MEAN CORDIAL LABELING OF BISTAR GRAPHS

Abstract

Let f be a mapping from V (G) to $\mathbb{Z}_2$. Give a numerical value (label) from {0, 1, 2} for every edge uv. f is called a mean- cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1, \forall\, i, j \in \mathbb{Z}_2$ where $v_f(a)$ and $e_f(a)$ represent the number of vertices and edges respectively labeled as a (a = 0,1,2). A mean cordial graph has mean-cordial labels. This study examines the mean-cordial labeling pattern of bistar graphs $B_{n,n}$.

Keywords: mean-cordial labeling, bistar graph.

Authors

Ansari Saima
Assistant professor
Maharashtra College
Mumbai.
saimaansari87@gmail.com

I. INTRODUCTION

The graphs under consideration are simple, finite, and undirected. $V(G)$ and $E(G)$ represent the set of vertices and set of edges of a graph G, respectively. The number of elements of $V(G)$ and $E(G)$ is called the order and size of G respectively. Tracking system, route design, broadband networks, astrographs, and coding-encoding are just a few of the fields where labeled graphs are used (Gallian, 2022)[4]. Cahit introduced the cordial labeling concept in 1987 (Cahit, 1987).

Definition 1: "A graph G is called bipartite if its vertex set V can be partitioned into two disjoined subsets V_1 and V_2 such that $V = V_1 \cup V_2$ and every edge of G has one end vertex in V_1 and one end vertex in V_2." (Bipartite Graphs, n.d.).

Definition 2: A graph G is said to be complete bipartite if it is bipartite and if every vertex of V_1 is adjacent to every vertex of V_2. If the number of vertices in V_1 is m and number of vertices in V_2 is n then $K_{m,n}$ represents the complete bipartite graph. (S. Khunti et al. , 2020).

Definition 3: Let $f : V(G) \rightarrow \mathbb{Z}_2$ be a function. Give a numerical value (label) $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ for every edge uv. f is called a mean-cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1, \forall\ i, j \in \mathbb{Z}_2$ where $v_f(a)$ and $e_f(a)$ represent the number of vertices and edges respectively labeled as $a\ (a = 0,1,2)$. A graph that admits mean-cordial labeling is called a mean-cordial graph (Ponraj et al., 2012).

If we reduce the codomain of f to $\{0,1\}$ then this definition becomes the definition of product cordial labeling. M. Sundaram, R. Ponraj, and Somasundaram established the concept of cordial product labelling (Sundaram et al., 2004).

I examine the mean cordial labeling pattern of bistar graphs. The sign $\lfloor x \rfloor$ denotes the largest integer less than or equal to x, and $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. Terminologies not described in this paper are used in Harary's context (Book 1 (Harary).Pdf, n.d.).

Definition 4: "A Bistar graph is the graph obtained by joining the centre (apex) vertices of two copies of $K_{1,n}$ by an edge and it is denoted by $B_{n,n}$." (S. Khunti et al., 2020).
Here $|V(B_{n,n})| = 2n + 2$ and $|E(B_{n,n})| = 2n + 1$.
Example 1:

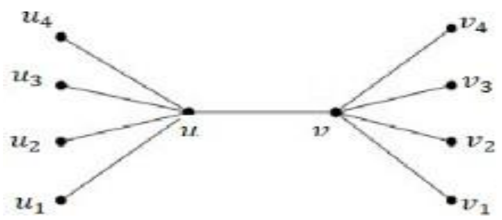

Figure 1: Bistar graph $B_{4,4}$

Definition 5: Let f be a mapping from $V(G)$ to $\mathbb{Z}_2$. For every edge uv of G, nominate the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ or $\left\lfloor \frac{f(u)+f(v)}{2} \right\rfloor$. f is called a mean-cordial labeling of G if $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$, $\forall\ i, j \in \mathbb{Z}_2 = \{0,1,2\}$ where $v_f(a)$ and $e_f(a)$ represent the number of vertices and edges respectively labeled as a $(a = 0,1,2)$. An illustration using mean-cordial labeling is referred to as a mean-cordial graph.

II. MAIN RESULT

Theorem: Let $B_{n,n}$ be a bipartite graph, then $B_{n,n}$ admits mean cordial labeling $\forall n \in$.

Proof. Define $f : V(G) \to \mathbb{Z}_2$ and $g : E(G) \to \mathbb{Z}_2$.

Case 1: $n \equiv 0 \pmod 3$: That is, $n = 3k$, where k is positive integer.
Define vertex labeling as follows: $f(u) = 0$; $f(v) = 2$

$$f(u_i) = \begin{cases} 1 & 1 \le i \le k \\ 0 & k+1 \le i \le 2k \\ 2 & 2k+1 \le i \le 3k. \end{cases}$$

$$f(v_i) = \begin{cases} 1 & 1 \le i \le k \\ 2 & k+1 \le i \le 2k \\ 0 & 2k+1 \le i \le 3k. \end{cases}$$

Define edge labeling as follows:

$$g(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$$

$$g(uu_i) = \left\lfloor \frac{f(u)+f(u_i)}{2} \right\rfloor \qquad 1 \le i \le 3k,$$

$$g(vv_i) = \left\lceil \frac{f(v)+f(v_i)}{2} \right\rceil \qquad 1 \le i \le 3k$$

In this case, we have $v_f(0) = v_f(2) = 2k+1$, $v_f(1) = 2k$ and $e_g(0) = e_g(2) = 2k$, $e_g(1) = 2k+1$.

Example 2. Figure 2 represents the mean cordial labeling of bistar graph $B_{3,3}$.

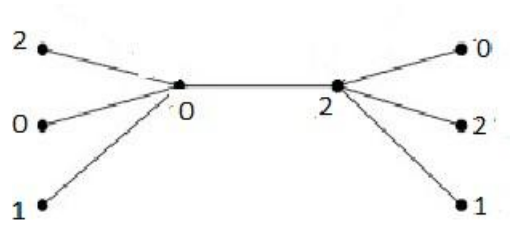

Figure 2: Bistar graph $B_{3,3}$

Case 2: $n \equiv 1 \pmod 3$: That is, $n = 3k+1$, where k is positive integer.
Define vertex labeling as follows: $f(u) = 0$; $f(v) = 2$

$$f(u_i) = \begin{cases} 1 & 1 \le i \le k \\ 0 & k+1 \le i \le 2k \\ 2 & 2k+1 \le i \le 3k. \end{cases}$$

$$f(v_i) = \begin{cases} 1 & 1 \le i \le k \\ 2 & k+1 \le i \le 2k \\ 0 & 2k+1 \le i \le 3k. \end{cases}$$

$$f(u_{3k+1}) = 1 \ ; \quad f(v_{3k+1}) = 1$$

Define edge labeling as follows:

$g(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$

$g(uu_i) = \left\lceil \frac{f(u)+f(u_i)}{2} \right\rceil \quad 1 \le i \le 3k+1,$

$g(vv_i) = \left\lceil \frac{f(v)+f(v_i)}{2} \right\rceil \quad 1 \le i \le 3k+1$

In this case, we have $v_f(0) = v_f(2) = 2k+1$, $v_f(1) = 2k+2$ and $e_g(0) = e_g(2) = e_g(1) = 2k+1$

Example 3. Figure 3 represents the mean cordial labeling of bistar graph $B_{4,4}$.

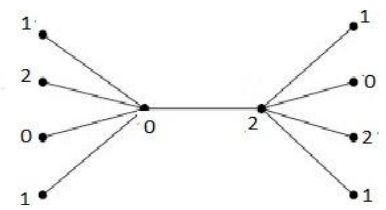

Figure 3: Bistar graph $B_{4,4}$

Case 3: $n \equiv 2 (\mathrm{mod}\ 3)$: That is, $n = 3k+2$, where k is positive integer.
Define vertex labeling as follows: $f(u) = 0$; $f(v) = 2$

$$f(u_i) = \begin{cases} 1 & 1 \le i \le k \\ 0 & k+1 \le i \le 2k \\ 2 & 2k+1 \le i \le 3k. \end{cases}$$

$$f(v_i) = \begin{cases} 1 & 1 \le i \le k \\ 2 & k+1 \le i \le 2k \\ 0 & 2k+1 \le i \le 3k. \end{cases}$$

$$f(u_{3k+1}) = 1 \ ; \ f(v_{3k+1}) = 1$$
$$f(u_{3k+2}) = 0 \ ; \ f(v_{3k+2}) = 2$$

We define edge labeling as follows:

$g(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$

$g(uu_i) = \left\lceil \frac{f(u)+f(u_i)}{2} \right\rceil \quad 1 \le i \le 3k+2,$

$g(vv_i) = \left\lceil \frac{f(v)+f(v_i)}{2} \right\rceil \quad 1 \le i \le 3k+2$

In this case, we have $v_f(0) = v_f(1) = v_f(2) = 2k+2$ and $e_g(0) = e_g(2) = 2k+2$, $e_g(1) = 2k+1$

Example 4. Figure 2 represents the mean cordial labeling of bistar graph $\square_{5,5}$.

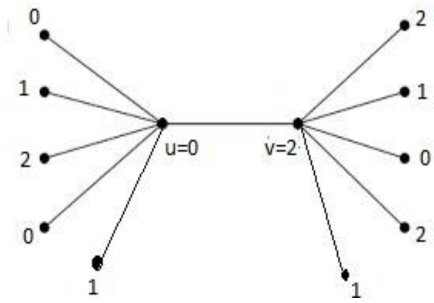

Figure 4: Bistar graph $B_{5,5}$

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