

SOFT J^c HOMOEMORPHISM IN SOFT TOPOLOGICAL SPACE

Abstract

The study of Soft J^c homeomorphism and Soft Strongly J^c homeomorphism in Soft Topological space is the subject of this article. We come up with a few of its properties and talk about a few composition theorems.

Keywords: Soft J^c homeomorphism, Soft Strongly J^c homeomorphism, Soft homeomorphism, Soft mappings.

Authors

S. Jackson

Assistant Professor
P.G and Research
Department of Mathematics
V. O. Chidambaram College
Manonmaniam Sundaranar University
Tirunelveli, Tamil Nadu, India.
1jacks.mat@voccollege.ac.in

J. Carlin

Research Scholar
P.G and Research
Department of Mathematics
V. O. Chidambaram College,
Manonmaniam Sundaranar University
Tirunelveli, Tamil Nadu, India.
carlinewilliams103@gmail.com

I. INTRODUCTION

Molodtsov D [3] introduced the Soft set concept in 1999 to deal uncertainty in a parametric fashion. Naim Cagman et al. [4] in 2011 created a soft topological space and established a soft topology on soft set. In 2002, P. K. Maji et al. [5] defined certain fundamental terminology for the theory, including the equality of two Soft sets, subset and superset of a Soft set, complement of a Soft set, null Soft set, and absolute soft set. C. G. Aras and H. Cakalli [1] presented soft mappings in soft topological spaces in 2013. Soft πgb -homeomorphism in Soft topological space was presented by C. Janaki and D. Sreeja [2] in 2014. We previously discussed the soft J^c closed set, soft J^c open set, soft J^c continuous and soft J^c open map. In this work, we look into soft J^c homeomorphism using an example. Theorems relating to their properties and composition were also examined. Additionally, we discover soft strongly J^c homeomorphisms and their characteristics. Throughout this paper Soft set has been represented as S-set.

II. PRELIMINARIES

Definition 2.1[3]: A **S-set** F_A on the universe G is defined by the set of ordered pairs $F_A = \{(x, f_a(x)): x \in E \text{ and } f_a(x) \in P(G)\}$, where $f_a: A \rightarrow P(G)$ such that $f_a(x) = \phi$ for all $x \notin A$. Hence f_a is called an approximate function of the S-set F_A . The value of f_a may be arbitrary, some of them may be empty, some may have non empty intersection.

Definition 2.2[3]

1. A S-set (F_A) over G is said to be **Null S-Set** denoted by F_ϕ or $\tilde{\phi}$ if for all $e \in A$, $F(e) = \phi$.
2. A S-set (F_E) over G is said to be an **Absolute S-Set** denoted by F_G or \tilde{G} if for all $e \in A$, $F(e) = G$.

Definition 2.3[3]: Let $x \in G$. Then (x_e) denotes the **S-point** over G , for which $x(\alpha) = \{x\} \forall \alpha \in E$. Also the **S-Singleton Set** corresponding to (x_e) is denoted by (x_E) .

Definition 2.4[4]: Let τ be a collection of S-sets over G with a fixed set E of parameters. Then τ is called a **S-topology** on G if

1. $\tilde{\phi}, \tilde{G}$ belongs to τ .
2. The union of any number of S-sets in τ belongs to τ .
3. The intersection of any two S-sets in τ belongs to τ .

The triplet (G, τ_E) is called **S-topological Space** over G . The members of τ are called **S-open** sets in G and complements of them are called **S-closed** sets in G .

Definition 2.5[4]: A S-map $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is said to be **S-open (closed)**, if the image of every S-open(closed) set in (G, τ_E) is S-open (closed) in (H, σ_K) .

Definition 2.6[6]: A S-Subset (F_E) of a S-Topological space (G, τ_E) is known as **S- J^c closed set** if $Scl^*(F_E) \subseteq (U_E)$ whenever $(F_E) \subseteq (U_E)$ and (U_E) is S-semi*open set. $SJ^cC(G)$ represents the collection of all S- J^c closed sets. The complement of S- J^c closed set is **S- J^c open set** and noted by $SJ^cO(G)$.

Definition 2.7[6]: A S-map $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is said to be **S- J^c continuous map** if $m^{-1}(V_K)$ is S- J^c closed set in (G, τ_E) for every s-closed set (V_K) in (H, σ_K) .

Definition 2.8[6]: A S-map $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is said to be **S- J^c closed (open)** if $m(U_E)$ is S- J^c closed (open) set in (H, σ_K) for every s-closed (open) set (U_E) in (G, τ_E) .

Definition 2.9[5]: A S-bijective map $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is called **S-homeomorphism** if m is both S-continuous map and S-open map.

Definition 2.10[6]: A S-topological space (X, τ_E) is said to be **S- T_{J^c} space** if every S- J^c closed set is S-closed.

III. S- J^c HOMEOMORPHISM

Definition 3.1: A S-bijective map m from (G, τ_E) to (H, σ_K) is called **S- J^c homeomorphism** if m is both S- J^c continuous map and S- J^c open map.

Example 3.2: Consider $G = \{x_1, x_2\}, H = \{y_1, y_2\}, E = \{e_1, e_2\}, K = \{k_1, k_2\}$. Define $u: G \rightarrow H$ and $v: E \rightarrow K$ as $u(x_1) = y_1, u(x_2) = y_2$ and $v(e_1) = k_1, v(e_2) = k_2$. Consider the S-topologies $\tau = \{\tilde{\phi}, \tilde{G}, (F_{E1}), (F_{E2})\}$ where $F_1(e_1) = \{x_1\}, F_1(e_2) = \{x_1\}, F_2(e_1) = \{x_2\}, F_2(e_2) = \{x_2\}$ and $\sigma = \{\tilde{\phi}, \tilde{H}, (X_{K1})\}$ where $X_1(k_1) = \{y_1, y_2\}, X_1(k_2) = \tilde{\phi}$. Therefore the s-mapping $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is both S- J^c continuous and S- J^c open map. Hence, m is S- J^c homeomorphism.

Theorem 3.3: A S-bijective map $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is S-homeomorphism. Then m is S- J^c homeomorphism.

Proof. Consider $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is a S-homeomorphism, then m is S-continuous and S-open map. Also consider (M_K) is a S-open set in (H, σ_K) . Therefore, $m^{-1}(M_K)$ is S-open set in (G, τ_E) . We know that all S-open sets are S- J^c open set, $m^{-1}(M_K)$ is S- J^c open set in (G, τ_E) . Thus m is S- J^c continuous. Again consider (N_E) is a S-open in (G, τ_E) . Since m is a S-open map, $m(N_E)$ is S-open set in (H, σ_K) . We know that all S-open sets are S- J^c open set, $m(N_E)$ is S- J^c open set in (H, σ_K) . Then m is a S- J^c open map. Hence m is S- J^c homeomorphism.

Remark 3.4: S- J^c homeomorphism need not be S-homeomorphism.

Example 3.5: Consider $G = \{x_1, x_2\}, H = \{y_1, y_2\}, E = \{e_1, e_2\}, K = \{k_1, k_2\}$. Define $u: G \rightarrow H$ and $v: E \rightarrow K$ as $u(x_1) = y_1, u(x_2) = y_2$ and $v(e_1) = k_1, v(e_2) = k_2$. Consider the S-topologies $\tau = \{\tilde{\phi}, \tilde{G}, (P_{E1}), (P_{E2})\}$ where $P_1(e_1) = \{x_2\}, P_1(e_2) = \{x_1\}, P_2(e_1) = \{x_2\}, P_2(e_2) = \{x_1, x_2\}$ and $\sigma = \{\tilde{\phi}, \tilde{H}, (Q_{K1}), (Q_{K2}), (Q_{K3})\}$ where $Q_1(k_1) = \{y_2\}, Q_1(k_2) = \tilde{\phi}, Q_2(k_1) = \tilde{\phi}, Q_2(k_2) = \{y_1\}, Q_3(k_1) = \{y_2\}, Q_3(k_2) = \{y_1\}$. Therefore the S-mapping $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is both S- J^c continuous and S- J^c open map, but m is not S-continuous and S-open map. Since, $m(\alpha_E) = \{(e_1, y_2), (e_2, y_1, y_2)\}$ is not S-open set in (H, σ_K) , also $m^{-1}(\beta_K) = \{(e_1, x_1), (e_2, x_1, x_2)\}$ is not S-closed set in (G, τ_E) . Hence, m is not S-homeomorphism.

Theorem 3.6: A S-bijective map $m : (G, \tau_E) \rightarrow (H, \sigma_K)$ is $S\text{-}J^c$ continuous, then the following are equivalent

- (i) m is $S\text{-}J^c$ open map
- (ii) m is $S\text{-}J^c$ homeomorphism
- (iii) m is $S\text{-}J^c$ closed map

Proof: (i) \Rightarrow (ii) Since a S-bijective map m is $S\text{-}J^c$ continuous, also m is $S\text{-}J^c$ open map. Then m is $S\text{-}J^c$ homeomorphism.

(ii) \Rightarrow (iii) Consider (U_E) is s-closed set in (G, τ_E) . Then $(U_E)^c$ is s-open set in (G, τ_E) . By hypothesis, $m((U_E)^c) = (m(U_E))^c$ is $S\text{-}J^c$ open set in (H, σ_K) . Therefore, $m(U_E)$ is $S\text{-}J^c$ closed set in (H, σ_K) . Hence, m is $S\text{-}J^c$ closed map.

(iii) \Rightarrow (i) Consider (V_E) is s-open set in (G, τ_E) . Then $(V_E)^c$ is s-closed set in (G, τ_E) . By hypothesis, $m((V_E)^c) = (m(V_E))^c$ is $S\text{-}J^c$ closed set in (H, σ_K) . Therefore, $m(V_E)$ is $S\text{-}J^c$ open set in (H, σ_K) . Hence, m is $S\text{-}J^c$ open map.

Theorem 3.7: Every $S\text{-}J^c$ homeomorphism from a $S\text{-}T_{J^c}$ space into another $S\text{-}T_{J^c}$ space is $S\text{-}J^c$ homeomorphism.

Proof: Consider a s-map $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is $S\text{-}J^c$ homeomorphism and (L_E) be a s-open set in (G, τ_E) . Since m is $S\text{-}J^c$ open and (H, σ_K) is a $S\text{-}T_{J^c}$ space, $m(L_E)$ is s-open set in (H, σ_K) . Thus, m is a s-open map. Since m is $S\text{-}J^c$ continuous and (G, τ_E) is a $S\text{-}T_{J^c}$ space, $m^{-1}(L_E)$ is s-closed in (G, τ_E) . Therefore, m is s-continuous. Hence, m is s-homeomorphism.

Theorem 3.8: Consider (G, τ_E) be a S-topological space, then $SJ^c h(G, \tau_E)$ is a group under the composition of maps.

Proof: Let us define a binary operation $*$: $SJ^c h(G, \tau_E) \times SJ^c h(G, \tau_E)$ by $m * n = (n \circ m)$ for all $m, n \in SJ^c h(G, \tau_E)$ and \circ is the usual operation of composition of maps. Then $(n \circ m) \in SJ^c h(G, \tau_E)$. We know that the composition of maps is associative, also the identity map $I: (G, \tau_E) \rightarrow (G, \tau_E)$ belonging to $SJ^c h(G, \tau_E)$ serves as the identity element. For every $m \in SJ^c h(G, \tau_E)$, $m \circ m^{-1} = m^{-1} \circ m = I$. Therefore the inverse exists for each element of $SJ^c h(G, \tau_E)$. Hence, $SJ^c h(G, \tau_E)$ forms a group under the operation of composition of maps.

Theorem 3.9: If $m : (G, \tau_E) \rightarrow (H, \sigma_K)$ is a $S\text{-}J^c$ homeomorphism. Then, m induces a S-isomorphism from the group $SJ^c h(G, \tau_E)$ onto the group $SJ^c h(H, \sigma_K)$.

Proof: Suppose $m \in SJ^c h(G, \tau_E)$. We define a function $\psi m: SJ^c h(G, \tau_E) \rightarrow SJ^c h(H, \sigma_K)$ by $\psi m = m \circ h \circ m^{-1}$ for every $h \in SJ^c h(G, \tau_E)$. Therefore m is a S-bijection. Now for all $n, h \in SJ^c h(G, \tau_E)$, we get $\psi m(n \circ h) = m \circ (n \circ h) \circ m^{-1} = (m \circ n \circ m^{-1}) \circ (m \circ h \circ m^{-1}) = \psi m(n) \circ \psi m(h)$.

Theorem 3.10: Consider $m : (G, \tau_E) \rightarrow (H, \sigma_K)$ be a $S\text{-}J^c$ homeomorphism and (U_E) be a $S\text{-}J^c$ closed subset of (G, τ_E) . Also, (V_K) be a s-closed subset of (H, σ_K) such that $m(U_E) =$

(V_K) . Assume that $SJ^cC(G, \tau_E)$ is closed under any s-intersection. Then the restriction $m_{(U_E)}: ((U_E), \tau_{(U_E),E}) \rightarrow ((V_K), \sigma_{(V_K),K})$ is a $S-J^c$ homeomorphism.

Proof:

- i. Consider $m : (G, \tau_E) \rightarrow (H, \sigma_K)$ is a $S-J^c$ homeomorphism. Since m is S-1-1 and s-onto, $m_{(U_E)}$ is S-1-1 and $m_{(U_E)}(U_E) = (V_K)$ such that $m_{(U_E)}$ is s-onto. Hence, $m_{(U_E)}$ is s-bijection.
- ii. Consider (L_E) be an s-open set of $((U_E), \tau_{(U_E),E})$. $(L_E) = (U_E) \tilde{\cap} (A_E)$, for some s-open set (A_E) in (G, τ_E) . Since, m is S-1-1 then $(L_E) = m((U_E) \tilde{\cap} (A_E)) = m(U_E) \tilde{\cap} m(A_E) = (V_K) \tilde{\cap} m(A_E)$. Since, m is $S-J^c$ open and (A_E) is s-open set in (G, τ_E) , then $m(A_E)$ is $S-J^c$ open in (H, σ_K) . Therefore $m(A_E)$ is a $S-J^c$ open in $((V_K), \sigma_{(V_K),K})$. Hence, $m_{(U_E)}$ is $S-J^c$ open map.
- iii. Consider (S_K) be a s-closed in $((V_K), \sigma_{(V_K),K})$. Then $(S_K) = (V_K) \tilde{\cap} (B_K)$ for some s-closed set (B_K) in (H, σ_K) . Since (V_K) is a s-closed set in (H, σ_K) , then (S_K) is a s-closed set in (H, σ_K) . By hypothesis and assumption, $m^{-1}((S_K) \tilde{\cap} (X_K)) = (Y_E)$ (say) is a $S-J^c$ closed set in (G, τ_E) . Since $m_A^{-1}(S_K) = (Y_E)$, It is adequate to prove that (Y_E) is $S-J^c$ closed in $((U_E), \tau_{(U_E),E})$. Let (C_E) be $S-\hat{g}$ -open set in $((A, E), \tau_{(A,E),E})$ such that $(Y_E) \tilde{\cap} (C_E)$. Then by hypothesis and by relativity of $S-\hat{g}$ -open set, (C_E) is $S-\hat{g}$ -open set in (G, τ_E) . Since (Y_E) is a $S-J^c$ closed set in (G, τ_E) , $s^*Cl_{\tilde{G}}(Y_E) \subseteq Int(C_E)$. Since (U_E) is S-open, $s^*Cl_{(U_E)}(Y_E) = s^*Cl_{\tilde{G}}(Y_E) \tilde{\cap} (U_E) \subseteq Int(C_E) \tilde{\cap} Int(U_E) = Int((C_E) \tilde{\cap} (U_E)) \subseteq Int(C_E)$ and so $(Y_E) = m_A^{-1}(S_K)$ is $S-J^c$ closed in $((U_E), \tau_{(U_E),E})$.

Therefore, $m_{(U_E)}$ is s-bijection, $S-J^c$ continuous and $S-J^c$ open map. Hence the restriction map $m_{(U_E)} : ((U_E), \tau_{(U_E),E}) \rightarrow ((V_K), \sigma_{(V_K),K})$ is a $S-J^c$ Homeomorphism.

Theorem 3.11: If $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is S-strongly J^c continuous and $n: (H, \sigma_K) \rightarrow (W, \rho_R)$ be a $S-J^c$ homeomorphism then $n \circ m: (G, \tau_E) \rightarrow (W, \rho_R)$ is $S-J^c$ homeomorphism.

Proof: Consider (D_E) be a s-open set in (G, τ_E) . Since every s-open set is $S-J^c$ open set, $m(D_E)$ is a s-open set in (H, σ_K) . Then $n(m(D_E))$ is $S-J^c$ open set in (W, ρ_R) . Also, consider (I_R) be a s-closed in (W, ρ_R) , $n^{-1}(I_R)$ is a $S-J^c$ closed set in (H, σ_K) . Since m is S-strongly J^c continuous and n is $S-J^c$ continuous, $m^{-1}(n^{-1}(I_R))$ is $S-J^c$ closed set in (G, τ_E) . Thus $n \circ m$ is $S-J^c$ open map and $S-J^c$ continuous. Hence, $n \circ m$ is $S-J^c$ homeomorphism.

IV. S-STRONGLY J^c HOMEOMORPHISM

Definition 4.1: A s-bijective map m from (G, τ_E) to (H, σ_K) is S-strongly J^c homeomorphism if m and m^{-1} are both $S-J^c$ irresolute.

Example 4.2: Consider $G = \{x_1, x_2\}, H = \{y_1, y_2\}, E = \{e_1, e_2\}, K = \{k_1, k_2\}$. Define $u : G \rightarrow H$ and $v : E \rightarrow K$ as $u(x_1) = y_1, u(x_2) = y_2$ and $v(e_1) = k_1, v(e_2) = k_2$. Consider the S-topologies $\tau = \{\tilde{\phi}, \tilde{G}, (S_{E1}), (S_{E2})\}$ where $S_1(e_1) = \{x_1\}, S_1(e_2) = \{x_1\}, S_2(e_1) = \{x_2\}, S_2(e_2) = \{x_2\}$ and $\sigma = \{\tilde{\phi}, \tilde{H}, (T_{K1}), (T_{K2})\}$ where $T_1(k_1) = \{y_1\}, T_1(k_2) = \{y_2\}$,

$T_2(k_1) = \{y_2\}$, $T_2(k_2) = \{y_1\}$. Therefore the s-mapping $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is $S\text{-}J^c$ irresolute also the s-mapping $m^{-1}: (H, \sigma_K) \rightarrow (G, \tau_E)$ is $S\text{-}J^c$ irresolute. Hence, m is S -strongly J^c homeomorphism.

Theorem 4.3: If a s-map $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is S -strongly J^c homeomorphism then m is a $S\text{-}J^c$ homeomorphism.

Proof: Consider $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ be a S -strongly J^c homeomorphism, then m and m^{-1} are $S\text{-}J^c$ irresolute. Since every $S\text{-}J^c$ irresolute map is $S\text{-}J^c$ continuous and m is $S\text{-}J^c$ open map if m^{-1} is $S\text{-}J^c$ continuous. Then m is $S\text{-}J^c$ continuous and $S\text{-}J^c$ open map. Hence, m is $S\text{-}J^c$ homeomorphism.

Remark 4.4: The reverse implication of the above theorem is not true.

Example 4.5: Let $G = \{x_1, x_2\}$, $H = \{y_1, y_2\}$, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$. Define $u: G \rightarrow H$ and $v: E \rightarrow K$ as $u(x_1) = y_1$, $u(x_2) = y_2$ and $v(e_1) = k_1$, $v(e_2) = k_2$. Consider the S -topologies $\tau = \{\tilde{\phi}, \tilde{G}, (B_E)\}$ where $B(e_1) = \phi$, $B(e_2) = \{x_2\}$ and $\sigma = \{\tilde{\phi}, \tilde{H}, (C_{K1}, C_{K2}, C_{K3})\}$ where $C_1(k_1) = \{y_2\}$, $C_1(k_2) = \{y_2\}$, $C_2(k_1) = \{y_2\}$, $C_2(k_2) = \{y_1, y_2\}$, $C_3(k_1) = \{y_1, y_2\}$, $C_3(k_2) = \{y_2\}$. Therefore the s-mapping $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is both $S\text{-}J^c$ continuous and $S\text{-}J^c$ open map, but m is $S\text{-}J^c$ irresolute and m^{-1} is not $S\text{-}J^c$ irresolute. Since, $(m^{-1})^{-1}(\gamma_E) = \{(e_1, y_2), (e_2, \phi)\}$ is not $S\text{-}J^c$ closed set in (H, σ_K) . Hence, m is not S -strongly J^c homeomorphism.

Theorem 4.6: If $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is a S -strongly J^c homeomorphism, then $SJ^c cl(m(X_K)) = m(SJ^c cl(X_K))$ for every s-subset (X_K) of (H, σ_K) .

Proof: Consider $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is a S -strongly J^c homeomorphism, then m is S -strongly J^c continuous. Since $SJ^c cl(m(X_K))$ is $S\text{-}J^c$ closed set in (H, σ_K) , $m^{-1}(SJ^c cl(m(X_K)))$ is $S\text{-}J^c$ closed in (G, τ_E) . Now, $m(X_K) \cong m(SJ^c cl(X_K))$ and so $J^c cl(m(X_K)) \cong m(SJ^c cl(X_K))$. Also, since m is a S -strongly J^c homeomorphism, then m^{-1} is $S\text{-}J^c$ irresolute. Since $SJ^c cl(m(X_K))$ is $S\text{-}J^c$ closed set in (G, τ_E) , $(m^{-1})^{-1}(SJ^c cl(m(X_K))) = m(SJ^c cl(m(X_K)))$ is $S\text{-}J^c$ closed in (H, σ_K) . Now, $(X_K) \cong (m^{-1})^{-1}(m(X_K)) \cong (m^{-1})^{-1}(SJ^c cl(m(X_K))) = m(SJ^c cl(m(X_K)))$ then $J^c cl(X_K) \cong m^{-1}(SJ^c cl(m(X_K)))$.

Therefore, $(SJ^c cl(X_K)) \cong m(m^{-1}(SJ^c cl(m(X_K)))) \cong SJ^c cl(m(X_K))$.

Hence, $SJ^c cl(m(X_K)) = m(SJ^c cl(X_K))$.

Theorem 4.7: If $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ is a S -strongly J^c homeomorphism, then $SJ^c cl(m^{-1}(V_K)) = m^{-1}(SJ^c cl(V_K))$ for all $(V_K) \cong (H, \sigma_K)$.

Proof: Consider m is a S -strongly J^c homeomorphism, then m is $S\text{-}J^c$ irresolute. Since $SJ^c cl(m(V_K))$ is a $S\text{-}J^c$ closed set in (H, σ_K) , $m^{-1}(SJ^c cl(V_K))$ is $S\text{-}J^c$ closed set in (G, τ_E) . Now, $m^{-1}(V_K) \cong m^{-1}(SJ^c cl(V_K))$ then $SJ^c cl(m^{-1}(V_K)) \cong m^{-1}(SJ^c cl(V_K))$. Also, since

m is a S-strongly J^c homeomorphism, m^{-1} is S- J^c irresolute. Since $SJ^c cl(m^{-1}(V_K))$ is S- J^c closed set in (G, τ_E) , $(m^{-1})^{-1}(SJ^c cl(m^{-1}(V_K))) = m(SJ^c cl(m^{-1}(V_K)))$ is S- J^c closed set in (H, σ_K) . Now $(V_K) \cong (m^{-1})^{-1}(m^{-1}(V_K)) \cong (m^{-1})^{-1}(SJ^c cl(m^{-1}(V_K))) = m(SJ^c cl(m^{-1}(V_K)))$, $J^c cl(V_K) \cong m(SJ^c cl(m^{-1}(V_K)))$. Thus $m^{-1}(SJ^c cl(V_K)) \cong m^{-1}(m(SJ^c cl(m^{-1}(V_K)))) \cong SJ^c cl(m^{-1}(V_K))$ and hence the equality holds.

Theorem 4.8: The composition of two S-strongly J^c homeomorphism is S-strongly J^c homeomorphism.

Proof: Consider the s-mappings $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ and $n: (H, \sigma_K) \rightarrow (W, \rho_R)$ be two S-strongly J^c homeomorphism and (Y_R) be a S- J^c closed set in (W, ρ_R) . Since m and n are S- J^c irresolute, $n^{-1}(Y_R) = m^{-1}(n^{-1}(Y_R))$ is S- J^c closed set in (G, τ_E) . Thus $(n \circ m)$ is S- J^c irresolute. Now, consider (Z_E) is S- J^c closed set in (G, τ_E) . Since m^{-1} and n^{-1} are S- J^c irresolute, $(m^{-1})^{-1}(Y_R) = (n^{-1})^{-1}[(m^{-1})^{-1}(Y_R)] = n(m(Y_R))$ is S- J^c closed set in (W, ρ_R) . Therefore, $(n \circ m)^{-1}$ is S- J^c irresolute. Hence, $(n \circ m)$ is S-strongly J^c homeomorphism.

Theorem 4.9: Consider $m: (G, \tau_E) \rightarrow (H, \sigma_K)$ be a S-strongly J^c homeomorphism and $n: (H, \sigma_K) \rightarrow (W, \rho_R)$ be a S- J^c homeomorphism then $n \circ m: (G, \tau_E) \rightarrow (W, \rho_R)$ is S- J^c homeomorphism.

Proof: Consider (P_K) be a S-closed set in (W, ρ_R) . Since, m is S- J^c irresolute and n is S- J^c continuous. $m^{-1}(n^{-1}(P_K))$ is S- J^c closed set in (G, τ_E) . Therefore, $n \circ m$ is S- J^c continuous. Now, consider (Q_E) be a s-open set in (G, τ_E) , $m(n(Q_E))$ is S- J^c open set in (W, ρ_R) . Since, m^{-1} is S- J^c irresolute and n is S- J^c open map. Thus $n \circ m$ is S- J^c open map. Hence, $n \circ m$ is S- J^c homeomorphism.

V. CONCLUSION

We have introduced S- J^c closed (open) maps with S- J^c closed (open) set in S-topological space. We have learned some of its properties and composition theorems. Then we have studied about S-strongly J^c closed maps and S-quasi J^c closed maps with some properties.

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