# EXPLORING SOME NEUTROSOPHIC STATISTICAL DISTRIBUTIONS: A COMPREHENSIVE REVIEW OF METHODOLOGY

**Abstract**

Neutrosophic statistics represent a pioneering and innovative approach to statistics, introducing a unique paradigm that accommodates indeterminate and vague information. This emerging discipline extends traditional statistical methods by incorporating neutrosophic logic, which deals with the indeterminacy arising from incomplete or uncertain data. The applications of neutrosophic statistics are diverse and impactful. Neutrosophic statistics provide a robust framework for decision support systems, enabling more informed and resilient decision-making in ambiguous situations. Neutrosophic statistics accommodates these uncertainties, offering a more realistic representation of the underlying data and improving the reliability of analytical results. This chapter delves into various neutrosophic statistical basic concepts, exploring their relevance in situations where traditional discrete and continuous distributions fall short in addressing incomplete or uncertain data.

**Keywords:** Neutrosophic random variables, descriptive measures, discrete distributions, continuous distributions.

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## I. INTRODUCTION

Neutrosophic statistics involves the analysis of a dataset where the data, or a segment of it, exhibits a certain degree of indeterminacy. The field employs techniques for scrutinizing such data, marking a departure from classical statistics, where all data points are definitively determined. Notably, when the level of indeterminacy is zero, neutrosophic statistics aligns with classical statistics. The application of the neutrosophic measure facilitates the quantification of indeterminate data. Through neutrosophic statistical methods, researchers gain the capacity to interpret and organize data that may harbor indeterminacies, unveiling latent patterns. Various approaches within neutrosophic statistics exist, with examples presented and subsequent generalizations made for classes of examples. Importantly, practitioners have the creative freedom to devise novel approaches to exploring neutrosophic statistics. It is crucial to underscore, akin to neutrosophic probability, that indeterminacy should not be conflated with randomness, unlike classical statistics, which solely addresses randomness, neutrosophic statistics account for both randomness and, notably, indeterminacy.

A conventional neutrosophic number[12] is conventionally expressed in the standard form: $a + bI$, where 'a' and 'b' denote real or complex coefficients, and 'I' represents indeterminacy, with the conditions $0 \cdot I = 0$ and $I^2 = I$. Consequently, it follows that $In = I$ for all positive integers 'n'. If both coefficients 'a' and 'b' are real, the expression $a + bI$ is termed a neutrosophic real number. Neutrosophic Random Numbers can also be generated by employing a pool of sets, rather than relying solely on crisp numbers. In this scenario, there are 100 balls, each labelled with an interval $[a, b]$, where $a$ and $b$ both belong to the set {1, 2, 3… 100}, with $a$ being less than or equal to $b$. If $a$ equals $b$, a crisp number $[a, a]$ is obtained, simplifying to $a$. However, if $a < b$, a set $[a, b]$ is generated. The process involves randomly extracting a ball, noting its interval, and then returning it to the pool. This iterative procedure results in a random sequence of intervals, diverging from the conventional approach of obtaining a random sequence of crisp numbers.

Neutrosophic probability [12] is situated within the broader context of the neutrosophic measure and involves estimating the likelihood of an event, distinct from indeterminacy, occurring, along with an acknowledgment that some degree of indeterminacy may also be present, and an estimation that the event will not occur. Neutrosophic probability serves as a generalization of classical probability, coinciding with it when the chance of determinacy in a stochastic process is zero. The neutrosophic probability for the occurrence of an event A is denoted as NP(A) = (ch(A), ch(indetA), ch($\underline{A}$)) = (T, I, F), where T, I, and F are subsets of [0, 1]. Here, T represents the chance of A occurring, I is the indeterminate chance related to A, and F is the chance that A does not occur. This formulation makes NP a generalization of Imprecise Probability as well. When T, I, and F are crisp numbers, the condition - $0 <= T + I + F <= 3^+$ holds, utilizing the same notations (T, I, F) as in neutrosophic logic and set theory.

## II. NEUTROSOPHIC UNIVARIATE AND MULTIVARIATE DATA

Neutrosophic data [12], encompassing data with inherent indeterminacy, undergoes classification similar to classical statistics. In the third person perspective, it is classified into discrete neutrosophic data, featuring isolated values like $8 + i_1$ (where $i_1 \in [0, 1]$), 12, and 21

+ $i_2$ (where $i_2 \in [3, 5]$). Alternatively, it can be continuous neutrosophic data, forming one or more intervals such as [0, 0.7] or [0.1, 1.0], indicating uncertainty about the exact value. Another categorization involves quantitative (numerical) neutrosophic data, exemplified by numbers within intervals like [1, 6], 23, 34, 45, or 73 where the exact values are unknown. On the qualitative (categorical) side, examples include color categories like blue or red (with uncertainty), and white, black, green, or yellow, where exact categorization is uncertain. A Neutrosophical statistical number N takes on the form: N=d+I, where d represents the determinate (sure) part of N, and i denotes the indeterminate part of N. For instance, consider the expression N=8+0.3I, where the determinate part is 8, ensuring certainty and the indeterminate part implies the possibility that the number may be slightly larger than 8. Furthermore, Neutrosophic data can be univariate, comprising observations on a single attribute, or multivariable, involving observations on two or more attributes. Specific cases include bivariate and tri-variate neutrosophic data.

## III. NEUTROSOPHIC DESCRIPTIVE MEASURES

Neutrosophic descriptive statistics[12] encompasses a range of techniques used to summarize and describe the characteristics of neutrosophic numerical data. Given the inherent indeterminacies in neutrosophic numerical data, representations such as neutrosophic line graphs and neutrosophic histograms are portrayed in three-dimensional spaces, deviating from the conventional two-dimensional spaces employed in classical statistics. The introduction of a third dimension, alongside the Cartesian System of two axes, accounts for indeterminacy (I). From these ambiguous graphical data displays, one can derive neutrosophic (unclear) information.

A Neutrosophic frequency distribution[12] is a tabular representation that showcases categories, frequencies, and relative frequencies, incorporating uncertainties arising from imprecise, incomplete, or unknown data associated with frequency. In such instances, the relative frequency is affected, becoming imprecise, incomplete, or unknown. When calculating the total for neutrosophic frequencies involving imprecise information, the minimum and maximum of estimated frequencies are computed. Similarly, when determining the neutrosophic relative frequency, the minimum and maximum values are considered across all potential scenarios. The neutrosophic relative frequencies are simply accumulated as the sum of intervals.

For discrete RV, the arithmetic mean of the neutrosophic RV is $\frac{\sum_{j=1}^{n} a_j}{n} + \frac{\sum_{j=1}^{n} b_j}{n} I$ ,if $a_j + b_j I$, for all $j \in \Re$ where $a$, $b$ are real numbers, and $I$ is indeterminacy, such that $I^2 = I$ and $0 \cdot I = 0$. For continuous RV, the arithmetic mean of the neutrosophic RV is $[\frac{\sum_{j=1}^{n} a_j}{n}, \frac{\sum_{j=1}^{n} b_j}{n}]$ ,if $(a_j, b_j)$, for all $j \in \Re$ where $a$, $b$ are real numbers(not sure which one). When determining the standard deviation, one computes the square root of a neutrosophic number, representing the result as $x + yI$. The values of $x$ and $y$ are then determined, contributing to the establishment of the neutrosophic standard deviation. The neutrosophic quartiles are defined similarly to classical statistics: the first quartile (lower quartile) is determined as the 1/4 $(n + 1)$ $th$, the second quartile as the 2/4 $(n + 1)th$, and the third quartile as the 3/4 $(n + 1)th$. In cases where $(n + 1)$ is not divisible by 4, the average of the two neutrosophic observations, whose ranks the quartile falls between, is taken. Another approach involves considering the inferior integer part of $i/4\ (n + 1)$, where $i = 1, 2, 3$. The neutrosophic mode involves identifying the neutrosophic value or values that have the highest frequency or occurrence, considering the inherent uncertainties.

## IV. NEUTROSOPHIC PROBABILITY DISTRIBUTIONS

In this section, the discrete and continuous distributions have been discussed in the presence of a neutrosophic scenario. The conventional probability distributions are relevant when a sample is drawn from a population with uncertain observations. However, it becomes crucial to incorporate probability models in situations characterized by indeterminacy [11]. Various authors have proposed neutrosophic probability distributions to address this need. Some Discrete neutrosophic probability distributions [8], some continuous neutrosophic probability distributions [9], Neutrosophic Weibull [3], Neutrosophic Uniform, Neutrosophic Exponential, and Neutrosophic Poisson [2] have been studied. Additionally, [12] presented Normal and binomial distributions, [9] introduced the Neutrosophic Rayeigh distribution, and [13] proposed the Neutrosophic Beta distribution.

### *1. Neutrosophic Uniform Discrete Distribution(NUDD)*

Consider the neutrosophic random variable $X_N$, has neutrosophic uniform discrete distribution denoted as $X_N$ ~ uniform{$x_{I,\ldots\ldots,}$ $x_n$} or $X$ ~ uniform{$x_{I\ +\ I,\ldots\ldots,}$ $x_{n+1}$} if the probability that $X_N$ takes any value in constant, specifically $\frac{1}{n+I}$. Neutrosophic probability function(PDF) is given by

$$f_X(x-1) = \begin{cases} \frac{1}{n+I}\ , & if\ x = x_1 + I, x_2 + I, \ldots\ldots, x_n + I. \\ 0\ , & otherwise. \end{cases}$$

***Properties***

- The mean and variance are given by $E(X_N) = \mu$ and
  $Var(X_N) = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu)$

### *2. Neutrosophic Bernoulli distribution(NBD)*

Let $X_N$ represent a neutrosophic random variable which follows a neutrosophic Bernoulli distribution, represented as $XN$~Ber($p_N$), where $p_N$ is a set with one or more elements (which may be an interval). The neutrosophic probability function [8] is defined as follows:

$$f_X(x-1) = \begin{cases} p_N^{x-I}(1-p_N)^{1-x+I}. & if\ x = I, 1+I. \\ 0\ , & otherwise\ . \end{cases}$$

***Properties:***

- The mean and variance is given by

$E(X_N) = p_N + I \, and \, Var(X_N) = p_N(1 - p_N)$

- Let $X_N$ be a neutrosophic random variable with neutrosophic Bernoulli distribution and let a,b∈ ℝ and a≠0. Consider RV, $Y_N = aX_N + b$, then the neutrosophic density function of $Y_N$ is given by

$$f_X(x-1) = \begin{cases} p_N^{(y-b-I)/a}(1-p_N)^{1-[(y-b-I)/a]}. \; if \; y = b+I, a+b+I. \\ 0 \;, \qquad otherwise \;. \end{cases}$$

and its mean is $E(Y_N) = ap + b + I$.

### 3. *Neutrosophic Binomial Distribution(NBD)*

Let $X_N$ be a neutrosophic random variable which is neutrosophic binomial distribution denoted by $X_N \sim$ bin($n$, $p_N$) where $p_N$ is set with one or more elements (may $p_N$ be an interval). Neutrosophic probability function[8] is given by

$$f_X(x-1) = \begin{cases} \binom{n}{x-I} p_N^{x-1}(1-p_N)^{n-x+I} & , if \; x = I, 1+I, 2+I, \dots\dots, n+I. \\ 0, \qquad otherwise. \end{cases}$$

***Properties:***

- The mean and variance is given by $E(X_N) = np_N + I$ and $Var(X_N) = np_N(1 - p_N)$.
- Let $X_N$ and $Y_N$ be two independent neutrosophic random variables with neutrosophic distribution bin(n,$p_N$) and bin(m,$p_N$) respectively. Then, $X_N$ + $Y_N$ ~ bin(n+m,$p_N$).
- Let $X_N$ be a neutrosophic random variables with $X_N$ ~ bin(n,$p_N$), then n-$X_N$ ~ bin(n,$1 - p_N$).

### 4. *Neutrosophic Geometric Distribution(NGD)*

Let $X_N$ be a neutrosophic random variable which has neutrosophic geometric distribution denoted by $X_N$~ $geo(p_N)$ where $p_N$ is set with one or more elements(may $p_N$ be an interval). The neutrosophic probability function[8] is given by

$$f_X(x-I) = \begin{cases} p_N(1-p_N)^{x-I}, if \; x = I, 1+I, 2+I, \dots., n+I \\ 0, \qquad otherwise \end{cases}$$

***Properties:***

- The mean and variance is $E(X_N) = \frac{1-p_N}{p_N} + I$ and $Var(X_N) = \frac{1-p_N}{p_N^2}$ respectively
- Let $Y_N$= 1+$X_N$ be a neutrosophic random variable where $X_N$ has neutrosophic geometric distribution. Then, the neutrosophic density function of $Y_N$ , mean and variance are given by

(i) $f_Y(y-I) = \begin{cases} p_N(1-p_N)^{y-I-1}, if \; y = 1+I, 2+I, \dots., n+I. \\ 0, \qquad otherwise \end{cases}$

(ii) $E(Y_N) = \frac{1}{p_N} + I$ *and* $Var(Y_N) = \frac{1-p_N}{p_N^2}$

### 5. *Neutrosophic Negative Binomial Distribution(NNBD)*

Let $X_N$ be a neutrosophic random variable with neutrosophic negative binomial distribution[8] denoted by $X_N \sim$ binneg($r_N$ , $p_N$) where $r_N$ and $p_N$ are sets with one or more elements(may $r_N$ and $p_N$ be an interval). The neutrosophic probability function is given by

$$f_X(x-I) = \begin{cases} \binom{r_N + x - I - 1}{x - I} p_N^{r_N} (1-p_N)^{x-I}, if\ x = I, 1+I, 2+I, \ldots., n+I \\ 0, \quad otherwise \end{cases}$$

***Properties:***

- The mean and variance is given $E(X_N) = r_N \frac{1-p_N}{p_N} + I$ and $Var(X_N) = r_N \frac{1-p_N}{p_N^2}$.
- Consider $Y_N = r_N + X_N$ be a neutrosophic random variable such that $X_N \sim binneg(r_N , p_N)$. Then the neutrosophic density function, mean and variance are given, respectively

(i) $f_Y(y-I) = \begin{cases} \binom{y-1-I}{y-I-r_N} p_N^{r_N} (1-p_N)^{y-I-r_N}, if\ y = r_N + I, r_N + 1 + I \\ 0, \quad otherwise \end{cases}$

(ii) $E(Y_N) = \frac{r_N}{p_N} + I$ and $Var(Y_N) = r_N \frac{1-p_N}{p_N^2}$.

### 6. *Neutrosophic Hypergeometric Distribution(NHGD)*

Let $X_N$ be a neutrosophic random variable that has neutrosophic hypergeometric distribution[8] denoted by $X_N \sim$ hypergeo($N_N$ , $K_N$, $n$) where $N_N$ and $K_N$ are sets with one or more elements(may $N_N$ and $K_N$ be intervals). The Neutrosophic PDF is given by

$$f_X(x-I) = \begin{cases} \frac{\binom{K_N}{x-1}\binom{N_N-K_N}{n-x-1}}{\binom{N_N}{n}}, \quad if\ x = I, 1+I, 2+I, \ldots., n+I \\ 0, \quad otherwise \end{cases}$$

***Property:***

- The mean and variance is given by $E(X_N) = n\frac{K_N}{N_N} + I$ and $\mathrm{Var}(X_N) = n\frac{K_N}{N_N}\frac{N_N-K_N}{N_N}\frac{N_N-n}{N_N-1}$

### 7. *Neutrosophic POISSON DISTRIBUTION(NPD)*

Let $X_N$ denote a neutrosophic random variable, which follows neutrosophic Poisson distribution[8] denoted by $X_N \sim$Poisson($\lambda_N$) where $\lambda_N$ is set with one or more elements (which may be an interval)[2]. The Neutrosophic PDF is given by

$$f_X(x-I) = \begin{cases} e^{-\lambda_N} \dfrac{\lambda_N^{x-I}}{(x-I)!}, & \text{if } x = I, 1+I, 2+I, \dots, n+I. \\ 0, & \textit{otherwise.} \end{cases}$$

***Property:***

- The mean and variance is given by $E(X_N) = \lambda_N + I$ and $\mathrm{Var}(X_N) = \lambda_N$
- Consider two independent neutrosophic random variables $X_N$ and $Y_N$, characterized by neutrosophic distributions Poisson ($\lambda_{N1}$) and Poisson ($\lambda_{N2}$) respectively. Then, $X_N + Y_N$ ~ Poisson ($\lambda_{N1} + \lambda_{N2}$)

### 8. *Neutrosophic Uniform Distribution*

The Neutrosophic Uniform distribution in neutrosophic random variables for a neutrosophic continuous variable, denoted as $X_N$, is an extension of the classical Uniform distribution. However, in this context, the distribution parameters a, b, or both are imprecise and determined by an indeterminacy factor I ∈ [0, 1]. For instance, a, b, or both may be sets containing two or more elements (potentially intervals), with the constraint that a < b.

### 9. *Neutrosophic Normal Distribution*

Let $X_N$ be a neutrosophic random variable, has neutrosophic normal distribution denoted by $X_N \sim N_N\ (\mu_N, \sigma_N^2)$ where $\mu_N$ and $\sigma_N^2$ are sets with one or more elements. Neutrosophic probability function[13] is given by

$$f_X(x-I) = \frac{1}{\sigma_N\sqrt{2\Pi}} \exp\left(-\frac{(x-\mu_N)^2}{2\sigma_N^2}\right)$$

### 10. *Neutrosophic Exponential Distribution(NED)*

The neutrosophic exponential distribution[2] is formulated as an extension of the classical exponential distribution. This distribution for neutrosophic random variables accommodates all data, including non-specific cases. The density function is articulated as:

$$f_X(x-I) = \begin{cases} \lambda_M e^{-(x-I)\lambda_M}; I < x < \infty \\ 0, \qquad otherwise \end{cases}$$

***Properties:***

- The mean and variance of the NED is $E(X_N) = \frac{1}{\lambda} + I$ and $Var(X_N) = \frac{1}{(\lambda)^2}$
- The area under the curve characterized by the NED is one.
- The neutrosophic reliability function of the NED is $[exp(-x\lambda_L), exp(-x\lambda_U)] = exp(-x\lambda_N)$.
- The hazard function $h_N(X)$ of the NED is $\lambda_N$.

 

- The median is given by $M_N = \left[\frac{\ln(2)}{\lambda_U}, \frac{\ln(2)}{\lambda_L}\right]$

### 11. *Neutrosophic Gamma Distribution*

Consider $X_N$ be a neutrosophic random variable which has neutrosophic gamma distribution with parameters $\alpha > 0$ and $\lambda > 0$ where $\alpha$ *or* $\lambda$ or both are sets with two or more elements (may α or λ or both are intervals), the neutrosophic PDF is defined as follows:

$$f_X(x-I) = \lambda e^{-\lambda(x-I)} \frac{(\lambda(x-I))^{\alpha-1}}{\Gamma(\alpha)}, if\ x \geq \text{I}.$$

***Property:***

- The mean of the density is $E(X_N) = \frac{\alpha}{\lambda} + I$
- Let $X_N$ with neutrosophic gamma RV's and $c > 0 \in \mathbb{R}$, then $cX_N \sim$ Ngamma($\alpha$, $\lambda/c$).

### 12. *Neutrosophic Beta Distribution*

The beta distribution [6] of the first kind and second kind comprises a group of continuous probability distributions defined on the intervals [0, 1] and [0, ∞] respectively, characterized by two positive shape parameters denoted as α and β. These parameters serve as exponents for the random variable, influencing the overall shape of the distribution. Commonly, the fundamental distribution is referred to as the Beta distribution of the first kind, while the secondary beta distribution is identified as that of the second kind.

***Neutrosophic Beta Distribution of First Kind:*** Let $X_N$ be a neutrosophic random variable that has a neutrosophic beta distribution[8] of first kind with parameters a > 0 and b > 0 where a or b or both are sets with two or more elements (may a or b or both are intervals), The neutrosophic PDF is defined as follows:

$$f_X(x-I) = \frac{1}{B(a,b)}(x-I)^{a-1}(1+I-x)^{b-1}, if\ I \leq x \leq 1+I$$

where, $B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$

***Property:***

- The first order moment is $E(X_N) = \frac{a}{a+b} + I$
- Let $X_N$ and $Y_N$ be two independence neutrosophic gamma($a$, $\lambda$) and gamma($b$, $\lambda$), RVs respectively. Then, $\frac{X_N}{X_N + Y_N} \sim Nbeta(a,b)$

***Neutrosphic Beta Distribution of Second Kind:*** Let X be a classical random variable characterized by a neutrosophic beta distribution of second kind[12] with neutrosophic parameters $\alpha_N$ and $\beta_N$, denoted as X → N-beta($x$; $\alpha_N$, $\beta_N$). In this case, the PDF and CDF are expressed as follows:

$$f_N(X=x)=f(x)=\begin{cases}\frac{X^{\alpha_N-1}(1-X)^{\beta_N-1}}{B(\alpha_N,\beta_N)},\ x>0\\ 0,\ otherwise\end{cases}$$ and

$$F_N(X)=I_X(\alpha_N,\beta_N)=\frac{B(X,\alpha_N,\beta_N)}{B(\alpha_N,\beta_N)}$$

where $\alpha_N$, $\beta_N$ are the neutrosophic shape parameters.

***Properties:***

- The rth non-central moments is given by
$$\mu_r'=E_N(X^r)=\prod_{i=0}^{r-1}\frac{(\alpha_N+i)}{(\alpha_N+\beta_N+i)}$$
- The mean and variance is $\mu_1=\frac{\alpha_N}{(\alpha_N+\beta_N)}$ & $\mu_2=\frac{\alpha_N\beta_N}{(\alpha_N+\beta_N+1)(\alpha_N+\beta_N)^2}$
- Hazard rate and survival functions are given as
$h_N(X)=\frac{X^{\alpha_N-1}(1-X)^{\beta_N-1}}{B(\alpha_N,\beta_N)-B(X,\alpha_N,\beta_N)}$ and $S_N(X)=1-\frac{B(X,\alpha_N,\beta_N)}{B(\alpha_N,\beta_N)}$
- The rth order statistic is give by
$$f_{N_{r,n}}(x)=\frac{1}{B(r,n-r-1)}\left[\frac{B(X,\alpha_N,\beta_N)}{B(\alpha_N,\beta_N)}\right]^{r-1}\left[1-\frac{B(X,\alpha_N,\beta_N)}{B(\alpha_N,\beta_N)}\right]^{n-r}\frac{X^{\alpha_N-1}(1-X)^{\beta_N-1}}{B(\alpha_N,\beta_N)}$$

### *13. Neutrosophic Weibull Distribution*

A Neutrosophic Weibull distribution[3] (NBD) for a continuous variable X is essentially a conventional Weibull distribution for X, except that its parameters α or β are considered indeterminate. The PDF and CDF for this distribution is then expressed as follows:

$$f(X,\alpha_N,\beta_N)=\begin{cases}\frac{\beta_N}{\alpha_N^{\beta_N}}X^{\beta_N-1}e^{-\left(\frac{X}{\alpha_N}\right)^{\beta_N}} & ;X,\alpha\ and\ \beta>0\\ 0, & otherwise\end{cases}$$ and

$$F(X,\alpha_N,\beta_N)=1-e^{-\left(\frac{X}{\alpha_N}\right)^{\beta_N}};\ X,\alpha\ and\ \beta>0$$

where $\alpha_N\ and\ \beta_N$ are the scale and shape parameter.

***Properties:***

- The r[th] moment about mean is $\alpha_N^r \Gamma\left(\beta_N + \frac{r}{\beta_N}\right)$
- The mean and variance are

  $E_N(X) == \alpha_N \Gamma\left(\frac{\beta_N+2}{\beta_N}\right)$ and $V_N(X) == \alpha_N^2 \left[\Gamma\left(\frac{\beta_N+2}{\beta_N}\right)\right] - \left[\Gamma\left(\frac{\beta_N+2}{\beta_N}\right)\right]^2$

- The reliability or survival function and hazard function are given as
  $F_N(X) = e^{-\left(\frac{X}{\alpha_N}\right)^{\beta_N}}$ $and\ h_N = \beta_N X^{\beta_N - 1} X^{\left(\beta_N - 1/\alpha_N\right)^{\beta_N}}$

***14. Neutrosophic Rayleigh Distribution***

Consider continuous random variable X, the Neutrosophic Rayleigh distribution(NRD) of a having indeterminacy in the scale parameter $\theta_N$[5]. Then the PDF and CDF is given by

$$f(X, \theta_N) = \begin{cases} \frac{X}{\theta_N^2} e^{-\frac{1}{2}\left(\frac{X}{\theta_N}\right)^2} ; X, \theta_N > 0 \\ 0, \qquad otherwise \end{cases}$$ and

$$F(X, \theta_N) = 1 - e^{-\frac{1}{2}\left(\frac{X}{\theta_N}\right)^2} ;\ X, \theta_N > 0$$

***15. Neutrosophic Pareto Distribution***

Let X be a continuous random variable is said to be Neutrosophic Pareto Distribution (NPD), with neutrosophic shape and scale parameters $\alpha_N$ and $\theta_N$ respectively, then the PDF and CDF is given by

$$f(X, \alpha_N, \theta_N) = \begin{cases} \frac{\alpha_N \theta_N^{\alpha_N}}{X^{\alpha_N+1}} ; X, \alpha_N\ and\ \theta_N > 0 \\ 0; otherwise \end{cases}$$ and

$$F(X, \alpha_N, \theta_N) = 1 - \left(\frac{\theta_N}{X}\right)^{\alpha_N} ;\ X, \alpha_N\ and\ \theta_N > 0$$

***Properties:***

- The mean and variance of the density is $E(X) = \frac{\alpha_N \theta_N}{\alpha_N} - 1$ and
  $V(X) = \frac{\alpha_N \theta_N^2}{(\alpha_N - 1)^2 (\alpha_N - 2)}$ for $\alpha_N > 2$

- The median is given by $2^{\frac{1}{\alpha_N}} \theta_N$

- The jth moment is given by $\frac{\alpha_N \theta_N^j}{(\alpha_N - j)}$

- The coefficient of skewness is given by $\left(2(\alpha_N + 1)/(\alpha_N - 3)\sqrt{\alpha_N - {}^{2}/_{\alpha_N}}\right)$

- The coefficient of kurtosis is given by $6(\alpha_N^3 + \alpha_N^2 - 6\alpha_N - 2/\alpha_N(\alpha_N - 3)(\alpha_N - 4)$

### *16. Neutrosophic Kumaraswamy Distribution*

The Kumaraswamy (Kw) distribution holds significant importance as a versatile tool for analyzing datasets within the unit interval (0, 1) [11]. Defined by two positive shape parameters, this distribution finds application in diverse fields such as reliability analysis, atmospheric temperatures, test scores, hydrological studies, and economic data. A neutrosophic Kumaraswamy distribution (NKw) of a continuous variable X is a classical Kumaraswamy distribution of x, but such parameters are imprecise. The PDF and CDF of NKw distribution are given as

$$f_N(X = x) = \begin{cases} \alpha_N \beta_N X^{\alpha_N - 1}(1 - X^{\alpha_N})^{\beta_N - 1}, X\epsilon(0,1) \\ 0, otherwise \end{cases}$$

and $F_N(X) = 1 - (1 - X^{\alpha_N})^{\beta_N}$

where $\alpha_N$ and $\beta_N$ are the shape parameters.

***Properties***

- The rth central moments of NKw distribution is $\mu_r' = E_N(X^r) = \beta_N\, B\left[1 + \frac{r}{\alpha_N}, \beta_N\right]$

- The mean and variance of NKW distribution are as follows;

  $\mu_1' = \beta_N\, B\left[1 + \frac{1}{\alpha_N}, \beta_N\right]$ and $\mu_2 = \beta_N\, B\left[1 + \frac{2}{\alpha_N}, \beta_N\right] - \left\{\beta_N\, B\left[1 + \frac{1}{\alpha_N}, \beta_N\right]\right\}^2$

- The Median of NKW distribution is $Q_{0.25} = \left(1 - \frac{1}{\beta_{N\sqrt{2}}}\right)^{\frac{1}{\alpha_N}}$

- The Shannon entropy of NKw distribution is $H_N(X) = (1 - 1/\alpha_N) + (1 + 1/\beta_N)\, H_{\beta N} - \log(\alpha_N \beta_N)$, where Hi is the harmonic number

- The survival function and hazard function for the NKw distribution are formulated as follows:

  $S_N(X) = (1 - X^{\alpha_N})^{\beta_N}$ and $F_N(X) = \frac{\alpha_N \beta_N X^{\alpha_N - 1}}{(1 - X^{\alpha_N})}$

## V. CONCLUSION

Neutrosophic probability distributions, in their exclusive treatment of explicitly stated undetermined values, form the subject of a comprehensive review in this chapter. The analysis encompasses alternatives to classical distributions within the context of neutrosophic random variables, spanning both discrete and continuous scenarios. As a result of this examination, it is concluded that increased attention is warranted for neutrosophic discrete distributions within neutrosophic random variables. The emphasis should be on exploring their inherent properties and their applicability in scenarios involving zero-inflated randomness. Additionally, neutrosophic continuous distributions require further scrutiny, particularly in the areas of an exponentiated family of distribution, truncated distribution, and transmuted map scenarios because of to analyze the lifetime data in uncertainty. A notable gap in the literature is identified concerning the exploration of censored random samples in uncertain environments. Furthermore, persistent challenges are acknowledged in finite mixture cases, both in the discrete and continuous domains of Neutrosophic Random Variables (NRVs). Looking ahead, there is an anticipation of future research delving into unexplored types of neutrosophic distributions in random variables.

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