QUADRIPARTITIONED SINGLE VALUED NEUTROSOPHIC REFINED VOLTERRA SPACES

Abstract

The focus of this paper is to present the concepts of quadripartitioned single valued neutrosophic refined volterra spaces and to investigate their characteristics. The ideas discussed in this paper are illustrated using a few examples.

Keywords: Quadripartitioned single valued neutrosophic refined nowhere dense, Quadripartitioned single valued neutrosophic refined G_δ -set,Quadripartitioned single valued neutrosophic refined F_σ -set, Quadripartitioned single valued neutrosophic refined volterra spaces.

Authors

Arokia Pratheesha S. V
Research Scholar
Nirmala College for Women
Coimbatore, Tamilnadu, India.
pratheesha555@gmail.com

Mohana. K
Assistant Professor
Nirmala College for Women
Coimbatore, Tamilnadu, India.
riyaraju1116@gmail.com

I. INTRODUCTION

L.A. Zadeh [19]was the first to explain fuzzy sets and fuzzy set operations. Fuzzy topological spaces were first introduced and developed by Chang [5]. The earliest publication of the "Intuitionistic fuzzy set" notion was made by Atanassov [1].Fuzzy sets and neurothosophic sets, an expansion of intuitionistic fuzzy sets, were first described by Smarandache[12]. Neutosophic set theory addresses the problem of uncertainty. As an extension of intuitionistic fuzzy sets, fuzzy sets, and the classical set, Wang [17] proposed single-valued neutrosphic sets.Four membership functions make up Chatterjee's quadripartitioned single valued neutrosophic sets: truth, contradiction, unknown, and falsity. Deli et al.'s [6] development of intuitionistic fuzzy multisets and fuzzy multisets was the introduction of neurosophic refined sets.The concept of Volterra spaces has been thoroughly studied in classical topology [4,7,8,9].Thangaraj and Soundararajan[15] introduce and research the idea of fuzzy Volterra space.By Soundararajan, Rizwan, and Syed Tahir Hussainy [14], the idea of intuitionistic fuzzy Volterra space was first suggested and researched.This paper is arranged as follows: quadripartitioned single valued neutrosophic refined nowhere dense set , quadripartitioned single valued neutrosophic refined volterra space and its characteristics is present and some results are made about the functions that preserve this context of images and preimages.

II. PRELIMINARIES

Definition 2.1 [2] A QSVNRS ϱ on Λ can be defined by

$$\varrho=\{\langle\kappa, T^J_\varrho(\kappa), D^J_\varrho(\kappa), Y^J_\varrho(\kappa), F^J_\varrho(\kappa)\rangle:\kappa\in\Lambda\}$$

where$T^J_\varrho(\kappa), D^J_\varrho(\kappa), Y^J_\varrho(\kappa), F^J_\varrho(\kappa):\Lambda\to[0,1]$ such that $0\le T^J_\varrho+D^J_\varrho+Y^J_\varrho+F^J_\varrho\le 4$ (J=1,2,...P) and for every $\kappa\in\Lambda$.$T^J_\varrho(\kappa), D^J_\varrho(\kappa), Y^J_\varrho(\kappa)andF^J_\varrho(\kappa)$ are the truth membership sequence,a contradiction membership sequence,an unknown membership sequence and falsity membership sequence of the element x respectively. P is also referred to as the QSVNRS(ϱ) dimension.

Definition 2.2 [2] Let ϱ, ζ ∈ QSVNRS(Λ) havimg the form

$$\varrho=\{\langle\kappa, T^J_\varrho(\kappa), D^J_\varrho(\kappa), Y^J_\varrho(\kappa), F^J_\varrho(\kappa)\rangle:\kappa\in\Lambda\}\ (J=1,2,...P)$$

$$\zeta=\{\langle\kappa, T^J_\zeta(\kappa), D^J_\zeta(\kappa), Y^J_\zeta(\kappa), F^J_\zeta(\kappa)\rangle:\kappa\in\Lambda\}\ (J=1,2,...P).\text{Then}$$

- $\varrho \,\tilde{\subseteq}\, \zeta$ if $T^J_\varrho(\kappa)\le T^J_\zeta(\kappa)$, $D^J_\varrho(\kappa)\le D^J_\zeta(\kappa)$, $Y^J_\varrho(\kappa)\le Y^J_\zeta(\kappa)$ and $F^J_\varrho(\kappa)\le F^J_\zeta(\kappa)$ (J=1,2,...P)
- $\varrho^{\tilde{c}}=\{\langle\kappa, F^J_\varrho(\kappa), Y^J_\varrho(\kappa), D^J_\varrho(\kappa), T^J_\varrho(\kappa)\rangle:\kappa\in\Lambda\}$ (J=1,2,...P)
- $\varrho\,\tilde{\cup}\,\zeta=\omega_1$ and is defined by
  $T^J_{\omega_1}(\kappa)=\max\{T^J_\varrho(\kappa),T^J_\zeta(\kappa)\}$, $D^J_{\omega_1}(\kappa)=\max\{D^J_\varrho(\kappa),D^J_\zeta(\kappa)\}$, $Y^J_{\omega_1}(\kappa)=\min\{Y^J_\varrho(\kappa),Y^J_\zeta(\kappa)\}$, $F^J_{\omega_1}(\kappa)=\min\{F^J_\varrho(\kappa),F^J_\zeta(\kappa)\}$ for all $\kappa\in\Lambda$ and J=1,2...P.

- $\varrho \tilde{\cap} \zeta = \omega_1$ and is defined by
  $T^J_{\omega_1}(\kappa)=\min\{T^J_{\varrho}(\kappa), T^J_{\zeta}(\kappa)\}$, $D^J_{\omega_1}(\kappa)=\min\{D^J_{\varrho}(\kappa), D^J_{\zeta}(\kappa)\}$, $Y^J_{\omega_1}(\kappa)=\max\{Y^J_{\varrho}(\kappa), Y^J_{\zeta}(\kappa)\}$,
  $F^J_{\omega_1}(\kappa)=\max\{F^J_{\varrho}(\kappa), F^J_{\zeta}(\kappa)\}$ for all $\kappa \in \Lambda$ and J=1,2...P.

Definition 2.3 [2] A QSVNRTS on Λ^* in a family $\mathfrak{T}$ of QSVNRS in Λ^* which satisfy the following axioms.

- $\tilde{\Phi}_{QNR}$, $\tilde{\mathbb{X}}_{QNR} \in \mathfrak{T}$.
- $H_1 \tilde{\cap} H_2 \in \mathfrak{T}$ for any H_1, $H_2 \in \mathfrak{T}$.
- $\tilde{\cup}\, H_i \in \mathfrak{T}$ for every$\{ H_i : i \in I\} \tilde{\subseteq} \mathfrak{T}$.

Here the pair $(\Lambda^*, \mathfrak{T})$ is called a QSVNRTS and any QSVNRS in $\mathfrak{T}$ is said to be quadripartitioned single valued neutrosophic refined open set (QNROS) in Λ^*. The complement of $\varrho^{\tilde{c}}$ of a QNROS ϱ in a QSVNRTS $(\Lambda^*,\mathfrak{T})$ is known as quadripartitioned single valued neutrosophic refined closed set (QNRCS) in Λ^*.

III. QUADRIPARTITIONED SINGLE VALUED NEUTROSOPHIC REFINED VOLTERRA SPACES

Definition 3.1 *A QSVNRS $\mathfrak{U}$ in a QSVNRTS $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is said to be QSVNR dense if there exists no QNRCS $\mathcal{V}$ in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ such that $\mathfrak{U} \tilde{\subset} \mathcal{V} \tilde{\subset} \tilde{\mathbb{X}}_{QNR}$.*

Definition 3.2 *A QSVNRS $\mathfrak{U}$ in a QSVNRTS $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is said to be QSVNR nowhere dense set if there exists no QNROS $\mathfrak{L}$ in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ such that $\mathfrak{L} \tilde{\subseteq}$ QNR$\mathfrak{cl}$($\mathfrak{U}$).That is QNR$\mathfrak{int}$(QNR$\mathfrak{cl}$($\mathfrak{U}$)) = $\tilde{\Phi}_{QNR}$.*

Proposition 3.3 *Let $\mathfrak{U}$ be a QSVNRS.If $\mathfrak{U}$ is a QNRCS in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ with QNR$\mathfrak{int}$($\mathfrak{U}$) = $\tilde{\Phi}_{QNR}$, then $\mathfrak{U}$ is a QSVNR nowhere dense in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$.*

Proof. Let $\mathfrak{U}$ is a QNRCS in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$. Then QNR$\mathfrak{cl}$($\mathfrak{U}$) = $\mathfrak{U}$. Now QNR$\mathfrak{int}$(QNR$\mathfrak{cl}$($\mathfrak{U}$)) = QNR$\mathfrak{int}$($\mathfrak{U}$) = $\tilde{\Phi}_{QNR}$ and hence $\mathfrak{U}$ is a QSVNR nowhere dense $\mathfrak{U}$ in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$.

Proposition 3.4 *Let $\mathfrak{U}$ be a QNRCS in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$,then U is a QSVNR nowhere dense set in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ iff QNR$\mathfrak{int}$(U) = $\tilde{\Phi}_{QNR}$.*

Proof. Let $\mathfrak{U}$ be a QNRCS in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$, with QNR$\mathfrak{int}$($\mathfrak{U}$) = $\tilde{\Phi}_{QNR}$. Then by Proposition 3.3, $\mathfrak{U}$ be QSVNR nowhere dense set $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$. Conversely,Let $\mathfrak{U}$ is a QSVNR nowhere dense set in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$.we have QNR$\mathfrak{int}$(QNR$\mathfrak{cl}$($\mathfrak{U}$)) = $\tilde{\Phi}_{QNR}$ implies that QNR$\mathfrak{int}$($\mathfrak{U}$) = $\tilde{\Phi}_{QNR}$.Since $\mathfrak{U}$ is a QNRCS,QNR$\mathfrak{cl}$(U) = $\mathfrak{U}$.

Proposition 3.5 *If $\mathfrak{U}$ is a QSVNR dense and QNROS in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$,then $U^{\tilde{c}}$ is a QSVNR nowheredense set in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$.*

Proof. Let $\mathfrak{U}$ is a QNROS in ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$),we have QNR$\mathfrak{cl}$($\mathfrak{U}$) = $\widetilde{\mathbb{X}}_{QNR}$. Now QNR$\mathfrak{int}$(QNR$\mathfrak{cl}$($\mathfrak{U}^{\tilde{c}}$)) = (QNR$\mathfrak{cl}$(QNR$\mathfrak{int}$($\mathfrak{U}$)$^{\tilde{c}}$ = (QNR$\mathfrak{cl}$($\mathfrak{U}$)) $^{\tilde{c}}$ = $\widetilde{\Phi}_{QNR}$. Hence $\mathfrak{U}^{\tilde{c}}$ is a QSVNR nowheredense set in ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$).

Proposition 3.6 *If* $\mathfrak{U}$ *be a QSVNR nowhere dense set in (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*), then QNR*$\mathfrak{cl}$*(*$\mathfrak{U}$*) is also a QSVNR nowhere dense set in (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*).*

Proof. Let QNR$\mathfrak{cl}$($\mathfrak{U}$)=$\mathfrak{V}$.Now QNR$\mathfrak{int}$(QNR$\mathfrak{cl}$($\mathfrak{V}$)) = QNR$\mathfrak{int}$(QNR$\mathfrak{cl}$(QNR$\mathfrak{cl}$($\mathfrak{U}$))) = QNR$\mathfrak{int}$(QNR$\mathfrak{cl}$($\mathfrak{U}$)) = $\widetilde{\Phi}_{QNR}$. Hence $\mathfrak{V}$ = QNR$\mathfrak{cl}$($\mathfrak{U}$) is a QSVNR nowheredense set in ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$).

Definition 3.7 *A QSVNRS* $\mathfrak{U}$ *in QSVNRTS (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*) is said to be QSVNR* G_{δ}*-set in (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*) if* $\mathfrak{U} = \bigcap_{j=1}^{\infty} \mathfrak{U}_j$ *where* $\mathfrak{U}_j$ *are QNROS in (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*).*

Definition 3.8 *A QSVNRS* $\mathfrak{U}$ *in a QSVNRTS (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*) is known as QSVNR* F_{σ}*-set in (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*) if* $\mathfrak{U} = \bigcup_{j=1}^{\infty} \mathfrak{U}_j$ *where* $\mathfrak{U}_j$ *are QNRCS in (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*).*

Definition 3.9 *A QSVNRTS (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*) is said to be QSVNR first category set if* $\mathfrak{U} = \bigcup_{j=1}^{\infty} \mathfrak{U}_j$ *where* $\mathfrak{U}_j$ *'s are QSVNR nowhere dense sets in (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*).Otherwise (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*) is known as QSVNR second category.*

Definition 3.10 *A QSVNRTS (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*) is known as Baire space if QNR*$\mathfrak{int}$*(*$\bigcup_{j=1}^{N} \mathfrak{U}_j$*)=* $\widetilde{\Phi}_{QNR}$*, where* $\mathfrak{U}_j$ *'s are QSVNR nowhere dense sets in (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*).*

Definition 3.11 *A QSVNRTS (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*) is known as QSVNR volterra space if QNR*$\mathfrak{cl}$*(*$\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j$*) =* $\widetilde{\mathbb{X}}_{QNR}$*, Here* $\mathfrak{U}_j$ *'s are QSVNR dense and QSVNR* G_{δ}*-sets in (*$Q^{\mathbb{X}}$*,*$Q^{\mathfrak{T}}$*).*

Example 3.12 *Let* $Q^{\mathbb{X}}$ *={*$\mathfrak{e},\mathfrak{f}$*}. Define the QSVNRS* $\ddot{\mathfrak{U}}$, $\ddot{\mathfrak{V}}$, $\ddot{\mathfrak{U}} \tilde{\cup} \ddot{\mathfrak{V}}$ *and* $\ddot{\mathfrak{U}} \tilde{\cap} \ddot{\mathfrak{V}}$ *as follows*

$\ddot{\mathfrak{U}}$ ={⟨𝔢,{0.5,0.3,0.4,0.2},{0.8,0.5,0.6,0.3},{0.5,0.3,0.2,0.4}⟩,

⟨𝔣,{0.2,0.4,0.3,0.1},{0.3,0.5,0.2,0.4}, {0.4,0.2,0.3,0.5}⟩}

$\ddot{\mathfrak{V}}$ ={⟨𝔢,{0.4,0.5,0.2,0.1},{0.7,0.9,0.4,0.5},{0.6,0.4,0.3,0.5}⟩,

⟨𝔣,{0.5,0.6,0.2,0.3},{0.1,0.6,0.3,0.2}, {0.5,0.1,0.2,0.6}⟩}

$\ddot{\mathfrak{U}} \tilde{\cup} \ddot{\mathfrak{V}}$ ={⟨𝔢,{0.5,0.5,0.2,0.1},{0.8,0.9,0.4,0.3},{0.6,0.4,0.2,0.4}⟩,

⟨𝔣,{0.5,0.6,0.2,0.1},{0.3,0.6,0.2,0.2}, {0.5,0.2,0.2,0.5}⟩}

$\ddot{\mathfrak{U}} \tilde{\cap} \ddot{\mathfrak{V}}$ ={⟨𝔢,{0.4,0.3,0.4,0.2},{0.7,0.5,0.6,0.5},{0.5,0.3,0.3,0.5}⟩,

⟨𝔣,{0.2,0.4,0.3,0.3},{0.1,0.5,0.3,0.4}, {0.4,0.1,0.3,0.6}⟩}

Then $Q^{\mathfrak{T}}$ = $\{\widetilde{\Phi}_{QNR}$, $\widetilde{\mathbb{X}}_{QNR}$, $\ddot{\mathfrak{U}}$, $\ddot{\mathfrak{V}}$, $\ddot{\mathfrak{U}} \,\widetilde{\cup}\, \ddot{\mathfrak{V}}$, $\ddot{\mathfrak{U}} \,\widetilde{\cap}\, \ddot{\mathfrak{V}}\}$ is a QSVNRT on $Q^{\mathbb{X}}$.Thus ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$) is a QSVNRTS.

Let $\mathfrak{N}$ = { $\ddot{\mathfrak{U}} \,\widetilde{\cap}\, \ddot{\mathfrak{V}} \,\widetilde{\cap}\, (\ddot{\mathfrak{U}} \,\widetilde{\cap}\, \ddot{\mathfrak{V}})$}, $\mathfrak{O}$ = { $\ddot{\mathfrak{U}} \,\widetilde{\cap}\, \ddot{\mathfrak{V}} \,\widetilde{\cap}\, (\ddot{\mathfrak{U}} \,\widetilde{\cup}\, \ddot{\mathfrak{V}})$},$\mathfrak{P}$ = { $\ddot{\mathfrak{U}} \,\widetilde{\cap}\, \ddot{\mathfrak{V}} \,\widetilde{\cap}\, (\ddot{\mathfrak{U}} \,\widetilde{\cap}\, \ddot{\mathfrak{V}}) \,\widetilde{\cap}\, (\ddot{\mathfrak{U}} \,\widetilde{\cup}\, \ddot{\mathfrak{V}})$},

where $\mathfrak{N}$, $\mathfrak{O}$, $\mathfrak{P}$ are QSVNR G_δ-set in ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$) and QNRcl($\mathfrak{N}$) = $\widetilde{\mathbb{X}}_{QNR}$, QNRcl($\mathfrak{O}$) = $\widetilde{\mathbb{X}}_{QNR}$, QNRcl($\mathfrak{P}$) = $\widetilde{\mathbb{X}}_{QNR}$.Then QNRcl($\mathfrak{N} \,\widetilde{\cap}\, \mathfrak{O} \,\widetilde{\cap}\, \mathfrak{P}$) = $\widetilde{\mathbb{X}}_{QNR}$. Hence ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$) is a QSVNR volterra space.

Example 3.13 *Let $Q^{\mathbb{X}}$ ={𝔢, 𝔣}. Define the QSVNRS $\ddot{\mathfrak{U}}$, $\ddot{\mathfrak{V}}$, $\ddot{\mathfrak{U}} \,\widetilde{\cup}\, \ddot{\mathfrak{V}}$ and $\ddot{\mathfrak{U}} \,\widetilde{\cap}\, \ddot{\mathfrak{V}}$ as follows*

$\ddot{\mathfrak{U}}$ ={⟨ 𝔢,{0.5,0.4,0.3,0.6},{0.2,0.7,0.4,0.3},{0.4,0.5,0.3,0.6}⟩,

⟨𝔣,{0.6,0.5,0.4,0.7},{0.5,0.6,0.3,0.6}, {0.5,0.3,0.4,0.6}⟩}

$\ddot{\mathfrak{V}}$ ={⟨𝔢,{0.7,0.3,0.4,0.2},{0.8,0.3,0.9,0.2},{0.5,0.2,0.6,0.1}⟩,

⟨𝔣,{0.7,0.4,0.5,0.6},{0.4,0.3,0.7,0.2}, {0.2,0.7,0.7,0.2}⟩}

$\ddot{\mathfrak{U}} \,\widetilde{\cup}\, \ddot{\mathfrak{V}}$ ={⟨𝔢,{0.7,0.4,0.3,0.2},{0.8,0.7,0.4,0.2},{0.5,0.5,0.3,0.1}⟩,

⟨𝔣,{0.7,0.5,0.4,0.6},{0.5,0.6,0.3,0.2}, {0.5,0.7,0.4,0.2}⟩}

$\ddot{\mathfrak{U}} \,\widetilde{\cap}\, \ddot{\mathfrak{V}}$ ={⟨𝔢,{0.5,0.3,0.4,0.6},{0.2,0.3,0.9,0.3},{0.4,0.2,0.6,0.6}⟩,

⟨𝔣,{0.6,0.4,0.5,0.7},{0.4,0.3,0.7,0.6}, {0.2,0.3,0.7,0.6}⟩}

Then $Q^{\mathfrak{T}}$ = $\{\widetilde{\Phi}_{QNR}$, $\widetilde{\mathbb{X}}_{QNR}$,$\ddot{\mathfrak{U}}$, $\ddot{\mathfrak{V}}$, $\ddot{\mathfrak{U}} \,\widetilde{\cup}\, \ddot{\mathfrak{V}}$, $\ddot{\mathfrak{U}} \,\widetilde{\cap}\, \ddot{\mathfrak{V}}\}$ is a QSVNRT on $Q^{\mathbb{X}}$.Thus ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$) is a QSVNRTS. But there is no QSVNR G_δ-set in ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$). Hence ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$) is not a QSVNR volterra space.

Proposition 3.14 *If $\mathfrak{U}$ = $\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j$, where $\mathfrak{U}_j$ are QSVNR dense and QSVNR G_δ-sets in QSVNR volterra space ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$), then $\mathfrak{U}$ is not a QNRCS.*

Proof. Let $\mathfrak{U}$ = $\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j$, where $\mathfrak{U}_j$'s are QSVNR dense and QSVNR G_δ sets in ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$). Since ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$) is a QSVNR volterra space, we have QNRcl($\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j$) = $\widetilde{\mathbb{X}}_{QNR}$.(i.e.,) QNRcl($\mathfrak{U}$) = $\widetilde{\mathbb{X}}_{QNR}$ which implies that QNRcl($\mathfrak{U}$) ≠ $\mathfrak{U}$. Therefore $\mathfrak{U}$ is not a QNRCS in ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$).

Proposition 3.15 *A QSVNRTS ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$) is a QSVNR volterra space, iff QNRint($\bigcup_{j=1}^{N} \mathfrak{U}_j^{\tilde{c}}$) = $\widetilde{\Phi}_{QNR}$, where $\mathfrak{U}_j$'s are QSVNR dense and QSVNR G_δ-sets in ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$).*

Proof. Let ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$) be a QSVNR volterra space and $\mathfrak{U}_j$'s are QSVNR dense and QSVNR G_δ-sets in ($Q^{\mathbb{X}}$,$Q^{\mathfrak{T}}$). Then we have QNRcl($\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j$) = $\widetilde{\mathbb{X}}_{QNR}$. Now QNRint($\bigcup_{j=1}^{\mathbb{N}} \mathfrak{U}_j)^{\tilde{c}}$ = (QNRcl($\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j$))$^{\tilde{c}}$ = $\widetilde{\Phi}_{QNR}$.

Conversely let QNR$\mathfrak{int}(\bigcup_{j=1}^{\mathbb{N}} \mathfrak{U}_j^{\tilde{c}}) = \widetilde{\Phi}_{QNR}$,where $\mathfrak{U}_j$'s are QSVNR dense and QSVNR G_δ-sets in $(Q^{\mathbb{X}}, Q^{\mathfrak{T}})$.Then QNR$\mathfrak{int}(\bigcup_{j=1}^{\mathbb{N}} \mathfrak{U}_j)^{\tilde{c}} = \widetilde{\Phi}_{QNR}$, this implis that (QNR$\mathfrak{cl}(\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j))^{\tilde{c}} = \widetilde{\Phi}_{QNR}$.Therefore $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is a QSVNR volterra space.

Proposition 3.16 *Let $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ be a QSVNRTS. If QNR$\mathfrak{int}(\bigcup_{j=1}^{\mathbb{N}} \mathfrak{U}_j) = \widetilde{\Phi}_{QNR}$, $\mathfrak{U}_j$'s are QSVNR nowhere dense and QSVNR F_σ sets in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$, then $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is a QSVNR volterra space.*

Proof. Let QNR$\mathfrak{int}(\bigcup_{j=1}^{\mathbb{N}} \mathfrak{U}_j) = \widetilde{\Phi}_{QNR}$, which implies (QNR$\mathfrak{int}(\bigcup_{j=1}^{\mathbb{N}} \mathfrak{U}_j))^{\tilde{c}} = \widetilde{\mathbb{X}}_{QNR}$ (i.e.,) QNR$\mathfrak{cl}(\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j) = \widetilde{\mathbb{X}}_{QNR}$. $\mathfrak{U}_j$'s are QSVNR nowhere dense and QSVNR F_σ sets implies that $\mathfrak{U}_j^{\tilde{c}}$ are QSVNR dense and QSVNR G_δ sets in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$.Therefore QNR$\mathfrak{cl}(\bigcap_{j=1}^{N} \mathfrak{U}_j) = \widetilde{\mathbb{X}}_{QNR}$. Hence $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is a QSVNR volterra space.

Definition 3.17 *A QSVNRTS $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is known as QSVNR p-space if countable intersection of QNROS in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ in QSVNR open in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$.*

Definition 3.18 *A QSVNRTS $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is said to be QSVNR hyperconnected space if every QNROS $\mathfrak{U}$ is QSVNR dense set in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ (i.e.,) QNR$\mathfrak{cl}(\mathfrak{U}) = \widetilde{\mathbb{X}}_{QNR}$, for all $\mathfrak{U} \in Q^{\mathfrak{T}}$.*

Proposition 3.19 *If the QSVNRTS $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ be QSVNR p-space and QSVNR hyperconnected space then $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is also a QSVNR volterra space.*

Proof. Let $\mathfrak{U}_j$'s $(j = 1 \text{ to } \mathbb{N})$ is QSVNR dense and QSVNR G_δ-sets in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$. Since $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is QSVNR p-space, $\mathfrak{U}_j$'s is QSVNR G_δ sets, this implies that $\mathfrak{U}_j$'s is QNROS in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$.Then $\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j \in Q^{\mathfrak{T}}$.Since $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is a QSVNR hyperconnected space $\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j \in Q^{\mathfrak{T}}$ implies that QNR$\mathfrak{cl}(\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j) = \widetilde{\mathbb{X}}_{QNR}$. Hence $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is a QSVNR volterra space.

Definition 3.20 *A QSVNRTS $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is said to be QSVNR submaximal space if for each QSVNRS $\mathfrak{U}$ in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ such that QNR$\mathfrak{cl}(\mathfrak{U}) = \widetilde{\mathbb{X}}_{QNR}$, then $\mathfrak{U} \in Q^{\mathfrak{T}}$.*

Proposition 3.21 *If the QSVNRTS $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ be QSVNR submaximal and QSVNR hyperconnected space then $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is a QSVNR volterra space.*

Proof. Let $\mathfrak{U}_j$'s $(j = 1 \text{ to } \mathbb{N})$ is QSVNR dense and QSVNR G_δ sets in $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$. Since $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is a QSVNR submaximal space QNR$\mathfrak{cl}(\mathfrak{U}_j) = \widetilde{\mathbb{X}}_{QNR}$, implies that $\mathfrak{U}_j$'s $\in Q^{\mathfrak{T}}$ for all$(j = 1 \text{ to } \mathbb{N})$ this implies that $\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j \in Q^{\mathfrak{T}}$.Since $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is a QSVNR hyperconnected space, $\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j \in Q^{\mathfrak{T}}$ implies that QNR$\mathfrak{cl}(\bigcap_{j=1}^{\mathbb{N}} \mathfrak{U}_j) = \widetilde{\mathbb{X}}_{QNR}$. Hence $(Q^{\mathbb{X}},Q^{\mathfrak{T}})$ is a QSVNR volterra space.

IV. CONCLUSION

aracterizations of these spaces is presented outcomes of functions that preserve quadripartitioned single valued neutrosophic refined volterra space in the context of images and The concepts of quadripartitioned single valued neutrosophic refined volterra space as well as chpreimages are obtained.

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