# MOMENT ESTIMATION AND WEIGHTED APPROXIMATION BY MODIFIED-BERNSTEIN KANTROVINCH OPERATORS

## Abstract

The present work deals with the Kantrovinch type modification of modified Bernstein operator. We discuss the rate of convergence of proposed operators by means of modulus of continuity and Peetre's K-functional for Hölder's class of functions. Further, we derive a Vornovskaya type asymptotic result and study weighted approxi- mation with polynomial growth. Also, numerical examples illustrating the error functions and the approximation of the proposed operators for some continuous as well as piecewise continuous functions are given.

**Keywords:** Modulus of continuity; Kantrovinch operator; Bernstein operator; Moment estimates

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## I. INTRODUCTION

Positive linear operators are widely used in various fields of science and engineering. A very famous poly- nomial in this regard, in the approximation theory of positive linear operators was studied by Bernstein [1]. Bernstein operator for every bounded function $\psi \in C[0,1]$, $n \geq 1$ and $t \in [0,1]$ is defined as

$$B_n(\psi;t) = \sum_{k=0}^{n} p_{n,k}(t)\psi\left(\frac{k}{n}\right),$$

and $p_{n,k}(t) = \binom{n}{k} t^{k-1}(1-t)^{n-k-1}$ is Bernstein basis function. F. Usta [2] presented a new modification for $\psi \in C[0,1]$, $n \in \mathbb{N}$, $t \in (0,1)$ as

$$\mathfrak{B}_n(\psi;t) = \sum_{k=0}^{n} \binom{n}{k} (k-nt)^2\, t^{k-1}(1-t)^{n-k-1}\psi\left(\frac{k}{n}\right). \tag{1}$$

Thereafter, different modifications of the above operator have become interest to many researchers. M. Sofyalıogˇlu [3] introduced a parametric generalization of (1). For more details on parametric genaralizations, we refer the readers to [4, 5, 6, 7, 8, 9, 10, 11]. Approximation theory is important tool in the field of robotics [12, 13], applied mathematics [14, 15, 16, 17, 18, 19] and Fourier approximation [20, 21]. Sonker and Priyanka [22] introduced parametrically bivariate Baskakov-Stancu operators. Kantrovinch [23] introduced a modification involving integral for the class of Lebesgue integrable functions on [0,1] given by

$$K_n(\psi;t) = (n+1)\sum_{k=0}^{n} p_{n,k}(t)\int_{\frac{k}{n+1}}^{\frac{k+1}{n+1}} \psi(u)du, \tag{2}$$

where $t \epsilon (0,1)$.

In the present work, we introduce Kantrovinch modification of the operator given by equation 1 as follows:

$$\mathcal{K}_n(\psi;t) = n\sum_{k=0}^{n} \binom{n}{k} (k-nt)^2\, t^{k-1}(1-t)^{n-k-1\int_{k/n}^{(k+1)/n} \psi(u)du},\ t \in (0,1). \tag{3}$$

Also, we introduce some numerical example using MATLAB in order to show the theoretical approach for approximation by newly defined operators.

## II. PRELIMINARIES

**Lemma 1:** *The modified-Bernstein operators* $\mathfrak{B}_n(.;t)$ [2], *for* $n \in$ N, *satisfy the following identities:*

1. $\mathfrak{B}_n(1;t) = 1;$
2. $\mathfrak{B}_n(y;t) = \left(\frac{n-2}{n}\right)t + \frac{1}{n};$
3. $\mathfrak{B}_n(y^2;t) = \left(\frac{n^2-7n+6}{n^2}\right)t^2 + \left(\frac{5n-6}{n^2}\right)t + \frac{1}{n^2};$
4. $\mathfrak{B}_n(y^3;t) = \left(\frac{n^3-15n^2+38n-24}{n^3}\right)t^3 + 12\left(\frac{n^2-4n+3}{n^3}\right)t^2 + \left(\frac{13n-14}{n^3}\right)t + \frac{1}{n^3}.$

## III. MOMENT ESTIMATION

Using the preliminaries, we can prove the following identities for Modified-Bernstein-Kantrovinch operators:

**Lemma 2:** *For $n \in \mathbb{N}$ the operator $\mathcal{K}_n(\psi(y);t)$ satisfies the followings:*

1. $\mathcal{K}_n(1;t) = 1;$
2. $\mathcal{K}_n(y;t) = \left(\frac{n-2}{n}\right)t + \frac{3}{2n};$
3. $\mathcal{K}_n(y^2;t) = \left(\frac{n^2-7n+6}{n^2}\right)t^2 + \left(\frac{6n-8}{n^2}\right)t + \frac{7}{3n^2};$
4. $\mathcal{K}_n(y^3;t) = \left(\frac{n^3-15n^2+38n-24}{n^3}\right)t^3 + \left(\frac{27n^2-117n+90}{2n^3}\right)t^2 + \left(\frac{42n-48}{n^3}\right)t + \frac{15}{4n^3}.$

***Proof:*** Using the linear property of $\mathcal{K}_n(\psi;t)$, we've

$$\mathcal{K}_n(y;t) = \mathfrak{B}_{n,a}(y;t) + \frac{1}{2n}\mathfrak{B}_{n,a}(1;t)$$

By using preliminaries, we can see part (2) is true. In a similar manner, we can prove other parts of above result.

Let us denote the $r^{th}$ order moment of $\mathcal{K}_n((y-t)^r;t)$ *by* $\gamma_{n,r}(t)$.

**Lemma 3:** *For $n \in \mathbb{N}$, the $r^{th}$ ($r = 1,2,4$) ordered moments of $\mathcal{K}_n(.;t)$ are given by*

1. $\gamma_{n,1}(t) = \left(\frac{-2}{n}\right)t + \frac{3}{2n};$
2. $\gamma_{n,2}(t) = \left(\frac{-3n+6}{n^2}\right)t^2 + \left(\frac{3n-8}{n^2}\right)t + \frac{7}{3n^2}.$

***Proof:*** Using the linear property of $\mathcal{K}_n(.;t)$ and lemma (2), above lemma can be derived easily. □

**Corollary 1:** *For $n \in \mathbb{N}$, operator $\mathcal{K}_n(.;t)$ satisfies the followings:*

1. $\lim 1_{n\to\infty} n\mathcal{K}_n\big((y-t);t\big) = \frac{3}{2} - 2t;$
2. $\lim 1_{n\to\infty} n\mathcal{K}_n((y-t)^2;t) = 3t(1-t).$

## IV. APPROXIMATION PROPERTIES OF $\mathcal{K}$n(.;t)

### A. Local Approximation

**Theorem 1:** *Let $\psi \in C(0,1)$, then*

$$\lim_{n\to\infty} \mathcal{K}_n(\psi;t) = \psi(t) \text{ uniformly on } (0,1).$$

***Proof:*** Using Lemma (2), we have

$\lim_{n\to\infty} \mathcal{K}_n(y^k;t) = t^k,\ k = 0,\ 1,\ 2,$ Uniformly on (0,1). The required result is immediately given by Korovkin type Theorem [24].

**B. Rate of Convergence.** For $\psi \in C(0,1)$, the modulus of continuity of $\psi$ is defined as

$$\omega(\psi,\zeta)=\sup_{|y-t|\leq\zeta}\left\{\sup_{t\in(0,1)}|\psi(y)-\psi(t)|\right\}.$$

Also, from [25], we can write

$$|\psi(y)-\psi(t)|\leq\left(1+\frac{(y-t)^2}{\zeta^2}\right)\omega(f,\zeta)$$

Also, the Peetre's K-functional is given by

$$K(\psi;\zeta)=\inf_{f\in C^2[0,1]}\{\|\psi-f+\zeta\|f\},\qquad \zeta>0,$$

where $C^2[0,1]=\{\psi\in C[0,1]:\psi',\psi''\in C[0,1]\}$. By [26], $\exists$ a constant $M>0$ such that

$$K(\psi;\zeta)\leq M\omega_2\left(\psi,\sqrt{\zeta}\right),\zeta>0, \tag{4}$$

where $\omega_2\left(\psi,\sqrt{\eta}\right)=\sup_{0<|h|<\sqrt{\eta}}\sup_{t,t+2h\in(0,1)}|\psi(t+2h)-2\psi(t+h)+\psi(t)|$ is the second ordered modulus of continuity of $\psi$ on (0,1).

**Theorem 2:** *Let $t\in(0,1)$ and $\psi\in C[0,1]$. Then we have*

$$\mathcal{K}_n(\psi;t)-\psi(t)|\leq 2\omega\left(\psi,\sqrt{\gamma_{n,2}(t)}\right),$$

where $\gamma_{n,2}^2(t)=\mathcal{K}_n((y-t)^2;t)$, is the second ordered central moment of nth proposed operator.

***Proof:*** For $\psi\in C[0,1]$, we obtain

$$\begin{aligned}|\mathcal{K}_n(\psi;t)-\psi(t)|&=n\sum_{k=0}^{n}p_{n,k}(t)\int_{k/n}^{(k+1)/n\backslash}|\psi(y)-\psi(t)|dy\\&\leq n\sum_{k=0}^{n}p_{n,k}(t)\int_{k/n}^{(k+1)/n}\left(1+\frac{(y-t)^2}{\zeta^2}\right)\omega(f,\zeta)dy\\&=\left(1+\frac{1}{\zeta^2}\backslash cK_n((y-t)^2;t)\right)\omega(\psi,\zeta)\end{aligned}$$

By taking $\zeta^2=\gamma_{n,2}(t)$, we reach the required result.
Next, we define Hölder's class of functions for $\alpha\in(0,1]$ as follows

$\mathcal{H}_\alpha(0,1)=\{\psi\in C(0,1):|\psi(y)-\psi(t|)\leq M_\psi|y-t|^\alpha;\ t\in(0,1)\}$ .
The following theorem gives the convergence rate for Hölder's class of functions:

**Theorem 3:** *Let $t\in(0,1)$ and $\psi\in\mathcal{H}_\alpha(0,1)$. Then we have*

 

$$|\mathcal{K}_n(\psi;t)-\psi(t)| \leq M\sqrt{\gamma_{n,2}^{\alpha}(t)},$$

*where $\gamma_{n,2}(t)$ is the second ordered central moment of nth proposed operator.*

***Proof:*** For $\psi \in \mathcal{H}_\alpha(0,1)$, consider

$$|\mathcal{K}_n(\psi;t)-\psi(t)| = n\sum_{k=0}^{n} p_{n,k}(t)\int_{k/n}^{(k+1)/n} |\psi(y)-\psi(t)|dy$$

On applying Hölder inequality with $p=\frac{2}{\alpha},\ q=\frac{2}{2-\alpha}$ twice, we are led to

$$|\mathcal{K}_n(\psi;t)-\psi(t)| \leq \left\{n\sum_{k=0}^{n} p_{n,k}(t)\int_{k/n}^{(k+1)/n} |\psi(y)-\psi(t)|^{\frac{2}{\alpha}}dy\right\}^{\frac{\alpha}{2}}$$

$$\leq M\left\{n\sum_{k=0}^{n} p_{n,k}(t)\int_{k/n}^{(k+1)/n} |y-t|^{2}\,dy\right\}^{\frac{\alpha}{2}}$$

$$= M\mathcal{K}_n((y-t)^2;t)^{\frac{\alpha}{2}},$$

which completes the result.

**Theorem 4:** *Let $\psi \in C[0,1]$ and $0<t<1$. Then for all $n \in \mathbb{N}$, $\exists$ an absolute constant M such that*

$$|\mathcal{K}_n(\psi;t)-\psi(t)| \leq M\omega_2\left(\psi;\sqrt{\left\{\gamma_{n,2}(t)+\frac{1}{2}\gamma_{n,1}^{2}(t)\right\}}\right)+2\omega\big(\psi,|\gamma_{n,1}(t)|\big).$$

***Proof:*** Firstly, we define an auxiliary operator

$$\mathcal{K}_n^*(g;t) = \mathcal{K}_n(g;t) - g\left(\frac{n-2}{n}t+\frac{3}{2n}\right)+g(t) \tag{5}$$

Then, we have $\mathcal{K}_n(1;t)=1$ and $\mathcal{K}_n(y-t;\ t)=0$. Now Taylor's expansion for $g \in C^2[0,1]$ is given by

$$g(y) = g(t)+(y-t)g'(t)+\int_t^y (y-u)g''(u)du, \qquad t \in (0,1).$$

Applying auxiliary operator to both sides of above expansion, we obtain

$$\mathcal{K}_n^*(g;t)-g(t) = \mathcal{K}_n\left(\int_t^y (y-u)g''(u)du\,;t\right)-\int_t^{\frac{n-2}{n}t+\frac{3}{2n}}\left(\frac{n-2}{n}t+\frac{3}{2n}-u\right)g''(u)du$$

(6)

Now,

$$\left|\int_t^y (y-u)g''(u)du\right| \leq g''(y-t)^2$$

and

$$\left|\int_t^{\frac{n-2}{n}t+\frac{3}{2n}} \left(\frac{n-2}{n}t+\frac{3}{2n}-u\right)g''(u)du\right| \leq \frac{1}{2}g''\left(\frac{-2}{n}t+\frac{3}{2n}\right)^2$$

Rewriting Equation 6, we obtain

$$|\mathcal{K}_n^*(g;t)-g(t)| \leq \left|\left|g''\right|\right|\mathcal{K}_n((y-t)^2;t)+\frac{1}{2}g''\left(\frac{-2}{n}t+\frac{3}{2n}\right)^2 = \|g''\|\gamma_{n,2}(t)+\frac{1}{2}\gamma_{n,1}^2(t) \tag{7}$$

Also,

$$|\mathcal{K}_n^*(g;t)| \leq 3\|g\| \tag{8}$$

In the view of Equations 7 and 8, we get

$$\begin{aligned}
&|\mathcal{K}_n(\psi;t)-\psi(t)| \\
&\quad = |\mathcal{K}_n^*(\psi;t)+\psi\left(\frac{n-2}{n}t+\frac{3}{2n}\right)-\psi(t)-\psi(t)+g(t)-g(t)+\mathcal{K}_n^*(g;t) \\
&\quad -\mathcal{K}_n^*(g;t) \\
&\quad \leq |\mathcal{K}_n^*(\psi-g;t)-(\psi-g)(t)|+|\mathcal{K}_n^*(g;t)-g(t)| \\
&\quad +\left|\psi\left(\frac{n-2}{n}t+\frac{3}{2n}\right)-\psi(t)\right| \\
&\quad \leq 4\|\psi-g\|+\|g''\|\{\gamma_{n,2}(t)+\frac{1}{2}\gamma_{n,1}^2(t)\}+\omega(\psi,\zeta)\left(1+\frac{1}{\zeta}\left|\frac{-2}{n}t+\frac{3}{2n}\right|\right)
\end{aligned}$$

Taking infimum to RHS of above equation over $g \in C^2[0,1]$ and $\zeta = |\gamma_{n,1}(t)|$, we are led to

$$|\mathcal{K}_n(\psi;t)-\psi(t)| \leq 4K\left(\psi;\{\gamma_{n,2}(t)+\frac{1}{2}\gamma_{n,1}^2(t)\}\right)+2\omega\left(\psi,\left|\gamma_{n,1}(t)\right|\right).$$

We reach the required result immediately by using equation 4

**C. Voronovskaya-Type Asymptotic Result:** In this subsection, we derive an asymptotic formula for the proposed operator as follows:

**Theorem 5:** *Let* $\psi \in C^2[0,1]$. *and* $t \in (0,1)$. *Then, we have*

$$\lim_{n\to\infty} n\left(\mathcal{K}_n(\psi;t)-\psi(t)\right) = \frac{1}{2}\{(3-4t)\psi'(t)+3t(1-t)\psi''(t)\}.$$

***Proof:*** From Peano form of remainder of Taylor's expansion, we can write

$$\psi(y) = \psi(t) + (y-t)\psi'(t) + \frac{1}{2}(y-t)^2\psi''(t) + (y-t)^2\epsilon(y,t), \tag{9}$$

where $\epsilon(y,t) = \frac{\psi''(\xi)-\psi''(t)}{2}$ for some $\xi$ lying between $t$ and $y$. Also, $\lim_{y\to t}\epsilon(y,t) = 0$. Now, operating the equation 9 by $\mathcal{K}_n(.;t)$, we get

$$\begin{aligned}\mathcal{K}_n(\psi;t) - \psi(t) \\ &= \mathcal{K}_n\big((y-t);t\big)\psi'(t) + \frac{1}{2}\mathcal{K}_n((y-t)^2;t)\psi''(t) + \mathcal{K}_n(\epsilon(y,t)(y-t)^2;t).\end{aligned}$$

Using corollary 1 and Cauchy-Schwartz inequality, we can deduce

$$\begin{aligned}\lim_{n\to\infty} n\big(\mathcal{K}_n(\psi;t) - \psi(t)\big) \\ &= \psi'(t)\lim_{n\to\infty} n\mathcal{K}_n\big((y-t);t\big) + \frac{1}{2}\psi''(t)\lim_{n\to\infty} n\mathcal{K}_n((y-t)^2;t) \\ &+ \lim_{n\to\infty}\big(n\mathcal{K}_n((y-t)^2\epsilon(y,t);t)\big) \\ &\le (3-4t)\psi'(t) + \frac{3}{2}t(1-t)\psi''(t) \\ &+ \lim_{n\to\infty}\sqrt{n^2\mathcal{K}_n((y-t)^4;t)}\sqrt{\mathcal{K}\_\{n\}(\epsilon\hat{}\{2\}(y,t);t)}.\end{aligned} \tag{10}$$

By Theorem 1, we have

$$\lim_{n\to\infty}\mathcal{K}_n(\epsilon^2(y,t);t) = \epsilon^2(t,t) = 0.$$

Using above equation in 10, we are led to the required result.

**D. Weighted Approximation:** Consider a weight function $\sigma(t) = 1 + t^2$ on (0,1). Let $B_\sigma(0,1)$ denotes the space of all functions $\phi$ on (0,1) such that

$$|\varphi(t)| \le M_\varphi\sigma(t)$$

and $C_\sigma(0,1)$ be the subspace of all continuous functions in $B_\sigma(0,1)$ endowed with norm $\|.\|_\sigma$ given by

$$\|\varphi\|_\sigma = \sup_{t\in(0,1)}\frac{\varphi(t)}{\sigma(t)}$$

Next, we prove an inequality and convergence for the operator $\mathcal{K}_n(.;t)$ in weighted space as follows:

**Lemma 4:** *Let $\psi \in C_\sigma(0,1)$. Then following inequality holds for $\mathcal{K}_n(\psi;t)$*

$$\|\mathcal{K}_n(\psi;t)\|_\sigma \le \frac{7}{3}\|\psi\|_\sigma.$$

***Proof:*** By using definition of proposed operator, we may write

$$\|\mathcal{K}_n(\psi;t)\|_\sigma = \sup_{t\in(0,1)} \frac{|\mathcal{K}_n(\psi;t)|}{\sigma(t)} \leq \|\psi\|_\sigma \sup_{t\in(0,1)} \frac{n}{1+t^2} \sum_{k=0}^{n} b_{n,k}(t) \int_{k/n}^{(k+1)/n} (1+u^2)du$$
$$= \|\psi\|_\sigma \sup_{t\in(0,1)} \frac{1}{1+t^2} \{1+\mathcal{K}_n(y^2;t)\} \leq \frac{7}{3} \|\psi\|_\sigma.$$

**Theorem 6:** *For $\psi \in C_\sigma(0,1)$, the newly modified operator $\mathcal{K}_n(.;t)$ satisfies*

$$\lim_{n\to\infty} \|\mathcal{K}_n(\psi;t) - \psi(t)\|_\sigma = 0.$$

***Proof:*** From lemma 2, we obtain

$$\|\mathcal{K}_n(y;t) - t\|_\sigma = \sup_{t\in(0,1)} \frac{|\mathcal{K}_n(y;t) - t|}{1+t^2} = \left|\frac{1}{2n}\right| \sup_{t\in(0,1)} \frac{|3-4t|}{1+t^2} \leq \frac{1}{n}.$$

Also,

$$\|\mathcal{K}_n(y^2;t) - t^2\|_\sigma = \sup_{t\in(0,1)} \frac{|\mathcal{K}_n(\psi;t) - \psi(t)|}{1+t^2} \leq \frac{-35}{3n^2} + \frac{13}{n}.$$

Thus, in limiting condition, we can write

$$\lim_{n\to\infty} \left\|\mathcal{K}_n\left(y^j;t\right) - t^j\right\|;\ j = 0,1,2.$$

Then, the weighted convergence holds for all $\psi \in C_\sigma(0,1)$ from the results given by Gadjiev [28].

## V. GRAPHICAL ANALYSIS

Now, we introduce some simulation results in order to substantiate the convergence behavior of $\mathcal{K}_n(\psi;t)$ for continuous as well as piece-wise continuous functions $\psi$ by using MATLAB. To test the approximation behavior of newly defined operators, let us consider a polynomial function $\psi(t) = t^3 - t^2 + \frac{t}{10} + 0.1$ and a piecewise continuous function $\varphi$ given by

$$\phi(t) = \begin{cases} e^{2t},\ 0 \leq t \leq 0.4 \\ cos(10t),\ 0.4 < t \leq 1. \end{cases}$$

As the new sequence of operators is defined on (0, 1), so for that we will consider approximation over equally spaced grids in [0.0005, 0.9995]. Figure 1 and 2 shows the approximation and error in the approximation by proposed operator to $\psi(t)$ respectively for n=20, 50 and 100. On the other hand, behavior of proposed operators towards $\varphi(t)$ is shown in figure 3 and we can observe from the graph that error of approximation near point of discontinuity is gradually increasing here.

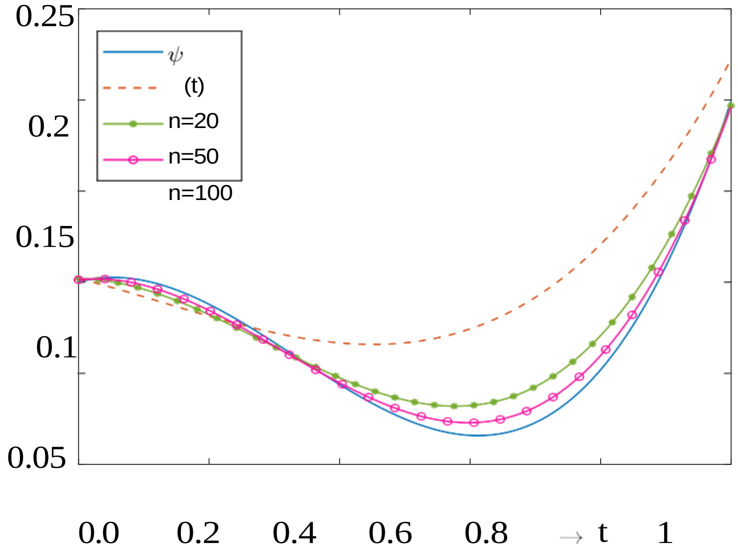


**Figure 1:** Approximation by proposed operator $\mathcal{K}_n(\psi;t)$ to $\psi$ at different values of n.

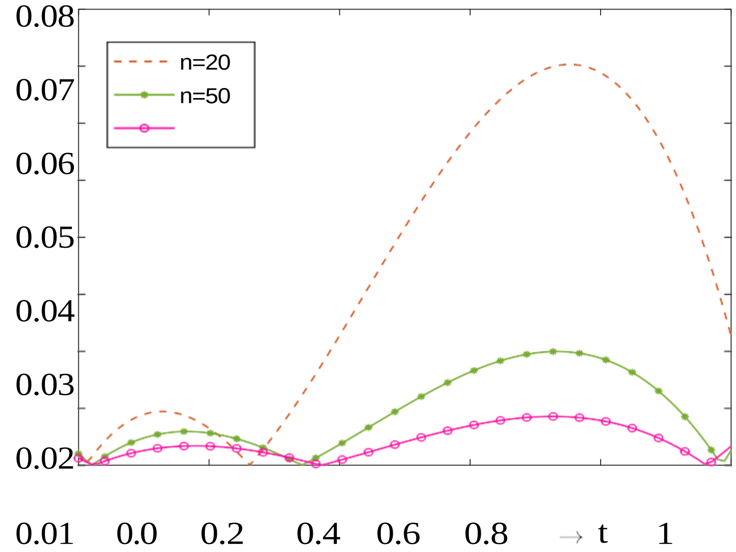


**Figure 2:** Error in the approximation by proposed operator $\mathcal{K}_n(\psi;t)$ to $\psi$ at different values of n.

## VI. CLOSING COMMENTS

In this manuscript, we presented modified-Bernstein-Kantrovinch operators and discussed the rate of convergence, asymptotic formula and weighted approximation of these operators. Also, we included some numerical simulations in order to test the newly defined operators.

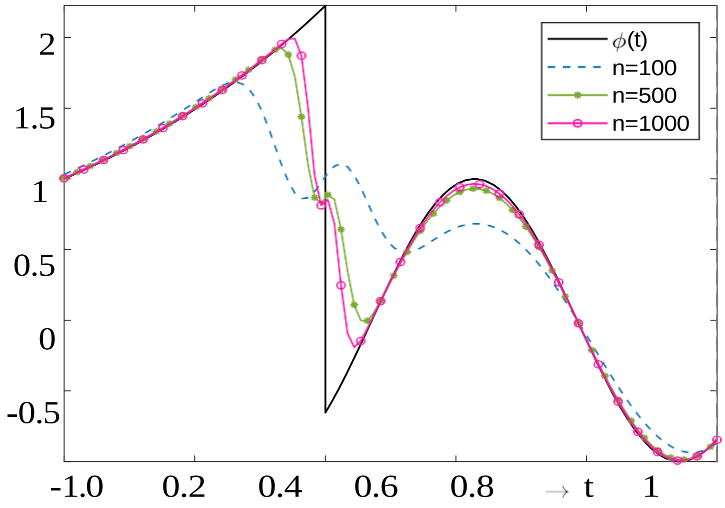


**Figure 3:** Approximation by Proposed Operators $K_n(\varphi;t)$ to Discontinuous Function $\varphi(t)$ at Different Values of n.

**Conflict of Interest:** There is no conflict of interest.

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## REFERENCES

[1] S. Bernstein, Démonstration du théoréme de weierstrass fondée surle calculde probabilités, Comm. Soc. Math. Kharkow13 (1912/13) 1–2.
[2] F. Usta, On new modification of bernstein operators: theory and applications, Iranian Journal of Science and Technology, Transactions A: Science 44 (4) (2020) 1119–1124.
[3] M. Sofyalıog˘lu, K. Kanat, B. Cekim, Parametric generalization of the modified bernstein operators, Filomat 36 (5) (2022).
[4] S. Sonker, Priyanka, Approximation properties of modified-bernstein operators having szász weight functions, in: International Con- ference on Soft Computing: Theories and Applications, Springer, 2023, pp. 177–185.
[5] S. Sonker, Priyanka, (c, 1, 1)-quasinormal convergence of double sequence of functions, in: Mathematical Modeling, Computational Intelligence Techniques and Renewable Energy: Proceedings of the Third International Conference, MMCITRE 2022, Springer, 2023,
[6] pp. 13–23.
[7] Q.-B. Cai, B.-Y. Lian, G. Zhou, Approximation properties of $\lambda$ -bernstein operators, Journal of inequalities and applications 2018 (1) (2018) 1–11.
[8] N. L. Braha, T. Mansour, H. M. Srivastava, A parametric generalization of the baskakov-schurer-szász-stancu approximation operators, Symmetry 13 (6) (2021) 980.
[9] P. Narain Agrawal, B. Baxhaku, R. Shukla, On q-analogue of a parametric generalization of baskakov operators, Mathematical Methods in the Applied Sciences 44 (7) (2021) 5989–6004.
[10] A. Kajla, M. Mursaleen, T. Acar, Durrmeyer-type generalization of parametric bernstein operators, Symmetry 12 (7) (2020) 1141.
[11] S. Mohiuddine, F. Özger, Approximation of functions by stancu variant of bernstein–kantorovich operators based on shape parameter $\alpha$, Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas 114 (2) (2020) 1–17.
[12] H. M. Srivastava, F. Özger, S. Mohiuddine, Construction of stancu-type bernstein operators based on bézier bases with shape parameter $\lambda$, Symmetry 11 (3) (2019) 316.
[13] K. S. Chaudhary, N. Kumar, Fractional order fast terminal sliding mode control scheme for tracking control of robot manipulators, ISA transactions 142 (2023) 57–69.

[14] N. Kumar, K. S. Chaudhary, Neural network based fractional order sliding mode tracking control of nonholonomic mobile robots., Journal of Computational Analysis & Applications 33 (1) (2024).
[15] L. Mohan, A. Prakash, Stability and numerical analysis of the generalised time-fractional cattaneo model for heat conduction in porous media, The European Physical Journal Plus 138 (3) (2023) 1–28.
[16] L. Mohan, A. Prakash, Analysing the conduction of heat in porous medium via caputo fractional operator with sumudu transform., Journal of Computational Analysis & Applications 33 (1) (2024).
[17] L. Mohan, A. Prakash, Stability and numerical analysis of fractional bbm-burger equation and fractional diffusion-wave equation with caputo derivative, Optical and Quantum Electronics 56 (1) (2024) 26.
[18] A. Prakash, L. Mohan, Application of caputo fractional operator to analyse the fractional model of brain tumour via modified technique, International Journal of Applied and Computational Mathematics 9 (5) (2023) 117. L. Mohan, A. Prakash, Two efficient techniques for analysis and simulation of time-fractional tricomi equation, Sa¯dhana¯ 49 (2) (2024) 1–13.
[19] A. Prakash, An efficient technique for solving fractional diffusion equation arising in oil pollution via natural transform., Waves in Random and Complex Media 2273323 (2023).
[20] S. Sonker, N. Devi, B. Bhusan Jena, S. Kumar Paikray, Approximation and simulation of signals via harmonic banach summable factors of fourier series, Mathematical Methods in the Applied Sciences 46 (12) (2023) 13411–13422.
[21] S. Sonker, N. Devi, Approximation of signals by product summability means of fourier–laguerre expansion, in: Mathematical Modeling, Computational Intelligence Techniques and Renewable Energy: Proceedings of the Third International Conference, MMCITRE 2022, Springer, 2023, pp. 49–58.
[22] S. Sonker, Priyanka, Rate of convergence of parametrically generalized bivariate baskakov-stancu operators, Filomat 37 (27) (2023) 9197–9214.
[23] L. V. Kantorovich, Sur certains développements suivant les polynômes de la forme de s, Bernstein, I, II, CR Acad. URSS 563 (568) (1930) 595–600.
[24] P. Korovkin, On convergence of linear positive operators in the space of continuous functions, in: Dokl. Akad. Nauk SSSR, Vol. 90, 1953, pp. 961–964.
[25] F. Altomare, M. Campiti, Korovkin-type approximation theory and its applications, in: Korovkin-type Approximation Theory and Its Applications, de Gruyter, 2011.
[26] R. A. DeVore, G. G. Lorentz, Constructive approximation, Vol. 303, Springer Science & Business Media, 1993.
[27] A. Gadjiev, Theorems of the type of pp korovkin's theorems, Mat. Zametki 20 (5) (1976) 781–786.