IN THE PHYSICAL SYSTEM, EXAMINE THE MOTION OF AN ELECTRON VIA UPADHYAYA TRANSFORM AND IMAN TRANSFORM

Abstract

Numerous mathematical techniques have been used over the past few decades to solve the particle motion problem. In this study, we investigate the motion of an electron in a physical system using the Upadhyaya and Iman transforms. For this, we describe a differential equation of motion for a particle and use both of these transforms to solve it. The results revealed that both methods produced similar results.

Keywords: Differential equations; Physical problems; Upadhyaya transform (UT); Iman transform; motion of electron.

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I. INTRODUCTION

Integral and differential equations have important roles in science and engineering [1-2]. Integral transforms have several applications in engineering and technology, as well as fundamental sciences and mathematics. The Upadhyaya transform and the Iman transform are integral transforms that may be used to solve any boundary value problem that appears as a differential equation describing a physical system. A variety of integral transforms, including Laplace, Fourier, and others, were used to solve the physical differential equations. The SEE transformation, a novel transformation that is extremely helpful in the solution of differential equations and systems of differential equations, has been illustrated in year 2021 [3]. Mousa [4] solved the Volterra integral equation of the first kind using the Upadhyaya transform. Shilpa and Pralahad [5] used integral transforms to investigate the motion of an electron in a physical system. Shilpa and Pralahad [6] determined the motion of a particle system of ordinary differential equations using the Elzaki and Laplace transforms. Iman Almardy [7-8] recently presented a novel transform known as the Iman transform and used it to solve a system of ordinary differential equations. To solve problems involving differential equations, Ali et al. [9] proposed two parametric SEE transformations. Dinesh and Prakash [10] solved the Volterra integral equation of the second kind by using the Upadhyaya transform in their recent study. In this work, we examined the Upadhyaya transform and Iman transform techniques for studying electron motion in physical system. To begin, we'll go over the basic formulae and properties shared by Upadhyaya and Iman transforms.

II. Definitions and Standard Results:

1. Upadhyaya Transform

Upadhyaya transformation of the function $f(t)$ is given by [11]:

$$U\{f(t)\} = \lambda_1 \int_0^\infty \exp(-\lambda_2 t) f(\lambda_3 t)\, dt = u(\lambda_1, \lambda_2, \lambda_3),\ (\lambda_1, \lambda_2, \lambda_3) > 0,\ t \geq 0 \quad \ldots.. \quad (1)$$

In the general formulation of the Upadhyaya transform, the parameters λ_1, λ_2 and λ_3 are complex parameters; however, for the sake of this research, we will assume that all of these parameters are positive real numbers (see Upadhyaya [11] and Upadhyaya et al. [12]).

- **Upadhyaya Transform of some functions** [11]:

$$U(1) = \frac{\lambda_1}{\lambda_2},\ U(t^m) = \frac{m!\lambda_1 \lambda_3^m}{\lambda_2^{m+1}} (m \in N),\ U(e^{at}) = \frac{\lambda_1}{\lambda_2 - a\lambda_3}$$

$$U[\sin(at)] = \frac{a\lambda_1\lambda_3}{\lambda_2^2 + a^2\lambda_3^2}, U[\cos(at)] = \frac{\lambda_1\lambda_2}{\lambda_2^2 + a^2\lambda_3^2}$$

- **Upadhyaya Transform of Derivatives:**

If $U[f(t)] = u(\lambda_1, \lambda_2, \lambda_3)$ then from [11]

$$U[f'(t);\lambda_1,\lambda_2,\lambda_3]=\left(\frac{\lambda_2}{\lambda_3}\right)U[f(t);\lambda_1,\lambda_2,\lambda_3]-\frac{\lambda_1}{\lambda_3}f(0),$$

$$U[f''(t);\lambda_1,\lambda_2,\lambda_3]=\left(\frac{\lambda_2}{\lambda_3}\right)^2U[f(t);\lambda_1,\lambda_2,\lambda_3]-\frac{\lambda_1\lambda_2}{\lambda_3^2}f(0)-\frac{\lambda_1}{\lambda_3}f'(0),$$

$$U[f^n(t);\lambda_1,\lambda_2,\lambda_3]=\left(\frac{\lambda_2}{\lambda_3}\right)^nU[f(t);\lambda_1,\lambda_2,\lambda_3]-\frac{\lambda_1\lambda_2^{n-1}}{\lambda_3^n}f(0)-\frac{\lambda_1\lambda_2^{n-2}}{\lambda_3^{n-1}}f'(0)-\frac{\lambda_1\lambda_2^{n-3}}{\lambda_3^{n-2}}f''(0)-.....-\frac{\lambda_1}{\lambda_3}f^{n-1}(0).$$

2. Iman Transform

Iman transformation of the function $f(t)$ is given by [7]

$$I[f(t)]=\frac{1}{v^2}\int_0^\infty \exp\left(-v^2t\right)f(t)\,dt=I(v),\ t\geq 0,\ k_1\leq v\leq k_2.\qquad .... \qquad (2)$$

The variable v in this transform is used to factor the variable t in the argument of the function f .This transformation is more closely related to the Laplace and Elzaki transformation.

- **Iman Transform of some functions** [7]:

$$I(1)=\frac{1}{v^4},\ I(t^m)=\frac{m!}{v^{2m+4}}(m\in N),\ I(e^{at})=\frac{1}{v^4-av^2},$$
$$I[\sin(at)]=\frac{a}{v^2(v^4+a^2)},\ I[\cos(at)]=\frac{1}{(v^4+a^2)}$$

For both the transform the sufficient condition for the function to exist is:

- Function should be piecewise continuous
- Function should be of exponential order

- **Iman Transform of Derivatives:**

If $I[f(t)]=I(v)$ then from [7]

$$I\left[\frac{\partial f(t)}{\partial t}\right]=v^2I(v)-\frac{1}{v^2}f(0),$$

$$I\left[\frac{\partial^2 f(t)}{\partial t}\right]=v^4I(v)-f(0)-\frac{1}{v^2}f'(0)$$

$$I\left[\frac{\partial^n f(t)}{\partial t^n}\right]=v^nI(v)-\sum_{k=0}^{n-1}\frac{1}{v^{4-2n+2k}}f^k(0).$$

III. NUMERICAL APPLICATIONS

Take into account the motion of an electron as given by the following equations [5]:

m d 2 x dt 2 +eh dy dt =eE.. .... (3)

m d 2 y dt 2 −eh dx dt =0 . .. . (4)

Let us consider the conditions

$$x(0)=0,\dot{x}(0)=0, y(0)=0, \dot{y}(0)=0$$

Applying the Upadhyaya Transform and the derivative property of the Upadhyaya Transform to both sides of equation (3), we get

$$mU\left[\frac{d^2x}{dt^2}\right]+ehU\left[\frac{dy}{dt}\right]=U[eE]$$

$$m\left\{\left(\frac{\lambda_2}{\lambda_3}\right)^2 U[x(t)]-\frac{\lambda_1\lambda_2}{\lambda_3^2}x(0)-\frac{\lambda_1}{\lambda_3}\dot{x}(0)\right\}+eh\left\{\left(\frac{\lambda_2}{\lambda_3}\right)U[y(t)]-\frac{\lambda_1}{\lambda_3}y(0)\right\}=\frac{eE\lambda_1}{\lambda_2}\ \ldots\ldots \quad (5)$$

Equation (5) can be expressed as follows by using the initial conditions:

$$m\left(\frac{\lambda_2}{\lambda_3}\right)^2 U[x(t)]+eh\left(\frac{\lambda_2}{\lambda_3}\right)U[y(t)]=\frac{eE\lambda_1}{\lambda_2}\ \ldots.. \quad (6)$$

Performing the Upadhyaya transform bilaterally to equation (4) and using the derivative property of Upadhyaya, we get

$$mU\left[\frac{d^2y}{dt^2}\right]-ehU\left[\frac{dx}{dt}\right]=0$$

$$m\left\{\left(\frac{\lambda_2}{\lambda_3}\right)^2 U[y(t)]-\frac{\lambda_1\lambda_2}{\lambda_3^2}y(0)-\frac{\lambda_1}{\lambda_3}\dot{y}(0)\right\}-eh\left\{\left(\frac{\lambda_2}{\lambda_3}\right)U[x(t)]-\frac{\lambda_1}{\lambda_3}x(0)\right\}=0\ \ldots.. \quad (7)$$

Equation (7) can be expressed as follows by using the initial conditions:

$$m\left(\frac{\lambda_2}{\lambda_3}\right)^2 U[y(t)]-eh\left(\frac{\lambda_2}{\lambda_3}\right)U[x(t)]=0\ \ldots.. \quad (8)$$

Now Solving Equations (6) and (8), we get

$$U[x(t)] = \frac{eE\lambda_1}{\lambda_2} \frac{\lambda_3^2\, m}{(m^2\lambda_2^2 + e^2h^2\lambda_3^2)}$$

$$U[x(t)] = \frac{eE}{m} \frac{\lambda_1\lambda_3^2}{\lambda_2(\lambda_2^2 + \omega^2\lambda_3^2)}$$

where $\omega = \frac{eh}{m}$

$$U[x(t)] = \frac{eE}{m\omega^2}\left[\frac{\lambda_1}{\lambda_2} - \frac{\lambda_1\lambda_2}{(\lambda_2^2 + \omega^2\lambda_3^2)}\right] \ldots.. \quad (9)$$

After applying bilaterally the inverse Upadhyaya transform to equation (9), we obtain

$$x(t) = \frac{eE}{m\omega^2}[1 - \cos\omega t]$$

$$x(t) = \frac{eE}{m\left(\frac{e^2h^2}{m^2}\right)}[1 - \cos\omega t]$$

$$x(t) = \frac{E}{h\left(\frac{eh}{m}\right)}[1 - \cos\omega t]$$

$$x(t) = \frac{E}{h\omega}[1 - \cos\omega t] \quad \ldots\ldots \quad (10)$$

Similarly, we can calculate the value of $y(t)$, as illustrated below.

$$U[y(t)] = \frac{e^2Eh}{m^2} \frac{\lambda_1\lambda_3^3}{(\lambda_2^2 + \omega^2\lambda_3^2)\lambda_2^2}$$

$$U[y(t)] = \frac{e^2Eh}{m^2\omega^3}\left[\frac{\omega\lambda_1\lambda_3}{\lambda_2^2} - \frac{\omega\lambda_1\lambda_3}{\lambda_2^2 + \omega^2\lambda_3^2}\right]$$

$$U[y(t)] = \frac{e^2Eh^2}{h\,m^2\omega^3}\left[\frac{\omega\lambda_1\lambda_3}{\lambda_2^2} - \frac{\omega\lambda_1\lambda_3}{\lambda_2^2 + \omega^2\lambda_3^2}\right]$$

$$U[y(t)] = \frac{E}{h\omega}\left[\frac{\omega\lambda_1\lambda_3}{\lambda_2^2} - \frac{\omega\lambda_1\lambda_3}{\lambda_2^2+\omega^2\lambda_3^2}\right] \;\ldots.. \qquad (11)$$

After applying bilaterally the inverse Upadhyaya transform to equation (11), we obtain

$$y(t) = \frac{E}{h\omega}\left[\omega t - \sin\omega t\right] \;\ldots\ldots \qquad (12)$$

Performing the Iman transform bilaterally to equation (3) and using the derivative property of Iman, we get

$$m\,I\left[\frac{d^2x}{dt^2}\right] + eh\,I\left[\frac{dy}{dt}\right] = I\left[eE\right]$$

$$m\left\{v^4 I[x(t)] - x(0) - \frac{1}{v^2}\dot{x}(0)\right\} + eh\left\{v^2 I[y(t)] - \frac{1}{v^2}y(0)\right\} = \frac{eE}{v^4}\ldots\ldots \qquad (13)$$

Equation (13) can be expressed with the initial condition as

$$m\,v^4 I[x(t)] + eh\,v^2 I[y(t)] = \frac{eE}{v^4} \;\ldots\ldots \qquad (14)$$

After executing the Iman transform bilaterally to equation (4) and utilizing the derivative property of the Iman transform, we obtain

$$m\,I\left[\frac{d^2y}{dt^2}\right] - eh\,I\left[\frac{dx}{dt}\right] = 0$$

$$m\left\{v^4 I[y(t)] - y(0) - \frac{1}{v^2}\dot{y}(0)\right\} - eh\left\{v^2[x(t)] - \frac{1}{v^2}x(0)\right\} = 0 \;\ldots. \qquad (15)$$

Equation (15) can be represented by the initial conditions as follows:

$$m\,v^4 I[y(t)] - eh\,v^2 I[x(t)] = 0 \;\ldots\ldots \qquad (16)$$

Now Solving Equations (14) and (16), we get

$$I[x(t)] = \frac{eE}{v^4}\frac{m}{(m^2v^4 + e^2h^2)}$$

$$I[x(t)] = \frac{eE}{v^4}\frac{m}{(m^2v^4 + \omega^2m^2)}$$

$$I[x(t)] = \frac{eE}{m^2 v^4} \frac{m}{(v^4 + \omega^2)}$$

$$I[x(t)] = \frac{eE}{m\omega^2} \left[\frac{1}{v^4} - \frac{1}{v^4 + \omega^2} \right]$$

$$I[x(t)] = \frac{eE}{m\left(\frac{e^2 h^2}{m^2}\right)} \left[\frac{1}{v^4} - \frac{1}{v^4 + \omega^2} \right]$$

$$I[x(t)] = \frac{E}{h\left(\frac{eh}{m}\right)} \left[\frac{1}{v^4} - \frac{1}{v^4 + \omega^2} \right]$$

$$I[x(t)] = \frac{E}{h\,\omega} \left[\frac{1}{v^4} - \frac{1}{v^4 + \omega^2} \right] \quad \ldots.. \qquad (17)$$

After applying the inverse Iman transform bilaterally, Equation (17) produces the results shown below:

$$x(t) = \frac{E}{h\omega} [1 - \cos \omega t] \quad \ldots.. \qquad (18)$$

Similarly, we can calculate the value of $y(t)$, as illustrated below.

$$I[y(t)] = \frac{e^2 E h}{m^2} \frac{1}{v^4 (v^6 + \omega^2 v^2)}$$

$$I[y(t)] = \frac{e^2 E h^2}{h m^2 \omega^3} \left[\frac{\omega}{v^6} - \frac{\omega}{v^2 (v^4 + \omega^2)} \right]$$

$$I[y(t)] = \frac{e^2 E h^2}{h m^2 \omega^3} \left[\frac{\omega}{v^6} - \frac{\omega}{v^2 (v^4 + \omega^2)} \right]$$

$$I[y(t)] = \frac{E}{h\omega} \left[\frac{\omega}{v^6} - \frac{\omega}{v^2 (v^4 + \omega^2)} \right] \ldots\ldots \qquad (19)$$

After applying the inverse Iman transform bilaterally, Equation (19) produces the results shown below:

$$y(t) = \frac{E}{h\omega} [\omega t - \sin \omega t]. \quad \ldots\ldots \qquad (20)$$

The findings of equations (10) and (12) of the Upadhyaya transform and equations (18) and (20) of the Iman transform are comparable.

IV. CONCLUSION

In this paper, we effectively generalized the differential equations to investigate the motion of an electron in a physical system using the Upadhyaya transform and the Iman transform technique. It clearly demonstrates that when both of these methodologies are applied to any physical system problem, the result is equivalent. Both methods are powerful and efficient for solving problems in many physical systems in science and engineering.

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