LOWER BOUNDS ON THE MULTIPLICATIVE S-INDEX OF VARIOUS OPERATIONS ON GRAPH

Abstract

The mathematical modeling of chemical phenomena is done using graph theory by the field of mathematical chemistry known as the theory of chemical graphs. In chemical graph theory, a molecular graph is a simple graph where the vertices and edges stand in for individual atoms and the chemical bonds that connect them. Based on a certain topological feature of the relevant molecular graph, it is discovered that there is a strong association between the qualities of chemical compounds and their molecular structure. Topological index is a numerical description of a molecule. We introduce the Multiplicative S-index of a graph, a brand-new graph invariant. We define the lower bounds for the multiplicative S-index of the graph operations Join, Cartesian product, Composition, Strong product, Corona product, and Corona in this work.

Keywords: Zagreb, S-index, Graph operations

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I. INTRODUCTION

Graph theory has provided chemists with a wealth of useful tools, such as topological indices. Molecular graphs are commonly used to represent molecules and molecular compounds. A topological index is analogous to converting a chemical structure into a real number. Topological and graph invariants based on graph vertex distances are widely used for characterizing molecular graphs, establishing structural and property relationships, predicting biological activities of chemical compounds, and developing chemical applications. Topological indices are significant because they can be utilized directly as simple numerical descriptors in Quantitative Structure Property Relationships (QSPR) and Quantitative Structure Activity Relationships (QSAR) in comparison with physical, chemical, or biological characteristics of molecules. Topological indices come in a variety of forms, such as degree-based, distance-based, counting-related polynomials 2, and graph indices, among others. The practice of numerically coding chemical structures using topological indices or topological coindices has recently gained popularity in medicinal chemistry and bioinformatics [6,12,19].

Let's take a look at two simple connected graphs with disjoint vertex and edge sets, G_m and G_n. p_i and q_i stand for the number of vertices and edges for $i = m, n$. For each vertex $v \in V(G)$, the degree is given as $d_G(v) = \beta_G(v)$, where $d_G(v)$ is the number of edges incident on the vertex v.

I. Gutman and N. Trinajstic [2] defined the first and second Zagreb indexes of a graph in 1972 as follows:

$$M_1(G) = \sum_{v \in V(G)} [\beta_G(v)^2] = \sum_{uv \in E(G)} [\beta_G(u) + \beta_G(v)]$$

$$M_2(G) = \sum_{uv \in E(G)} [\beta_G(u)\beta_G(v)]$$

The F-index was [5] according to B. Furtula and I. Gutman in 2015:

$$F(G) = \sum_{v \in V(G)} [\beta_G(v)^3] = \sum_{uv \in E(G)} [\beta_G(u)^2 + \beta_G(v)^2]$$

The Y-index was first introduced in 2020 by Abdu Alameri and Noman AI-Naggar [10]. It is defined as:

$$Y(G) = \sum_{v \in V(G)} [\beta_G(v)^4] = \sum_{uv \in E(G)} [\beta_G(u)^3 + \beta_G(v)^3]$$

The S-index was established as [20] in 2021 by S. Nagarajan, G. Kayalvizhi, and G. Priyadharsini:

$$S(G) = \sum_{v \in V(G)} [\beta_G(v)^5] = \sum_{uv \in E(G)} [\beta_G(u)^4 + \beta_G(v)^4]$$

R. Todeschini and D. Ballabio [8] established the first and second Multiplicative Zagreb indices of a graph in 2010.

$$\prod_{1}(G) = \prod_{v\in V(G)} \beta_G(v)^2 \; and \; \prod_{2}(G) = \prod_{uv\in E(G)} \beta_G(u)\beta_G(v)$$

The Multiplicative Forgotten Topological Index was introduced in 2019 by Asghar Yousefi and Ali Iranmanesh [1]. It is defined as:

$$\prod_{F}(G) = \prod_{v\in V(G)} \beta_G(v)^3$$

In (2013) C.D. Kinkar and Y. Aysum [9] derived graph operations in Multiplicative Zagreb indices . The Multiplicative Zagreb coindices were calculated by K. Xu and K.C. Das [3] in (2013). In [1] Y. Asghar and Ali Iranmanesh (2019) derived the Multiplicative F-index of graph operations. M. Radhakrishnan and M. Suresh [15] derived the F-sum operations on graphs in multiplicative zagreb indices in [2020]. Liu J and Q. Zhang [16] defined upper bounds on multiplicative zagreb of connected graphs in [2012]. In [2015], M. Azari and A. Iranmanesh [17] presented lower bounds for the multiplicative sum zagreb index of graphs. M. Eliasi and D. Vukicevic [11] compared multiplicative indices in [2013]. In 2014, M. Azari [14] published the lower bounds on the narumi-katayama index of graphs. We investigated the lower bounds for the Multiplicative S-index of several graph operations in this article. Researchers who want to study more about graph operations might refer to [4,,7,10,13].

Definition 1.1: The Multiplicative S-index of a graph G is defined as the product of graph's four degree vertices and is denoted by:

$$\prod_{S}(G) = \prod_{v\in V(G)} \beta_G(v)^5$$

II. MAIN RESULTS

We developed various graph operations in the lower bound on the multiplicative S-index in this section.

Lemma 2.1: [18] (AM-GM inequality)

Let $x_1, \dots, x_n$ be a nonnegative numbers. Then

$\frac{x_1+\cdots+x_n}{n} \geq \sqrt[k]{x_1, \dots, x_n}$ holds with equality if and only if $x_1 = x_2 = \cdots = x_n$.

Join: "The *join* $G_m + G_n$ of graphs G_m and G_n with vertex sets $V(G_m)$ and $V(G_n)$ and edge sets $E(G_m)$ and $E(G_n)$ is the graph union $G_m \cup G_n$ together with all the edges between $V(G_m)$ and $V(G_n)$". Obviously,
$|V(G_m + G_n)| = |V(G_m)| + |V(G_n)| = p_m + p_n$ and $|E(G_m + G_n)| = |E(G_m)| + |E(G_n)| + |V(G_m)||V(G_n)| = q_m + q_n + p_m p_n$.

$$\beta_{G_m+G_n}(v) = \begin{Bmatrix} \beta_{G_m}(v) + p_n, & v \in V(G_m) \\ \beta_{G_n}(v) + p_m, & v \in V(G_n) \end{Bmatrix}$$

Theorem 2.2: The Multiplicative S-index of $G_m + G_n$ satisfies the inequality,

$$\prod_S(\mathrm{G_m} + \mathrm{G_n}) > \left(6\sqrt[3]{50}\right)^{p_m+p_n} \left(\sqrt{p_n^5}\right)^{p_m} \left(\sqrt{p_m^5}\right)^{p_n} \sqrt{\prod_S(G_m)\prod_S(G_n)}$$

Proof: Utilizing the definition 1, we've

$$\prod_S(G_m + G_n) = \prod_{v\in V(G_m+G_n)} \beta_{G_m+G_n}(v)^5$$
$$= \prod_{v\in V(G_m)}\left(\beta_{G_m}(v) + p_n\right)^5 \prod_{v\in V(G_n)}\left(\beta_{G_n}(v) + p_m\right)^5$$
$$= \prod_{v\in V(G_m)}\left[\beta_{G_m}(v)^5 + 5\beta_{G_m}(v)^4 p_n + 10\beta_{G_m}(v)^3 p_n^2 + 10\beta_{G_m}(v)^2 p_n^3 +\right.$$
5βGmvpn4+pn5
$$\prod_{v\in V(G_m)}\left[\beta_{G_n}(v)^5 + 5\beta_{G_n}(v)^4 p_m + 10\beta_{G_n}(v)^3 p_m^2 + 10\beta_{G_n}(v)^2 p_m^3 +\right.$$
5βGnvpm4+pm5

We now have, according to lemma 2.1,

$\prod_S(G_m + G_n) >$
v∈VGm66βGmv5× 5βGmv4pn× 10βGmv3pn2× 10βGmv2pn3× 5βGmvpn4× pn5

$\times$
$$\prod_{v\in V(G_m)} 6\sqrt[6]{\beta_{G_n}(v)^5 \times 5\beta_{G_n}(v)^4 p_m \times 10\beta_{G_n}(v)^3 p_m^2 \times 10\beta_{G_n}(v)^2 p_m^3 \times 5\beta_{G_n}(v) p_m^4 \times p_m^5}$$

We receive the complete result.

Cartesian Product: "The *Cartesian product* $G_m \times G_n$ of graphs G_m and G_n has the vertex set $V(G_m \times G_n) = V(G_m) \times V(G_n)$ and $(u,x)(v,y)$ is an edge of $G_m \times G_n$ if $uv \in E(G_m)$ and $= y$, or $u = v$ and $xy \in E(G_n)$". Obviously, $|V(G_m \times G_n)| = |V(G_m)||V(G_n)| = p_m p_n$ and $|E(G_m \times G_n)| = |E(G_m)||V(G_n)| + |E(G_n)||V(G_m)| = q_m p_n + q_n p_m$.

$$\beta_{G_m\times G_n}(x_1,x_2) = \beta_{G_m}(x_1) + \beta_{G_n}(x_2)$$

Theorem 2.3: The Multiplicative S-index of $G_m \times G_n$ satisfies the inequality,

$$\prod_S(\mathrm{G_m} \times \mathrm{G_n}) > \left(6\sqrt[3]{50}\right)^{p_m p_n} \sqrt{\left[\prod_S(G_m)\right]^{p_n} \left[\prod_S(G_n)\right]^{p_m}}$$

Proof: Utilizing the definition 1, we've

$$\prod_S(G_m \times G_n) = \prod_{(x_1,x_2)\in V(G_m\times G_n)} \beta_{G_m\times G_n}\left((x_1,x_2)\right)^5$$
$$= \prod_{x_1\in V(G_m)}\prod_{x_2\in V(G_n)}\left(\beta_{G_m}(x_1) + \beta_{G_n}(x_2)\right)^5$$

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$= \prod_{x_1 \in V(G_m)} \prod_{x_2 \in V(G_n)} [\beta_{G_m}(x_1)^5 + 5\beta_{G_m}(x_1)^4 \beta_{G_n}(x_2) + 10\beta_{G_m}(x_1)^3 \beta_{G_n}(x_2)^2 +$
10βGmx12βGnx23+5βGmx1βGnx22+βGnx25

We now have, according to lemma 2.1,

$\prod_S (G_m \times G_n) >$
x1∈VGmx2∈VGn6βGmx15× 5βGmx14βGnx2× 10βGmx13βGnx22×
10βGmx12βGnx23× 5βGmx1βGnx22× βGnx2516

We receive the complete result.

Composition:"The *Composition* $G_m[G_n]$ of graphs G_m and G_n with disjoint vertex sets $[V(G_m), V(G_n)]$ and edge sets $[E(G_m), E(G_n)]$ is the graph with vertex set $V(G_m) \times V(G_n)$ and $u = (u_1, v_1)$ is adjacent to $v = (u_2, v_2)$ whenever u_1 is adjacent to u_2 or $u_1 = u_2$ and v_1 is adjacent to v_2". $|V(G_m[G_n])| = |V(G_m)||V(G_n)| = p_m p_n$, $|E(G_m[G_n])| = |E(G_m)||V(G_n)|^2 + |V(G_m)||E(G_n)| = q_m {p_n}^2 + q_n p_m$.

$$\beta_{G_m[G_n]}(x_1, x_2) = p_n \beta_{G_m}(x_1) + \beta_{G_n}(x_2)$$

Theorem 2.4: The Multiplicative S-index of $G_m[G_n]$ satisfies the inequality,

$$\prod_S (\mathrm{G_m}[\mathrm{G_n}]) > \left(6\sqrt[3]{50}\right)^{p_m p_n} \left(\sqrt{p_n^5}\right)^{p_m p_n} \sqrt{\left[\prod_S (G_m)\right]^{p_n} \left[\prod_S (G_n)\right]^{p_m}}$$

Proof: Utilizing the definition 1, we've

$$\prod_S (G_m[G_n]) = \prod_{(x_1,x_2) \in V(G_m[G_n])} \beta_{G_m[G_n]}\big((x_1, x_2)\big)^5$$
$$= \prod_{x_1 \in V(G_m)} \prod_{x_2 \in V(G_n)} \left(p_n \beta_{G_m}(x_1) + \beta_{G_n}(x_2)\right)^5$$

$= \prod_{x_1 \in V(G_m)} \prod_{x_2 \in V(G_n)} [p_n^5 \beta_{G_m}(x_1)^5 + 5p_n^4 \beta_{G_m}(x_1)^4 \beta_{G_n}(x_2) + 10p_n^3 \beta_{G_m}(x_1)^3 \beta_{G_n}(x_2)^2 +$
10pn2βGmx12βGnx23+5pnβGmx1βGnx22+βGnx25

We now have, according to lemma 2.1,

$\prod_S (G_m[G_n]) >$
x1∈VGmx2∈VGn6pn5βGmx15× 5pn4βGmx14βGnx2× 10pn3βGmx13βGnx22×
10pn2βGmx12βGnx23× 5pnβGmx1βGnx22× βGnx2516

We receive the complete result.

Strong product:"The *Strong product* $G_m * \mathrm{G_n}$ of a graphs G_m and $\mathrm{G_n}$ is a graph with vertex set $V(G_m) \times V(\mathrm{G_n})$ and any two vertices (u_p, v_r) and (u_q, v_s) are adjacent if and only if $[u_p = u_q \ and \ v_r v_s \in E(\mathrm{G_n})]$ or $[v_r = v_s \ and \ u_p u_q \in E(G_m)]$ or $[u_p u_q \in E(G_m) \ and \ v_r v_s \in E(\mathrm{G_n})]$". $|V(G_m * \mathrm{G_n})| = |V(G_m)||V(\mathrm{G_n})| = p_m p_n$,

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$|E(G_m * \mathrm{G_n})| = |E(G_m)||V(\mathrm{G_n})| + |V(G_m)||E(\mathrm{G_n})| + 2|E(G_m)||E(\mathrm{G_n})| = q_m p_n + p_m q_n + 2q_m q_n.$

$$\beta_{G_m * H}\left((a,b)\right) = \beta_{G_m}(a) + \beta_H(b) + \beta_{G_m}(a)\beta_H(b)$$

Theorem 2.5: The Multiplicative S-index of $G_m * G_n$ satisfies the inequality,

$$\prod_{\mathrm{S}}(\mathrm{G_m} * \mathrm{G_n}) > \left(2100\sqrt[7]{33750}\right)^{p_m p_n} \sqrt{\left[\prod_S(G_m)\right]^{2p_n}\left[\prod_S(G_n)\right]^{2p_m}}$$

Proof: Utilizing the definition 1, we've

$$\prod_S(G_m * G_n) = \prod_{(x_1,x_2)\in V(G_m * G_n)} \beta_{G_m * G_n}\left((x_1,x_2)\right)^5$$
$$= \prod_{x_1\in V(G_m)}\prod_{x_2\in V(G_n)}\left(\beta_{G_m}(x_1) + \beta_{G_n}(x_2) + \beta_{G_m}(x_1)\beta_{G_n}(x_2)\right)^5$$

We now have, according to lemma 2.1,

$\prod_S(G_m * G_n) > \prod_{x_1\in V(G_m)}\prod_{x_2\in V(G_n)} 21\big[\beta_{G_m}(x_1)^5 \times \beta_{G_n}(x_2)^5 \times \beta_{G_m}(x_1)^5\beta_{G_n}(x_2)^5 \times$
5βGmx15βGnx2× 5βGmx14βGnx2× 5βGmx1βGnx24× 5βGmx1βGnx25× 5βGmx15βGn x24× 5βGmx14βGnx25× 10βGmx13βGnx22× 20βGmx14βGnx22× 10βGmx15βGnx22× 10βGmx12βGnx23× 20βGmx12βGnx24× 10βGmx12βGnx25× 10βGmx15βGnx23× 20β Gmx14βGnx24× 10βGmx13βGnx25× 30βGmx13βGnx23× 30βGmx14βGnx23× 30βGm x13βGnx24121

We receive the complete result.

Corona product: "The *Corona product* $G_m \odot \mathrm{G_n}$ of graphs G_m and $\mathrm{G_n}$ with disjoint vertex sets $V(G_m)$ and $V(\mathrm{G_n})$ and edge sets $E(G_m)$ and $E(\mathrm{G_n})$ is the graph derived by one copy of G_m and k_1 copies of $\mathrm{G_n}$ and joining the i^{th} vertex of G_m to each vertex in i^{th} copy of $\mathrm{G_n}$". Obviously, $|V(G_m \odot \mathrm{G_n})| = |V(G_m)| + |V(G_m)||V(\mathrm{G_n})| = p_m + p_m p_n$, $|E(G_m \odot \mathrm{G_n})| = |E(G_m)| + |V(G_m)||E(\mathrm{G_n})| + |V(G_m)||V(\mathrm{G_n})| = q_m + p_m q_n + p_m p_n$.

$$\beta_{G_m \odot \mathrm{G_n}}(v) = \begin{Bmatrix} \beta_{G_m}(v) + p_n\ , & v \in V(G_m) \\ \beta_{\mathrm{G_n}}(v) + 1\ , & v \in V(\mathrm{G_n}) \end{Bmatrix}$$

Theorem 2.6: The Multiplicative S-index of $G_m \odot G_n$ satisfies the inequality,

$$\prod_{\mathrm{S}}(\mathrm{G_m} \odot \mathrm{G_n}) > \left(6\sqrt[3]{50}\right)^{p_m(p_n+1)}\left(\sqrt{p_n^5}\right)^{p_m}\sqrt{\prod_S(G_m)\left(\prod_S(G_n)\right)^{p_m}}$$

Proof: Utilizing the definition 1, we've

$$\prod_S(G_m \odot G_n) = \prod_{v\in V(G_m \odot G_n)} \beta_{G_m \odot G_n}(v)^5$$

$= \prod_{v\in V(G_m)}\left(\beta_{G_m}(v)+p_n\right)^5 \prod_{v\in V(G_n)}\left(\beta_{G_n}(v)+1\right)^5$

$= \prod_{v\in V(G_m)}[\beta_{G_m}(v)^5 + 5\beta_{G_m}(v)^4 p_n + 10\beta_{G_m}(v)^3 p_n^2 + 10\beta_{G_m}(v)^2 p_n^3 +$
5βGmvpn4+pn5

$[\prod_{v\in V(G_m)}[\beta_{G_n}(v)^5 + 5\beta_{G_n}(v)^4 + 10\beta_{G_n}(v)^3 + 10\beta_{G_n}(v)^2 + 5\beta_{G_n}(v) +$
1pm

We now have, according to lemma 2.1,

$\prod_S(G_m \odot G_n) >$
v∈VGm66βGmv5× 5βGmv4pn× 10βGmv3pn2× 10βGmv2pn3× 5βGmvpn4×pn5

$\times$

$$\left[\prod_{v\in V(G_m)} 6\sqrt[6]{\beta_{G_n}(v)^5\times 5\beta_{G_n}(v)^4\times 10\beta_{G_n}(v)^3\times 10\beta_{G_n}(v)^2\times 5\beta_{G_n}(v)\times 1}\right]^{p_m}$$

We receive the complete result.

Corona join product: "Let $\mathrm{G_m}(k_1, j_1)$ and $\mathrm{G_n}(k_2, j_2)$ be simple connected graphs, and the Corona join graph of $\mathrm{G_m}$ and $\mathrm{G_n}$ is obtained by taking one copy of $\mathrm{G_m}$, k_1 copies of $\mathrm{G_n}$, and joining each vertex of the i^{th} copy of $\mathrm{G_n}$ with all vertices of $\mathrm{G_m}$". The *Corona join product* of $\mathrm{G_m}$ and $\mathrm{G_n}$ is denoted by

$$\beta_{\mathrm{G_m}\oplus\mathrm{G_n}}(v) = \begin{Bmatrix} \beta_{\mathrm{G_m}}(v) + p_m p_n\,, & if\ v\in V(\mathrm{G_m}) \\ \beta_{\mathrm{G_n}}(v) + p_m\,, & if\ v\in V(\mathrm{G_n}) \end{Bmatrix}$$

Theorem 2.7: The Multiplicative S-index of $G_m \oplus G_n$ satisfies the inequality,

$$\prod_{\mathrm{S}}(\mathrm{G_m}\oplus\mathrm{G_n}) > \left(6\sqrt[3]{50}\right)^{p_m+p_n}\left(\sqrt{p_m^5 p_n^5}\right)^{p_m}\left(\sqrt{p_m^5}\right)^{p_n}\sqrt{\prod_S(G_m)\prod_S(G_n)}$$

Proof: Utilizing the definition 1, we've

$\prod_S(G_m\oplus G_n) = \prod_{v\in V(G_m\oplus G_n)}\beta_{G_m\oplus G_n}(v)^5$

$= \prod_{v\in V(G_m)}\left(\beta_{G_m}(v)+p_m p_n\right)^5 \prod_{v\in V(G_n)}\left(\beta_{G_n}(v)+p_m\right)^5$

$= \prod_{v\in V(G_m)}[\beta_{G_m}(v)^5 + 5\beta_{G_m}(v)^4 p_m p_n + 10\beta_{G_m}(v)^3 p_m^2 p_n^2 + 10\beta_{G_m}(v)^2 p_m^3 p_n^3 +$
5βGmvpm4pn4+pm5pn5

$\prod_{v\in V(G_m)}[\beta_{G_n}(v)^5 + 5\beta_{G_n}(v)^4 p_m + 10\beta_{G_n}(v)^3 p_m^2 + 10\beta_{G_n}(v)^2 p_m^3 +$
5βGnvpm4+pm5

We now have, according to lemma 2.1,

$\prod_S(G_m\oplus G_n) >$
v∈VGm66βGmv5× 5βGmv4pmpn× 10βGmv3pm2pn2× 10βGmv2pm3pn3× 5βGmvpm4 pn4×pm5pn5

$$\times \prod_{v\in V(G_m)} 6\sqrt[6]{\beta_{G_n}(v)^5 \times 5\beta_{G_n}(v)^4 p_m \times 10\beta_{G_n}(v)^3 p_m^2 \times 10\beta_{G_n}(v)^2 p_m^3 \times 5\beta_{G_n}(v) p_m^4 \times p_m^5}$$

We receive the complete result.

III. CONCLUSION

In numerous disciplines, topological indices are defined and employed to study the characteristics of diverse things, such as atoms and molecules. Numerous topological indices have been defined and researched by mathematicians and chemists. In this paper, we looked at the lower bound of the Multiplicative S-index for a number of graph operations, including join, Cartesian product, composition, strong product, corona product, and corona join product.

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