X}|u_A(x) - u_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|v_A(x) - v_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|w_A(x) - w_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|x_A(x) - x_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|y_A(x) - y_B(x)| \end{pmatrix} \leq 1 - sup_{\alpha\epsilon j}\delta_\alpha$$

Hence, $A = (sup_{\alpha\epsilon j}\delta_\alpha)B$.

7. Since $A = (\delta_1)B$, we have

$$d_{CFPNS}(A,B) = max\begin{pmatrix} \max(sup_{x\epsilon X}|p_A(x) - p_B(x)|, sup_{x\epsilon X}|q_A(x) - q_B(x)|, sup_{x\epsilon X}|r_A(x) - r_B(x)|, sup_{x\epsilon X}|s_A(x) - s_B(x)|, sup_{x\epsilon X}|t_A(x) - t_B(x)|) \\ \max(\frac{1}{2\pi} sup_{x\epsilon X}|u_A(x) - u_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|v_A(x) - v_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|w_A(x) - w_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|x_A(x) - x_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|y_A(x) - y_B(x)| \end{pmatrix} \leq 1 - \delta_1$$

Which implies

$sup_{x\epsilon X}|p_A(x) - p_B(x)| \leq 1 - \delta_1$,

$sup_{x\epsilon X}|q_A(x) - q_B(x)| \leq 1 - \delta_1$,

$sup_{x\epsilon X}|r_A(x) - r_B(x)| \leq 1 - \delta_1$,

$sup_{x\epsilon X}|s_A(x) - s_B(x)| \leq 1 - \delta_1$,

$sup_{x\epsilon X}|t_A(x) - t_B(x)| \leq 1 - \delta_1$, and

$\frac{1}{2\pi} sup_{x\epsilon X}|u_A(x) - u_B(x)| \leq 1 - \delta_1$,

$\frac{1}{2\pi} sup_{x\epsilon X}|v_A(x) - u_B(x)| \leq 1 - \delta_1$,

$\frac{1}{2\pi} sup_{x\epsilon X}|w_A(x) - w_B(x)| \leq 1 - \delta_1$,

$\frac{1}{2\pi} sup_{x\epsilon X}|x_A(x) - x_B(x)| \leq 1 - \delta_1$,

$\frac{1}{2\pi} sup_{x\epsilon X}|y_A(x) - y_B(x)| \leq 1 - \delta_1$,

Also we have $B = (\delta_2)C$, so

$$d_{CFPNS}(A,B) = max\begin{pmatrix} \max(sup_{x\epsilon X}|p_A(x) - p_B(x)|, sup_{x\epsilon X}|q_A(x) - q_B(x)|, sup_{x\epsilon X}|r_A(x) - r_B(x)|, sup_{x\epsilon X}|s_A(x) - s_B(x)|, sup_{x\epsilon X}|t_A(x) - t_B(x)|) \\ \max(\frac{1}{2\pi} sup_{x\epsilon X}|u_A(x) - u_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|v_A(x) - v_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|w_A(x) - w_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|x_A(x) - x_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|y_A(x) - y_B(x)| \end{pmatrix} \leq 1 - \delta_2$$

Which implies

$sup_{x\epsilon X}|p_A(x) - p_B(x)| \leq 1 - \delta_2$,

$sup_{x\epsilon X}|q_A(x) - q_B(x)| \leq 1 - \delta_2$,
$sup_{x\epsilon X}|r_A(x) - r_B(x)| \leq 1 - \delta_2$,
$sup_{x\epsilon X}|s_A(x) - s_B(x)| \leq 1 - \delta_2$,
$sup_{x\epsilon X}|t_A(x) - t_B(x)| \leq 1 - \delta_2$, and
$\frac{1}{2\pi} sup_{x\epsilon X}|u_A(x) - u_B(x)| \leq 1 - \delta_2$,
$\frac{1}{2\pi} sup_{x\epsilon X}|v_A(x) - u_B(x)| \leq 1 - \delta_2$,
$\frac{1}{2\pi} sup_{x\epsilon X}|w_A(x) - w_B(x)| \leq 1 - \delta_2$,
$\frac{1}{2\pi} sup_{x\epsilon X}|x_A(x) - x_B(x)| \leq 1 - \delta_2$,
$\frac{1}{2\pi} sup_{x\epsilon X}|y_A(x) - y_B(x)| \leq 1 - \delta_2$,

Now

$$d_{CFPNS}(A,B) = max\begin{pmatrix} \max(sup_{x\epsilon X}|p_B(x) - p_C(x)|, sup_{x\epsilon X}|q_B(x) - q_C(x)|, sup_{x\epsilon X}|r_B(x) - r_C(x)|, sup_{x\epsilon X}|s_B(x) - s_C(x)|, sup_{x\epsilon X}|t_B(x) - t_C(x)|) \\ \max(\frac{1}{2\pi} sup_{x\epsilon X}|u_B(x) - u_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|v_B(x) - v_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|w_B(x) - w_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|x_B(x) - x_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|y_B(x) - y_C(x)| \end{pmatrix}$$

Now

$$d_{CFPNS}(A,C) = max\begin{pmatrix} \max(sup_{x\epsilon X}|p_A(x) - p_C(x)|, sup_{x\epsilon X}|q_A(x) - q_C(x)|, sup_{x\epsilon X}|r_A(x) - r_C(x)|, sup_{x\epsilon X}|s_A(x) - s_C(x)|, sup_{x\epsilon X}|t_A(x) - t_C(x)|) \\ \max(\frac{1}{2\pi} sup_{x\epsilon X}|u_A(x) - u_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|v_A(x) - v_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|w_A(x) - w_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|x_A(x) - x_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|y_A(x) - y_C(x)| \end{pmatrix}$$

$$\leq max\begin{pmatrix} \max(sup_{x\epsilon X}|p_A(x) - p_B(x)|, sup_{x\epsilon X}|q_A(x) - q_B(x)|, sup_{x\epsilon X}|r_A(x) - r_B(x)|, sup_{x\epsilon X}|s_A(x) - s_B(x)|, sup_{x\epsilon X}|t_A(x) - t_B(x)|) + \\ \max(sup_{x\epsilon X}|p_B(x) - p_C(x)|, sup_{x\epsilon X}|q_B(x) - q_C(x)|, sup_{x\epsilon X}|r_B(x) - r_C(x)|, sup_{x\epsilon X}|s_B(x) - s_C(x)|, sup_{x\epsilon X}|t_B(x) - t_C(x)|) \\ \max\left(\frac{1}{2\pi} sup_{x\epsilon X}|u_A(x) - u_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|v_A(x) - v_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|w_A(x) - w_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|x_A(x) - x_B(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|y_A(x) - y_B(x)|\right) + \\ \max(\frac{1}{2\pi} sup_{x\epsilon X}|u_B(x) - u_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|v_B(x) - v_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|w_B(x) - w_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|x_B(x) - x_C(x)|, \frac{1}{2\pi} sup_{x\epsilon X}|y_B(x) - y_C(x)| \end{pmatrix}$$

$$\leq Max((1 - \delta_1) + (1 - \delta_2), (1 - \delta_1) + (1 - \delta_2)$$

$$= (1 - \delta_1) + (1 - \delta_2) = 1 - (\delta_1 + \delta_2 \text{-} 1),$$

From definition 4.3,

$d_{CFPNS}(A, C) \leq 1$. Therefore, $d_{CFPNS}(A, C) \leq$ 1- $\delta_1 * \delta_2 =$ 1- δ where $\delta = \delta_1 * \delta_2$. Then $A = (\delta) C$.

Theorem 4.7

If $A = (\delta)B$, Where A&B are two complex fermatean pentapartitioned neutrosophic sets on X then to prove that $c(A) = (\delta)c(B)$ where $c(A)$ and $c(B)$ are the complement of the complex fermatean pentapartitioned neutrosophic sets.

Proof: Since

$$d_{CfpNS}(c(A), c(B))$$

$$= max\begin{pmatrix} \max(sup_{x\epsilon X}\left|p_{c(A)}(x) - p_{c(B)}(x)\right|, sup_{x\epsilon X}\left|q_{c(A)}(x) - q_{c(B)}(x)\right|, sup_{x\epsilon X}\left|r_{c(A)}(x) - r_{c(B)}(x)\right|, sup_{x\epsilon X}\left|s_{c(A)}(x) - s_{c(B)}(x)\right|, sup_{x\epsilon X}\left|t_{c(A)}(x) - t_{c(B)}(x)\right|) \\ \max(\frac{1}{2\pi} sup_{x\epsilon X}\left|u_{c(A)}(x) - u_{c(B)}(x)\right|, \frac{1}{2\pi} sup_{x\epsilon X}\left|v_{c(A)}(x) - v_{c(B)}(x)\right|, \frac{1}{2\pi} sup_{x\epsilon X}\left|w_{c(A)}(x) - w_{c(B)}(x)\right|, \frac{1}{2\pi} sup_{x\epsilon X}\left|x_{c(A)}(x) - x_{c(B)}(x)\right|, \\ \frac{1}{2\pi} sup_{x\epsilon X}\left|y_{c(A)}(x) - y_{c(B)}(x)\right| \end{pmatrix}$$

$$\leq max\begin{pmatrix} \max(sup_{x\epsilon X}|t_A(x) - t_B(x)|, sup_{x\epsilon X}|s_A(x) - s_B(x)|, sup_{x\epsilon X}|(1 - r_A(x)) - (1 - r_B(x))|, sup_{x\epsilon X}|q_A(x) - q_B(x)|, sup_{x\epsilon X}|p_A(x) - p_B(x)|) \\ \max(\frac{1}{2\pi} sup_{x\epsilon X}|(2\pi - u_A(x)) - (2\pi - u_B(x))|, \frac{1}{2\pi} sup_{x\epsilon X}|(2\pi - v_A(x)) - (2\pi - v_B(x))|, \frac{1}{2\pi} sup_{x\epsilon X}|(2\pi - w_A(x)) - (2\pi - w_B(x))|, \\ \frac{1}{2\pi} sup_{x\epsilon X}|(2\pi - x_A(x)) - (2\pi - x_B(x))|, \frac{1}{2\pi} sup_{x\epsilon X}|(2\pi - y_A(x)) - (2\pi - y_B(x))| \end{pmatrix}$$

$$= d_{CFPNS}(A, B) \leq 1 - \delta.$$

V. RELATIONS BETWEEN THE CFPNS AND THEIR APPLICATIONS IN DECISION MAKING

Definition 5.1

Consider A_N (n = 1, 2, 3… N) be N CFPNSs on X and $T_{A_n}(x) = p_s(x).e^{ju_s(x)}$ be a complex-valued truth membership function, $C_A(x) = q_s(x).e^{jv_s(x)}$ be a complex-valued contradiction membership function, $K_A(x) = r_s(x).e^{jw_s(x)}$ be a complex-valued ignorance membership function,

$U_A(x) = s_s(x).e^{jx_s(x)}$ be a complex-valued unknown membership function and
$F_A(x) = t_s(x).e^{jy_s(x)}$ be a complex-valued false membership function. Then the Cartesian products of A_n indicated as $A_1 \times A_2 \times \dots \dots \times A_N$ and it can be described as

$$T_{A_1 \times A_2 \times \dots\dots \times A_N}(x) = p_{A_1 \times A_2 \times \dots\dots \times A_N}(x).e^{ju_{A_1 \times A_2 \times \dots\dots \times A_N}(x)}$$

$$= \min\left(P_{A_1}(x_1), P_{A_2}(x_2), \dots P_{A_N}(x_N)\right).e^{jmin\left(u_{A_1}(x_1), u_{A_2}(x_2), \dots u_{A_N}(x_N)\right)},$$

$$C_{A_1 \times A_2 \times \dots\dots \times A_N}(x) = q_{A_1 \times A_2 \times \dots\dots \times A_N}(x).e^{jv_{A_1 \times A_2 \times \dots\dots \times A_N}(x)}$$

$$= \min\left(q_{A_1}(x_1), q_{A_2}(x_2), \dots q_{A_N}(x_N)\right).e^{jmin\left(v_{A_1}(x_1), v_{A_2}(x_2), \dots v_{A_N}(x_N)\right)},$$

$$K_{A_1 \times A_2 \times \dots\dots \times A_N}(x) = r_{A_1 \times A_2 \times \dots\dots \times A_N}(x).e^{jw_{A_1 \times A_2 \times \dots\dots \times A_N}(x)}$$

$$= \max\left(r_{A_1}(x_1), r_{A_2}(x_2), \dots r_{A_N}(x_N)\right).e^{jmax\left(w_{A_1}(x_1), w_{A_2}(x_2), \dots w_{A_N}(x_N)\right)},$$

$$U_{A_1 \times A_2 \times \dots\dots \times A_N}(x) = s_{A_1 \times A_2 \times \dots\dots \times A_N}(x).e^{jx_{A_1 \times A_2 \times \dots\dots \times A_N}(x)}$$

$$= \max\left(s_{A_1}(x_1), s_{A_2}(x_2), \dots s_{A_N}(x_N)\right).e^{jmax\left(x_{A_1}(x_1), x_{A_2}(x_2), \dots x_{A_N}(x_N)\right)},$$

$$F_{A_1\times A_2\times \ldots\ldots \times A_N}(x) = t_{A_1\times A_2\times \ldots\ldots \times A_N}(x).\, e^{jy_{A_1\times A_2\times \ldots\ldots \times A_N}(x)}$$

$$= \min\left(t_{A_1}(x_1), t_{A_2}(x_2), \ldots\ldots t_{A_N}(x_N)\right).\, e^{jmax\left(y_{A_1}(x_1), y_{A_2}(x_2), \ldots\ldots y_{A_N}(x_N)\right)}$$

5.2. Complex fermatean pentapartitioned neutrosophic relation

In this section, we discussed the definition of the Cartesian product of two CFPNSs and also solve an MCDM problem using the CFPNR approach.
The Cartesian product of two CFPNSs is defined as follows:

Definition 5.3.

The two universal sets U & V and let X and Y be two CFPNSs, then the Cartesian product of X and Y is denoted by X × Y is defined as

X × Y =
$\{\langle (u,v), T_{X\times Y}(u,v), C_{X\times Y}(u,v), K_{X\times Y}(u,v), U_{X\times Y}(u,v), F_{X\times Y}(u,v) \rangle : (u,v) \epsilon\, U \times V\}$,

where $T_{X\times Y}(u,v)$ is a complex valued truth membership function, $C_{X\times Y}(u,v)$ is a complex valued contradiction membership function, $K_{X\times Y}(u,v)$ is a complex valued ignorance membership function, $U_{X\times Y}(u,v)$ is a complex valued unknown membership function and $F_{X\times Y}(u,v)$ is a complex- valued falsity membership function. Then$\forall (u,v)\epsilon\, U \times V$,

$$T_{X\times Y}(u,v) = \min\big(p_X(u).\, p_Y(v)\big).\, e^{jmin\,(u_X(u)\,(u_Y(v)}\big),$$

$$C_{X\times Y}(u,v) = \min\big(q(u).\, q_Y(v)\big).\, e^{jmin\,(v_X(u)\,(v_Y(v)}\big),$$

$$K_{X\times Y}(u,v) = \max\big(r_X(u).\, r_Y(v)\big).\, e^{jmax\,(w_X(u)\,(w_Y(v)}\big),$$

$$U_{X\times Y}(u,v) = \max\big(s_X(u).\, s_Y(v)\big).\, e^{jmax\,(x_X(u)\,(x_Y(v)}\big),$$

$$F_{X\times Y}(u,v) = \max\big(t_X(u).\, t_Y(v)\big).\, e^{jmax\,(y_X(u)\,(y_Y(v)}\big).$$

Now, let's define the term CFPNR as follows:

Definition 5.4.

The two universal sets U & V and let X and Y be two CFPNSs, Then the connection of two complex fermatean pentapartitioned neutrosophic set from X to Y of X×Y is represented by

R(X, Y), where R(X, Y) ⊆ X×Y.

Here R(X, Y) is the set of ordered sequences

$\mathrm{R(X,Y)} = \{\langle (u,v), T_{X\times Y}(u,v), C_{X\times Y}(u,v), K_{X\times Y}(u,v), U_{X\times Y}(u,v), F_{X\times Y}(u,v) \rangle : (u,v)\epsilon\, U \times V\}$, where $\forall u \,\epsilon\, U$ & $\forall v \,\epsilon\, V$,$\mathrm{T}_R(\mathrm{u,v}) = p_R(\mathrm{u,v}).\, e^{ju_R(\mathrm{u,v})}$,

$\mathrm{C}_R(\mathrm{u},\mathrm{v}) = q_R(\mathrm{u},\mathrm{v}).e^{jv_R(\mathrm{u},\mathrm{v})}$, $\mathrm{K}_R(\mathrm{u},\mathrm{v}) = r_R(\mathrm{u},\mathrm{v}).e^{jw_R(\mathrm{u},\mathrm{v})}$,

$\mathrm{U}_R(\mathrm{u},\mathrm{v}) = s_R(\mathrm{u},\mathrm{v}).e^{jx_R(\mathrm{u},\mathrm{v})}$, $\mathrm{F}_R(\mathrm{u},\mathrm{v}) = t_R(\mathrm{u},\mathrm{v}).e^{jy_R(\mathrm{u},\mathrm{v})}$.

The values $\mathrm{T}_R(\mathrm{u},\mathrm{v})$, $\mathrm{C}_R(\mathrm{u},\mathrm{v})$, $\mathrm{K}_R(\mathrm{u},\mathrm{v})$,$\mathrm{U}_R(\mathrm{u},\mathrm{v})$& $\mathrm{F}_R(\mathrm{u},\mathrm{v})$ are in the complex plane and both the amplitude terms $p_R(\mathrm{u},\mathrm{v}), q_R(\mathrm{u},\mathrm{v}), r_R(\mathrm{u},\mathrm{v})$, $s_R(\mathrm{u},\mathrm{v})$, $t_R(\mathrm{u},\mathrm{v})$ and the phase terms $u_R(\mathrm{u},\mathrm{v}), v_R(\mathrm{u},\mathrm{v}), w_R(\mathrm{u},\mathrm{v}), x_R(\mathrm{u},\mathrm{v}), y_R(\mathrm{u},\mathrm{v})$ are real valued such that

$\mathrm{T}_R(\mathrm{u},\mathrm{v})$, $\mathrm{C}_R(\mathrm{u},\mathrm{v})$, $\mathrm{K}_R(\mathrm{u},\mathrm{v})$,$\mathrm{U}_R(\mathrm{u},\mathrm{v})$& $\mathrm{F}_R(\mathrm{u},\mathrm{v}) \in [0,1]$ and
$0 \leq T_R(\mathrm{u},\mathrm{v}), \mathrm{C}_R(\mathrm{u},\mathrm{v}), \mathrm{K}_R(\mathrm{u},\mathrm{v}), \mathrm{U}_R(\mathrm{u},\mathrm{v}), \mathrm{F}_R(\mathrm{u},\mathrm{v}) \leq 3$.

VI. COMPLEX FERMATEAN PENTAPARTITIONED NEUTROSOPHIC RELATION EDUCATION

When time is a significant component and indeterminacy is inevitable, CFPNR can be used to quantify the interaction between various education variables efficiently. We now present a relation between two CFPNSs as an example.

Example 6.1

Assume assessment is done to find the most effective teaching method that raises student achievement. Let U denote the collection of educational strategies used with a particular set of pupils, where

U = $\{u_1 = cooperative\ leaning, u_2 = self - learning, u_3 = traditional\ teaching\}$.

Let V represent a collection of metrics measuring the interaction and academic accomplishment of the student, where

V= $\{v_1 = academic\ ahievement, v_2 = emotional\ interaction, v_3 = social\ interaction\}$.

Consider two CFPNSs over U and V, respectively, X and Y, which can be described as follows:

$$X = \left\{ \begin{array}{c} \frac{u_1}{\langle 0.9\mathrm{e}^{\mathrm{j}.2\pi(11/12)}, 0.2\mathrm{e}^{\mathrm{j}.2\pi(1/12)}, 0.1\mathrm{e}^{\mathrm{j}.2\pi(1/48)}, 0.6\mathrm{e}^{\mathrm{j}.2\pi(1/12)}, 0.2\mathrm{e}^{\mathrm{j}.2\pi(3/12)} \rangle}, \frac{u_2}{\langle 0.3\mathrm{e}^{\mathrm{j}.2\pi(4/12)}, 0.6\mathrm{e}^{\mathrm{j}.2\pi(4/12)}, 0.1\mathrm{e}^{\mathrm{j}.2\pi(4/12)}, 0.5\mathrm{e}^{\mathrm{j}.2\pi(5/12)}, 0.8\mathrm{e}^{\mathrm{j}.2\pi(2/12)} \rangle}, \\ \frac{u_3}{\langle 0.2\mathrm{e}^{\mathrm{j}.2\pi(3/12)}, 0.6\mathrm{e}^{\mathrm{j}.2\pi(4/12)}, 0.9\mathrm{e}^{\mathrm{j}.2\pi(7/12)}, 0.6\mathrm{e}^{\mathrm{j}.2\pi(6/12)}, 0.3\mathrm{e}^{\mathrm{j}.2\pi(5/12)} \rangle} \end{array} \right\}$$

$$Y = \left\{ \begin{array}{c} \frac{v_1}{\langle 0.9\mathrm{e}^{\mathrm{j}.2\pi(12/12)}, 0.3\mathrm{e}^{\mathrm{j}.2\pi(3/12)}, 0.1\mathrm{e}^{\mathrm{j}.2\pi(1/24)}, 0.1\mathrm{e}^{\mathrm{j}.2\pi(2/12)}, 0.4\mathrm{e}^{\mathrm{j}.2\pi(11/12)} \rangle}, \frac{v_2}{\langle 0.4\mathrm{e}^{\mathrm{j}.2\pi(5/12)}, 0.5\mathrm{e}^{2\pi(4/12)}, 0.3\mathrm{e}^{\mathrm{j}.2\pi(8/12)}, 0.3\mathrm{e}^{\mathrm{j}.2\pi(4/12)}, 0.01\mathrm{e}^{\mathrm{j}.2\pi(5/12)} \rangle}, \\ \frac{v_3}{\langle 0.8\mathrm{e}^{\mathrm{j}.2\pi(12/12)}, 0.5\mathrm{e}^{\mathrm{j}.2\pi(9/12)}, 0.2\mathrm{e}^{\mathrm{j}.2\pi(2/12)}, 0.7\mathrm{e}^{\mathrm{j}2\pi(7/12)}, 0.5\mathrm{e}^{\mathrm{j}2\pi(3/12)} \rangle} \end{array} \right\}$$

We now calculate the relationship between the two CFPNSs, X and Y, to look into how students' performance is impacted by contemporary teaching techniques. It is represented by R(X, Y), such that R(X, Y) ⊆ X×Y and it can be expressed as

$$\mathrm{R}(\mathrm{X},\mathrm{Y}) = \left\{ \frac{(u_1, v_1)}{\langle 0.9\mathrm{e}^{\mathrm{j}.2\pi(11/12)}, 0.2\mathrm{e}^{\mathrm{j}.2\pi(1/12)}, 0.1\mathrm{e}^{\mathrm{j}.2\pi(1/24)}, 0.6\mathrm{e}^{\mathrm{j}.2\pi(2/12)}, 0.4\mathrm{e}^{\mathrm{j}.2\pi(11/12)} \rangle}, \frac{(u_1, v_2)}{\langle 0.4\mathrm{e}^{\mathrm{j}.2\pi(5/12)}, 0.2\mathrm{e}^{\mathrm{j}.2\pi(1/12)}, 0.3\mathrm{e}^{\mathrm{j}.2\pi(8/12)}, 0.6\mathrm{e}^{\mathrm{j}.2\pi(4/12)} 0.2\mathrm{e}^{\mathrm{j}.2\pi(5/12)} \rangle}, \right.$$

$$\frac{(u_1,v_3)}{\langle 0.8e^{j.2\pi(11/12)}, 0.2e^{j.2\pi(1/12)}, 0.2e^{j.2\pi(2/12)}, 0.7e^{j2\pi(7/12)}, 0.5e^{j2\pi(3/12)}\rangle}, \frac{(u_2,v_1)}{\langle 0.3e^{j.2\pi(4/12)}, 0.3e^{j.2\pi(3/12)}, 0.1e^{j.2\pi(4/12)}, 0.5e^{j.2\pi(5/12)}, 0.8e^{j.2\pi(11/12)}\rangle},$$

$$\frac{(u_2,v_2)}{\langle 0.3e^{j.2\pi(4/12)}, 0.5e^{2\pi(4/12)}, 0.3e^{j.2\pi(8/12)}, 0.5e^{j.2\pi(5/12)}, 0.8e^{j.2\pi(5/12)}\rangle}, \frac{(u_2,v_3)}{\langle 0.3e^{j.2\pi(4/12)}, 0.5e^{j.2\pi(4/12)}, 0.2e^{j.2\pi(4/12)}, 0.7e^{j2\pi(7/12)}, 0.8e^{j2\pi(3/12)}\rangle},$$

$$\frac{(u_3,v_1)}{\langle 0.2e^{j.2\pi(3/12)}, 0.3e^{j.2\pi(3/12)}, 0.9e^{j.2\pi(7/12)}, 0.6e^{j.2\pi(6/12)}, 0.4e^{j.2\pi(11/12)}\rangle}, \frac{(u_3,v_2)}{\langle 0.2e^{j.2\pi(3/12)}, 0.5e^{2\pi(4/12)}, 0.9e^{j.2\pi(8/12)}, 0.6e^{j.2\pi(6/12)}, 0.3e^{j.2\pi(5/12)}\rangle},$$

$$\left.\frac{(u_3,v_3)}{\langle 0.2e^{j.2\pi(3/12)}, 0.5e^{j.2\pi(4/12)}, 0.9e^{j.2\pi(8/12)}, 0.7e^{j2\pi(7/12)}, 0.5e^{j2\pi(5/12)}\rangle}\right\}$$

Assume that a 12-month period is used to measure the relationship between X and Y. In our example, the terms of truth amplitude, contradiction amplitude, ignorance amplitude, unknown amplitude and false amplitude of R(X, Y) is to measure the truth membership degree of the impact of the modern methods in education on the student's performance, the contradiction membership degree of the effect of the recent techniques in education on the student's effectiveness, the ignorance membership degree of the effect of the recent techniques in education on the student's effectiveness, the unknown membership degree of the effect of the recent techniques in education on the student's effectiveness, and false membership degree of the effect of the recent techniques in education on the student's effectiveness respectively. Furthermore the truth phase term, the contradiction phase term, the ignorance phase term, the unknown phase term and false phase term of R(X, Y) represent the period of time in which the recent techniques influence the student's effectiveness, the time frame within which we cannot say whether the use of contemporary methods affects students' performance and the time frame within which such methods have no effect on the achievement of learners, respectively. Given that the phase terms of R(X, Y) denote time intervals and it depicts the relationship between contemporary educational practices and students' performance over a 12-month period, the values of each complex fermatean pentapartitioned neutrosophic value, the value of each of the truth, contradiction, ignorance, unknown, and false phase terms should be in between 0 and 1.

6.2 Operations of the relations between the complex fermatean pentapartitoned neutrosophic sets

In this section, we discuss about some fundamental CFPNS operations complement, inverse and composition of CFPNRs.

Definition 6.3

Consider R be a CFPNR on $U \times V$,

$\mathrm{R} = \{\langle (u,v), T_R(u,v), C_R(u,v), K_R(u,v), U_R(u,v), F_R(u,v)\rangle : (u,v) \epsilon\, U \times V\}$ .Then the complex fermatean pentapartitioned neutrosophic complement relation R represented byR^c, and it can be written as

$\mathrm{R}^c = \{\langle\, (u,v), T_{R^c}(u,v), C_{R^c}(u,v), K_{R^c}(u,v), U_{R^c}(u,v), F_{R^c}(u,v)\,\rangle : (u,v) \epsilon\, U \times\, V\}$, where

$T_{R^c}(u,v) = p_{R^c}(\mathrm{u,v}).\, e^{ju_{R^c}(\mathrm{u,v})} = t_R(\mathrm{u,v}).\, e^{j(2\pi - y_R(\mathrm{u,v}))}$.
$C_{R^c}(u,v) = q_{R^c}(\mathrm{u,v}).\, e^{jv_{R^c}(\mathrm{u,v})} = s_R(\mathrm{u,v}).\, e^{j(2\pi - x_R(\mathrm{u,v}))}$

$K_{R^c}(u,v) = r_{R^c}(\text{u,v}).\, e^{jw_{R^c}(\text{u,v})} = 1 - r_R(\text{u,v}).\, e^{j(2\pi - w_R(\text{u,v}))}$
$U_{R^c}(u,v) = s_{R^c}(\text{u,v}).\, e^{jx_{R^c}(\text{u,v})} = q_R(\text{u,v}).\, e^{j(2\pi - v_R(\text{u,v}))}$
$F_{R^c}(u,v) = t_{R^c}(\text{u,v}).\, e^{jy_{R^c}(\text{u,v})} = p_R(\text{u,v}).\, e^{j(2\pi - u_R(\text{u,v}))}$.

Definition 6.4. Let R be a CFPNR from X to Y and let the inverse of R is R^{-1} from Y to X, then it can be written as

$$R^{-1} = \{\langle (u,v), T_{R^{-1}}(u,v), C_{R^{-1}}(u,v), K_{R^{-1}}(u,v), U_{R^{-1}}(u,v), F_{R^{-1}}(u,v)\rangle : (v,u) \epsilon\, V \times U\}.$$

Where $\forall u \,\epsilon\, U$ & $\forall v \,\epsilon\, V$.

$T_{R^{-1}}(u,v) = T_R(\text{v,u}), C_{R^{-1}}(u,v) = C_R(\text{v,u}), K_{R^{-1}}(u,v) = K_R(\text{v,u}),$

$U_{R^{-1}}(u,v) = U_R(\text{v,u}), F_{R^{-1}}(u,v) = F_R(\text{v,u}).$

Preposition 6.5

Let X and Y be two CFPNSs over U and V and let R and S be two CFPNS relations from X to Y,

Then to prove that (i) $(R^{-1})^{-1} = R$. (ii) If $R \subseteq S$ then $R^{-1} \subseteq S^{-1}$.

Proof: For proving $\forall u \,\epsilon\, U$ and $\forall v \,\epsilon\, V$, we have

$(R^{-1})^{-1} = \{\langle\, (u,v), T_{R^{-1}}(u,v), C_{R^{-1}}(u,v), K_{R^{-1}}(u,v), U_{R^{-1}}(u,v), F_{R^{-1}}(u,v)\,\rangle : (v,u) \epsilon\, V \times U\}$,

Where $T_{(R^{-1})^{-1}}(u,v) = T_{R^{-1}}(\text{v,u}) = T_R(\text{u,v})$,

$C_{(R^{-1})^{-1}}(u,v) = C_{R^{-1}}(\text{v,u}) = T_R(\text{u,v}), K_{(R^{-1})^{-1}}(u,v) = K_{R^{-1}}(\text{v,u}) = K_R(\text{u,v}),$

$U_{(R^{-1})^{-1}}(u,v) = U_{R^{-1}}(\text{v,u}) = U_R(\text{u,v}), F_{(R^{-1})^{-1}}(u,v) = F_{R^{-1}}(\text{v,u}) = F_R(\text{u,v}).$

Which implies that $(R^{-1})^{-1} = R$.

If $R \subseteq S$, then we prove that $T_R(u,v) \leq T_s(u,v)$

$\Longrightarrow T_R(u,v) = T_{R^{-1}}(\text{v,u}) \leq T_s(u,v) = T_{S^{-1}}(\text{v,u})$

$\Longrightarrow T_{R^{-1}}(\text{v,u}) \geq T_{S^{-1}}(\text{v,u}),$

Then, we can demonstrate that

$C_R(u,v) \leq C_s(u,v) \Longrightarrow C_{R^{-1}}(\text{v,u}) \geq C_{S^{-1}}(\text{v,u}),$

$K_R(u,v) \leq K_s(u,v) \Longrightarrow K_{R^{-1}}(\text{v,u}) \geq K_{S^{-1}}(\text{v,u}),$

$U_R(u,v) \leq U_s(u,v) \Longrightarrow U_{R^{-1}}(\text{v,u}) \geq U_{S^{-1}}(\text{v,u}),$

$F_R(u,v) \leq F_s(u,v) \Longrightarrow F_{R^{-1}}(\mathrm{v},\mathrm{u}) \geq F_{S^{-1}}(\mathrm{v},\mathrm{u})$.

Definition 6.6.

Let U,V&W are Universal sets and let X, Y, & Z are three CFPNSs. Assume that S is a CFPNR connecting Y & Z, R is a CFPNR connecting X &Y. A CFPNR spanning from X to Z can be defined as the combination of CFPNRs R and S in the following manner:

$$R \circ S = \{\langle (u,w), T_{R\circ S}(u,w), C_{R\circ S}(u,w), K_{R\circ S}(u,w), U_{R\circ S}(u,w), F_{R\circ S}(u,w)\rangle : (u,w) \epsilon\, U \times W\},$$

$\forall (u,w) \epsilon\, U \times W$ and $\forall v\ \epsilon\ V$, $T_{R\circ S}(u,w)= p_{R\circ S}(\mathrm{u},\mathrm{w}).\, e^{ju_{R\circ S}(\mathrm{u},\mathrm{w})}$

Where

$p_{R\circ S}(\mathrm{u},\mathrm{w}) = \max[p_R(\mathrm{u},\mathrm{v}), p_S(\mathrm{v},\mathrm{w})] = \max[\min\big(p_X(\mathrm{u}), p_Y(\mathrm{v})\big), \min(p_Y(\mathrm{v}), p_Z(\mathrm{w})]$
and $u_{R\circ S}(\mathrm{u},\mathrm{w}) = \max[u_R(\mathrm{u},\mathrm{v}), p_S(\mathrm{v},\mathrm{w})]\ \max[\min\big(u_X(\mathrm{u}), u_Y(\mathrm{v})\big), \min(u_Y(\mathrm{v}), u_Z(\mathrm{w})]$.
$C_{R\circ S}(u,w)= q_{R\circ S}(\mathrm{u},\mathrm{w}).\, e^{jv_{R\circ S}(\mathrm{u},\mathrm{w})}$,

Where $q_{R\circ S}(\mathrm{u},\mathrm{w}) = \max[q_R(\mathrm{u},\mathrm{v}), q_S(\mathrm{v},\mathrm{w})] =$

$\max[\min\big(q_X(\mathrm{u}), q_Y(\mathrm{v})\big), \min(q_Y(\mathrm{v}), q_Z(\mathrm{w})]]$

and $v_{R\circ S}(\mathrm{u},\mathrm{w}) = \max[v_R(\mathrm{u},\mathrm{v}), v_S(\mathrm{v},\mathrm{w})] = \max[\min\big(v_X(\mathrm{u}), v_Y(\mathrm{v})\big), \min(v_Y(v), v_Z(w)]$.

$K_{R\circ S}(u,w)= r_{R\circ S}(\mathrm{u},\mathrm{w}).\, e^{jw_{R\circ S}(\mathrm{u},\mathrm{w})}$,
Where

$r_{R\circ S}(\mathrm{u},\mathrm{w}) = \min[r_R(\mathrm{u},\mathrm{v}), r_S(\mathrm{v},\mathrm{w})] = \min[\max\big(r_X(\mathrm{u}), r_Y(\mathrm{v})\big), \max(r_Y(\mathrm{v}), r_Z(\mathrm{w})$ and

$w_{R\circ S}(\mathrm{u},\mathrm{w}) = \min[w_R(\mathrm{u},\mathrm{v}), w_S(\mathrm{v},\mathrm{w})] = \min[\max\big(w_X(\mathrm{u}), w_Y(\mathrm{v})\big), \max(w_Y(\mathrm{v}), w_Z(\mathrm{w})]$.

$U_{R\circ S}(u,w)= s_{R\circ S}(\mathrm{u},\mathrm{w}).\, e^{jx_{R\circ S}(\mathrm{u},\mathrm{w})}$,

Where

$s_{R\circ S}(\mathrm{u},\mathrm{w}) = \min[s_R(\mathrm{u},\mathrm{v}), s_S(\mathrm{v},\mathrm{w})] = \min[\max\big(s_X(\mathrm{u}), s_Y(\mathrm{v})\big), \max(s_Y(\mathrm{v}), s_Z(\mathrm{w})]$ and

$x_{R\circ S}(\mathrm{u},\mathrm{w}) = \min[x_R(\mathrm{u},\mathrm{v}), x_S(\mathrm{v},\mathrm{w})] = \min[\max\big(x_X(\mathrm{u}), x_Y(\mathrm{v})\big), \max(x_Y(\mathrm{v}), x_Z(\mathrm{w})]$.

$F_{R\circ S}(u,w)= t_{R\circ S}(\mathrm{u},\mathrm{w}).\, e^{jy_{R\circ S}(\mathrm{u},\mathrm{w})}$,

Where

$t_{R\circ S}(\mathrm{u,w}) = \min[t_R(\mathrm{u,v}), t_S(\mathrm{v,w})] = \min[\max(t_X(\mathrm{u}), t_Y(\mathrm{v})), \max(t_Y(\mathrm{v}), t_Z(\mathrm{w})]$ and

$y_{R\circ S}(\mathrm{u,w}) = \min[y_R(\mathrm{u,v}), y_S(\mathrm{v,w})] = \min[\max(y_X(\mathrm{u}), xy_Y(\mathrm{v})), \max(xy_Y(\mathrm{v}), xy_Z(\mathrm{w})]$.

This association can be written as $R \circ S(u,w) = R(u,v) \cap S(v,w)$.

The application of the CFPNRs' composition in practical situations is shown in the example that follows.

Example 6.7

Consider that X, Y & Z are three CFPNSs that reflect the sets of financial and public opinion indicators from Malaysia, China, and Malaysia. Let us assume that the CFPNRs R & S are used to measure the interactions between these sets over a 12-month period. R(X, Y) denotes the impact of Chinese financial indicators on Malaysian financial indicators, S(Y, Z) represents the impact of Japanese financial indicators on Malaysian public perception indicators.

A new CFPNR T(X,Z) is created by combining CFPNRs R(X,Y) & S(Y,Z) and it shows that how Malaysian public opinion indicators are impacted by Chinese financial indicators.

For the sake of illustration, consider the composition of the following two approximations in the CFPNRs R(X,Y) &S(Y, Z).

$T_R(u,v), C_R(u,v), K_R(u,v), U_R(u,v), F_R(u,v)) =$

$\langle 0.8e^{j.(11/12)\pi}, 0.4e^{j.(1/12)\pi}, 0.2e^{j(1/48)\pi}, 0.6e^{j(3/12)\pi}, 0.2e^{j(3/12)\pi}\rangle,$

Where u ϵ X and v ϵ Y.

The Malaysian inflation rate and the Chinese Yuvan trade. This assesses the degree of phase (period) of the Chinese Yuvan's exchange rates influence on Malaysia's inflation rate for truth, contradiction, ignorance, unknown, and falsity information.

$\{(T_R(v,w), C_R(v,w), K_R(v,w), U_R(v,w), F_R(v,w)) =$
$\langle 0.7e^{j(10/12)\pi}, 0.8e^{j(3/12)\pi}, 0.1e^{j(1/24)\pi}, 0.5e^{j(6/12)\pi}, 0.4e^{j(5/12)\pi}\rangle\}$

Where w ϵ Z & v ϵ Y are the Malaysian economy and the country's inflation rate respectively. This approximation assesses the degree and phase (period) of the influence of the Chinese Yuvan exchange rate on the confidence in the Malaysian economy, accounting for truth, contradiction, ignorance, unknown, and misleading information. The result of this composition is:

$T_{R\circ S}(u,w), C_{R\circ S}(u,w), K_{R\circ S}(u,w), U_{R\circ S}(u,w), F_{R\circ S}(u,w) =$
$\langle 0.8e^{j(11/12)\pi}, 0.8e^{j(3/12)\pi}, 0.1e^{j(1/48)\pi}, 0.5e^{j(3/12)\pi}, 0.2e^{j(3/12)\pi}\rangle$

The components $T_{R\circ S}(u,w), C_{R\circ S}(u,w), K_{R\circ S}(u,w), U_{R\circ S}(u,w), F_{R\circ S}(u,w)$ measure respectively the truth, contradiction, ignorance, unknown and falsity for both degree and phase (period) of the of the inflation rate in Malaysian financial system.

Theorem 6.8.

Let X, Y& Z are three complex fermatean pentapartitioned neutrosophic sets over the universal set U, V& W and a CFPNR R from X to Y, and a CFPNR S from Y to Z, Then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

Proof: For all $(u, v) \epsilon\, U \times V$ & $(v, w) \epsilon\, V \times W$, Then

$(R \circ S)^{-1}$
$= \left\{\langle (u, w), T_{(R\circ S)^{-1}}(u, w), C_{(R\circ S)^{-1}}(u, w), K_{(R\circ S)^{-1}}(u, w), U_{(R\circ S)^{-1}}(u, w), F_{(R\circ S)^{-1}}(u, w) \rangle : (u, w) \epsilon\, U \times W\right\}$
and
$S^{-1} \circ R^{-1}$
$= \{\langle (u, w), T_{S^{-1}\circ R^{-1}}(u, w), C_{S^{-1}\circ R^{-1}}(u, w), K_{S^{-1}\circ R^{-1}}(u, w), U_{S^{-1}\circ R^{-1}}(u, w), F_{S^{-1}\circ R^{-1}}(u, w)\rangle : (u, w) \epsilon\, U \times W\}$

To demonstrate the equality, we have to prove that $T_{(R\circ S)^{-1}}(u, w) = T_{S^{-1}\circ R^{-1}}(u, w)$, $C_{(R\circ S)^{-1}}(u, w) = C_{S^{-1}\circ R^{-1}}(u, w)$,

$K_{(R\circ S)^{-1}}(u, w) = K_{S^{-1}\circ R^{-1}}(u, w)$, $U_{(R\circ S)^{-1}}(u, w) = U_{S^{-1}\circ R^{-1}}(u, w)$ and $F_{(R\circ S)^{-1}}(u, w) = F_{S^{-1}\circ R^{-1}}(u, w)$.

Therefore, $p_{(R\circ S)^{-1}}(u, w) = p_{R\circ S}(\mathrm{w}, \mathrm{u})$

$= \max\,[p_R(\mathrm{w}, \mathrm{v}), p_S(\mathrm{v}, \mathrm{u})] = \max\,[p_S(\mathrm{v}, \mathrm{u}), p_R(\mathrm{w}, \mathrm{v})]$

$= \max\left[\min\left(p_Y(\mathrm{v}), p_X(\mathrm{u})\right), \min\left(p_Z(\mathrm{w}), p_Y(\mathrm{v})\right)\right]$

$= \max\,[\min(p_X(\mathrm{u}), p_Y(\mathrm{v})), \min(p_Y(\mathrm{v}), p_Z(\mathrm{w})]$

$= \max\,[p_{S^{-1}}(\mathrm{u}, \mathrm{v}), p_{R^{-1}}(\mathrm{v}, \mathrm{w})]$

$= p_{S^{-1}\circ R^{-1}}(u, w)$,

Which implies $p_{(R\circ S)^{-1}}(u, w) = p_{S^{-1}\circ R^{-1}}(u, w)$, Similarly we can show that $u_{(R\circ S)^{-1}}(u, w) = u_{S^{-1}\circ R^{-1}}(u, w)$, to proving that
$T_{(R\circ S)^{-1}}(u, w) = T_{S^{-1}\circ R^{-1}}(u, w)$.

In a similar way, we can demonstrate that it also holds for the terms contradiction, ignorance, unknown, and falsity, completing the proof.

Theorem 6.9

Let X, Y, Z & W be CFPNSs over the universal sets U, V, L & M respectively. Let R be a CFPNR from X to Y, S be a CFPNR from Y to Z & T be a CFPNR from Z to W. Then $R \circ (S \circ T) = (R \circ S) \circ T$.

Proof: For all $(u,v) \epsilon U \times V$ and $(v,l) \epsilon V \times L$, $(l,m) \epsilon L \times M$.
Let

$$R \circ (S \circ T) = \left\{\langle (u,m), T_{R\circ(S\circ T)}(u,m), C_{R\circ(S\circ T)}(u,m), K_{R\circ(S\circ T)}(u,m), U_{R\circ(S\circ T)}(u,m), F_{R\circ(S\circ T)}(u,m) \rangle : (u,m) \epsilon U \times M\right\}$$

$$(R \circ S) \circ T = \left\{\langle (u,m), T_{.(R\circ S)\circ T}(u,m), C_{.(R\circ S)\circ T}(u,m), K_{.(R\circ S)\circ T}(u,m), U_{.(R\circ S)\circ T}(u,m), F_{.(R\circ S)\circ T}(u,m) \rangle : (u,m) \epsilon U \times M\right\}$$

To show that $T_{R\circ(S\circ T)}(u,m) = T_{(R\circ S)\circ T}(u,m)$, $C_{R\circ(S\circ T)}(u,m) = C_{(R\circ S)\circ T}(u,m)$
$K_{R\circ(S\circ T)}(u,m) = K_{(R\circ S)\circ T}(u,m)$, $U_{R\circ(S\circ T)}(u,m) = U_{(R\circ S)\circ T}(u,m)$
and $F_{R\circ(S\circ T)}(u,m) = F_{(R\circ S)\circ T}(u,m)$.

Therefore, $p_{R\circ(S\circ T)}(\mathrm{u,m})$ = max $[p_R(\mathrm{u,v}), p_{S\circ T}(\mathrm{v,m})]$ = max $[p_R(\mathrm{u,v}), \max(p_S(\mathrm{v,l}), p_T(\mathrm{l,m})]$

= max [min $(p_X(\mathrm{u}), p_Y(\mathrm{v}))$, max [min $(p_Y(\mathrm{v}), p_Z(\mathrm{l}), \min(p_Z(\mathrm{l}), p_W(\mathrm{m})\,)]$,

= max [min $(p_X(\mathrm{u}), p_Y(\mathrm{v}))$, min $(\,p_Y(\mathrm{v}), p_Z(\mathrm{l})), \min(p_Z(\mathrm{l}), p_W(\mathrm{m}))]$

= max [max [min $(p_X(\mathrm{u}), p_Y(\mathrm{v}), \min\big(p_Y(\mathrm{v}), p_Z(\mathrm{l})\big), \min(p_Z(\mathrm{l}), p_W(\mathrm{m}))]$

= max [max $[p_R(\mathrm{u,v}), p_S(\mathrm{v,l}), p_T(\mathrm{l,m})]$,

= max $[p_{(R\circ S)}(\mathrm{u,l}), p_T(\mathrm{l,m})]$,

$= p_{(R\circ S)\circ T}(u,m)$

Which implies that $p_{R\circ(S\circ T)}(\mathrm{u,m}) = p_{(R\circ S)\circ T}(u,m)$. Similarly we can show that

$u_{R\circ(S\circ T)}(\mathrm{u,m}) = u_{(R\circ S)\circ T}(u,m)$. Implies that$T_{R\circ(S\circ T)}(u,m) = T_{(R\circ S)\circ T}(u,m)$.
This completes the evidence because the proofs for the terms contradiction, ignorance, unknown, and falsity can all be demonstrated in a similar manner.

To gain a better understanding of this topic, we define projection and CFPNSs.

Definition 6.10.

Let U and V be two universal set and R be a CFPNR on $U \times V$. Then for all $u \epsilon U$ and $v \epsilon V$,

The projection of R on U is a CFPNS R_{P_U}. Defined respectively by the complex valued truth, contradiction, ignorance, unknown and falsity membership functions:

$$T_{R_{P_U}}(\mathrm{u}) = \max_{v} p_R(\mathrm{u},\mathrm{v}).\, e^{\mathrm{j}\max_{v} u_R(\mathrm{u},\mathrm{v}))},$$

$$C_{R_{P_U}}(\mathrm{u}) = \max_{v} q_R(\mathrm{u},\mathrm{v}).\, e^{\mathrm{j}\max_{v} v_R(\mathrm{u},\mathrm{v}))}$$

$$K_{R_{P_U}}(\mathrm{u}) = \min_{v} r_R(\mathrm{u},\mathrm{v}).\, e^{\mathrm{j}\min_{v} w_R(\mathrm{u},\mathrm{v}))}$$

$$U_{R_{P_U}}(\mathrm{u}) = \min_{v} s_R(\mathrm{u},\mathrm{v}).\, e^{\mathrm{j}\min_{v} x_R(\mathrm{u},\mathrm{v}))}$$

$$F_{R_{P_U}}(\mathrm{u}) = \min_{v} t_R(\mathrm{u},\mathrm{v}).\, e^{\mathrm{j}\min_{v} y_R(\mathrm{u},\mathrm{v}))}$$

The projection of R on V is a CFPNS R_{P_V} named respectively by the complex valued truth, contradiction, ignorance, unknown and falsity functions:

$$T_{R_{P_V}}(\mathrm{v}) = \max_{u} p_R(\mathrm{u},\mathrm{v}).\, e^{\mathrm{j}\max_{v} u_R(\mathrm{u},\mathrm{v}))},$$

$$C_{R_{P_V}}(\mathrm{v}) = \max_{u} q_R(\mathrm{u},\mathrm{v}).\, e^{\mathrm{j}\max_{u} v_R(\mathrm{u},\mathrm{v}))}$$

$$K_{R_{P_V}}(\mathrm{v}) = \min_{u} r_R(\mathrm{u},\mathrm{v}).\, e^{\mathrm{j}\min_{u} w_R(\mathrm{u},\mathrm{v}))}$$

$$U_{R_{P_V}}(\mathrm{v}) = \min_{u} s_R(\mathrm{u},\mathrm{v}).\, e^{\mathrm{j}\min_{u} x_R(\mathrm{u},\mathrm{v}))}$$

$$F_{R_{P_V}}(\mathrm{v}) = \min_{u} t_R(\mathrm{u},\mathrm{v}).\, e^{\mathrm{j}\min_{u} y_R(\mathrm{u},\mathrm{v}))}$$

Example 6.11

Let $U = \{u_1, u_2, u_3\}$ and $V = \{v_1, v_2\}$ be two universal set and let R be a CFPNR on $U \times V$ it can be written as follows:

$$\mathrm{R(U,V)} = \left\{\frac{(u_1,v_1)}{\langle 0.9\mathrm{e}^{\mathrm{j}.2\pi(11/12)}, 0.2\mathrm{e}^{\mathrm{j}.2\pi(1/12)}, 0.1\mathrm{e}^{\mathrm{j}.2\pi(1/24)}, 0.6\mathrm{e}^{\mathrm{j}.2\pi(2/12)}, 0.4\mathrm{e}^{\mathrm{j}.2\pi(11/12)}\rangle}, \frac{(u_1,v_2)}{\langle 0.4\mathrm{e}^{\mathrm{j}.2\pi(5/12)}, 0.2\mathrm{e}^{\mathrm{j}.2\pi(1/12)}, 0.3\mathrm{e}^{\mathrm{j}.2\pi(8/12)}, 0.6\mathrm{e}^{\mathrm{j}.2\pi(4/12)} 0.2\mathrm{e}^{\mathrm{j}.2\pi(5/12)}\rangle},\right.$$

$$, \frac{(u_2,v_1)}{\langle 0.3\mathrm{e}^{\mathrm{j}.2\pi(4/12)}, 0.3\mathrm{e}^{\mathrm{j}.2\pi(3/12)}, 0.1\mathrm{e}^{\mathrm{j}.2\pi(4/12)}, 0.5\mathrm{e}^{\mathrm{j}.2\pi(5/12)}, 0.8\mathrm{e}^{\mathrm{j}.2\pi(11/12)}\rangle}, \frac{(u_2,v_2)}{\langle 0.3\mathrm{e}^{\mathrm{j}.2\pi(4/12)}, 0.5\mathrm{e}^{2\pi(4/12)}, 0.3\mathrm{e}^{\mathrm{j}.2\pi(8/12)}, 0.5\mathrm{e}^{\mathrm{j}.2\pi(4/12)}, 0.4\mathrm{e}^{\mathrm{j}.2\pi(5/12)}\rangle}$$

$$, \frac{(u_3,v_1)}{\langle 0.2\mathrm{e}^{\mathrm{j}.2\pi(11/12)}, 0.3\mathrm{e}^{\mathrm{j}.2\pi(3/12)}, 0.9\mathrm{e}^{\mathrm{j}.2\pi(7/12)}, 0.6\mathrm{e}^{\mathrm{j}.2\pi(6/12)}, 0.4\mathrm{e}^{\mathrm{j}.2\pi(11/12)}\rangle}, \frac{(u_3,v_2)}{\langle 0.4\mathrm{e}^{\mathrm{j}.2\pi(3/12)}, 0.5\mathrm{e}^{2\pi(4/12)}, 0.8\mathrm{e}^{\mathrm{j}.2\pi(8/12)}, 0.7\mathrm{e}^{\mathrm{j}.2\pi(4/12)}, 0.3\mathrm{e}^{\mathrm{j}.2\pi(5/12)}\rangle},$$

The projection R on U is given by:

$$R_{P_U}(\mathrm{u}) = \left\{\frac{u_1}{\langle \max(0.9,0.4)\mathrm{e}^{\mathrm{j}.2\pi\max((11/12),(5/12))}, \max(0.2,0.2)\mathrm{e}^{\mathrm{j}.2\pi\max((1/12),(1/12))}, \min(0.1,0.3)\mathrm{e}^{\mathrm{j}.2\pi\min((1/24),(8/12))}, \min(0.6,0.6)\mathrm{e}^{\mathrm{j}.2\pi\min((2/12),(4/12))}, \min(0.4,0.2)\mathrm{e}^{\mathrm{j}.2\pi\min((11/12),(5/12))}\rangle},\right.$$

$$\frac{u_2}{\langle \max(0.3,0.3)\mathrm{e}^{\mathrm{j}.2\pi\max((4/12),(4/12))}, \max(0.3,0.5)\mathrm{e}^{\mathrm{j}.2\pi\max((3/12),(4/12))}, \min(0.1,0.3)\mathrm{e}^{\mathrm{j}.2\pi\min((4/12),(8/12))}, \min(0.5,0.5)\mathrm{e}^{\mathrm{j}.2\pi\min((5/12),(4/12))}, \min(0.8,0.4)\mathrm{e}^{\mathrm{j}.2\pi((11/12),(15/12))}\rangle}$$

$$\left.\frac{u_3}{\langle \max(0.2,0.4)\mathrm{e}^{\mathrm{j}.2\pi\max((11/12),(3/12))}, \max(0.3,0.5)\mathrm{e}^{\mathrm{j}.2\pi\max((3/12),(4/12))}, \min(0.9,0.8)\mathrm{e}^{\mathrm{j}.2\pi\min((7/12),(8/12))}, \min(0.6,0.7)\mathrm{e}^{\mathrm{j}.2\pi\min((6/12),(4/12))}, \min(0.4,0.3)\mathrm{e}^{\mathrm{j}.2\pi((11/12),(5/12))}\rangle}\right\}$$

$$R_{P_U}(\mathrm{u}) = \left\{\frac{u_1}{\langle 0.9\mathrm{e}^{\mathrm{j}.2\pi(11/12)}, 0.2\mathrm{e}^{\mathrm{j}.2\pi(1/12))}, 0.1\mathrm{e}^{\mathrm{j}.2\pi(1/24)}, 0.6\mathrm{e}^{\mathrm{j}.2\pi((2/12)}, 0.2\mathrm{e}^{\mathrm{j}.2\pi(5/12)}\rangle},\right.$$

$$\frac{u_2}{\langle(0.3e^{j.2\pi(11/12)}, 0.2e^{j.2\pi,(1/12))}, 0.1e^{j.2\pi(1/24)}, 0.6e^{j.2\pi(2/12))}, 0.2e^{j.2\pi((11/12)}\rangle}$$

$$\left.\frac{u_3}{\langle\, 0.4e^{j.2\pi(11/12)}, 0.5e^{j.2\pi(4/12))}, 0.8e^{j.2\pi(7/12)}, 0.6e^{j.2\pi((4/12)}, 0.3e^{j.2\pi(5/12)}\rangle}\right\},$$

The projection R on V is given by:

R_{P_V}(v)=

$$\left\{\frac{v_1}{\langle \max(0.9,0.3,0.2)e^{j.2\pi\max\left(\left(\frac{11}{12}\right),\left(\frac{4}{12}\right),\left(\frac{11}{12}\right)\right)}, \max(0.2,0.3,0.3)e^{j.2\pi\max\left(\left(\frac{1}{12}\right),\left(\frac{3}{12}\right),\left(\frac{3}{12}\right)\right)}, \min(0.1,0.1,0.9)e^{j.2\pi\min\left(\left(\frac{1}{24}\right),\left(\frac{4}{12}\right),\left(\frac{7}{12}\right)\right)}, \min(0.6,0.5,0.6)e^{j.2\pi\min\left(\left(\frac{2}{12}\right),\left(\frac{5}{12}\right),\left(\frac{6}{12}\right)\right)}, \min(0.6,0.5,0.6)e^{j.2\pi\min\left(\left(\frac{11}{12}\right),\left(\frac{11}{12}\right),\left(\frac{11}{12}\right)\right)}\rangle},\right.$$

$$\left.\frac{v_2}{\langle \max(0.4,0.3,0.4)e^{j.2\pi\max\left(\left(\frac{5}{12}\right),\left(\frac{4}{12}\right),\left(\frac{3}{12}\right)\right)}, \max(0.2,0.5,0.5)\, e^{j.2\pi\max\left(\left(\frac{1}{12}\right),\left(\frac{4}{12}\right),\left(\frac{4}{12}\right)\right)}, \min(0.3,0.3,0.8)\, e^{j.2\pi\min\left(\left(\frac{8}{12}\right),\left(\frac{8}{12}\right),\left(\frac{8}{12}\right)\right)}, \min(0.6,0.5,0.7)\, e^{j.2\pi\min\left(\left(\frac{4}{12}\right),\left(\frac{4}{12}\right),\left(\frac{4}{12}\right)\right)}, \min(0.2,0.4,0.3)\, e^{j.2\pi\min\left(\left(\frac{5}{12}\right),\left(\frac{5}{12}\right),\left(\frac{5}{12}\right)\right)}\rangle}\right\},$$

$$R_{P_V}(\mathrm{v})=\left\{\frac{v_1}{\langle 0.9, e^{j.2\pi\left(\frac{11}{12}\right)}, 0.3e^{j.2\pi\left(\frac{3}{12}\right)}, 0.1e^{j.2\pi\left(\frac{1}{24}\right)}, 0.5e^{j.2\pi\left(\frac{2}{12}\right)}, 0.5e^{j.2\pi\left(\frac{11}{12}\right)}\rangle}, \frac{v_2}{\langle 0.4, e^{j.2\pi\left(\frac{5}{12}\right)}, 0.5e^{j.2\pi\left(\frac{4}{12}\right)}, 0.3e^{j.2\pi\left(\frac{8}{12}\right)}, 0.5e^{j.2\pi\left(\frac{4}{12}\right)}, 0.52e^{j.2\pi\left(\frac{5}{12}\right)}\rangle}\right\}$$

VII. CONCLUSION

Complex valued membership functions for truth, contradiction, ignorance, unknown, and falsehood characterize a complicated fermatean pentapartitioned neutrosophic set. Consequently a complex valued truth membership function is a truth membership function plus one more term.

The additional term is referred to as the phase term the conventional truth membership function is known as the truth amplitude term. Thus, uncertainty is represented by the truth amplitude term in this sense, periodicity in the uncertain state is represented by the phase term. Therefore, uncertainty with periodicity as a whole is represented a complex-valued truth membership function. Comparably, a complex-valued membership function for contradiction denotes a periodicity, while a complex-valued membership function for ignorance denotes periodicity, a complex-valued membership function for unknown denotes a periodicity, and a complex-valued membership function for falsehood denotes periodicity for falsity. Here we know, explored various fundamental features of sets, including complement, union, intersection, complex fermatean pentapartitioned neutrosophic product and cartesean product. Additionally, provided an interpretation of the complex fermatean pentapartitioned neutrosophic set. Additionally, we have explored here the δ– equalities of complex fermatean pentapartitioned neutrosophic sets. The CFPNSs are derived, examined and utilized in this study to explain and address a real-world decision-making challenge. Before defining the CFPNR, the Cartesian product between two CFPNSs be defined. Next, we introduced several basic operators on the CFPNR, including the inverse and complement of the CFPNR. The axioms, definition of CFPNR composition is also provided along with an example that shows how to apply this idea to combine two CFPNRs to obtain practical information. We deduced certain properties using an instructive example and offered some theorems on the previous operation. Additionally, the notion of projection for CFPNRs is described and given examples.

REFERENCES

[1] Alkouri A and Salleh A (2012) Complex fuzzy sets, in: International conference on fundamental and applied sciences, AIP conference proceedings. Vol 1482, pp 464-470
[2] Atanassov TK (1986) Intuitionistic fuzzy sets. Fuzzy sets syst 20:87-96
[3] Chen Z, Aghakhani S, Man J, Dick S (2011) ANCFIS: a neuro-fuzzy architecture employing complex fuzzy sets. IEEE Tran Fuzzy syst 19(2):305-322
[4] Deschirijive G, Kerre EE (2207) on the position of intuitionistic fuzzy set theory in the framework of theories modeling imprecision. Inf Sci 177(8):1860-1866
[5] Guo Y, Cheng DH (2009) new neutrosophic approach to image segmentaion. Pattern Recognit 42:587-595
[6] Ramot D, Milo R, Friedman M, Kandel A (2202) Complex fuzzy sets, IEEE Trans Fuzzy syst 10(2):171-186
[7] Subha R., K.Mohana (2023), Fermatean pentapartitioned nuetrosophic sets, Indian Journal of Natural Sciences, vol.14/Issue 77/April/2023, international Bimonthly (Print) – Open Access. ISSN:0976-0997
[8] Zhang G, Dillon ST, Cai YK< Ma J, Lu J (2009) Operation properties and $\delta-$ equalities of complex fuzzy sets. Int J Approx Reson 50:1227-1249

