

# Performance Analysis of Transmit Antenna Selection with MRC Receiver MIMO System under Generalized- $K$ Fading Channels

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**Abstract:** The outage probability and the average bit error rate (ABER) for a multiple-input multiple-output (MIMO) system under generalized- $K$  fading channel is presented in this paper. Transmit antenna selection (TAS) at the base station is performed and maximal ratio combining (MRC) at the receiver is considered for the downlink transmission. Depending on the channel state information (CSI), the antenna at the transmit end that maximizes the MRC output SNR is selected for transmission. The effect of fading parameters on the outage and ABER of the system has been observed analytically. The effect of the number of transmit and receive antennas on the system has been studied. Computer simulations are performed to assess the accuracy of the proposed mathematical analysis.

**Keywords:** ABER, Generalized- $K$  Distribution, Maximal Ratio Combining, MIMO, Outage Probability, Transmit Antenna Selection.

## I. INTRODUCTION

In wireless communication, fading occurs due to multiple transmission paths, irregularities of earth surfaces or varying terrains or from man-made disturbances such as buildings or concrete walls etc. As a result the receiver perceives multiple copies of transmitting signal. Each signal copy will experience differences in attenuation, delay and phase shift which causes the received signal to fluctuate.

The multiple-input-multiple-output (MIMO) is an antenna system that can be used to provide improvements in both channel robustness as well as channel throughput. The MIMO system provides the best diversity gain of the signals received at the receiving end. However, in MIMO schemes, for a large number of antennas, the hardware complexity as well as the price of the system goes high. Simultaneous transmissions from multiple antennas have the inherent disadvantages of inter-antenna interference, the requirement of synchronization etc. The TAS is one of the most popular MIMO system to overcome these disadvantages. In TAS scheme, the CSI of all links has been sent back to the base station and based on CSI information the transmitter allots the best antenna for the user. The TAS scheme has been investigated over various flat fading channels in the past. In [1], the expression of ABER for TAS/MRC wireless systems under Hoyt fading channels have been derived and in [2] the expression for both outage probability and exact BER for the TAS / MRC scheme have been shown.

The generalized- $K$  ( $K_G$ ) distribution can be used to model the fading, shadowing and the propagation path-loss experience in mobile communication channels [3].  $K_G$  fading is a composite fading that consists of Nakagami- $m$  and Gamma distribution. The  $K_G$  fading model is a generalized model as it can be used to approximate many other fading models, such as  $K$  fading, Nakagami- $m$  and Rayleigh-Lognormal (R-L) [3][4][5]. It can usually cover more communications

scenarios encountered in real mobile wireless systems, than the other composite channel models [6]. In [4], the outage probability and the channel capacity over  $K_G$  fading channel is analyzed. However, a detailed performance analysis of more general receiver structures such as TAS/MRC operating over  $K_G$  channel is not available in the open technical literature. In this paper the outage probability and ABER performance of the TAS technique with MRC at receiver under  $K_G$  fading channel are realized.

In Section II, the system and channel model is described. The outage probability and ABER are analyzed in Section III and Section IV, respectively. The numerical results and discussions are provided in Section V. Finally, the conclusions are given in Section VI.

## II. SYSTEM AND CHANNEL MODEL DESCRIPTION

The MIMO wireless communication system with  $N_t$  transmit antennas at the base station and  $N_r$  receiver antennas with the user is shown in Figure 1. The scheduler at the base station selects the best transmit antenna which maximizes the post-processing SNR at the output of MRC receiver based on channel state information (CSI). The channel between the transmit antenna and the user is modeled as a slow flat fading  $K_G$  fading channels. MRC is performed with the user of the system to improve the quality of the downlink information. In the MRC receiver, the received signals from all diversity antennas are co-phased, multiplied by a weight factor proportional to the branch SNR and add together.

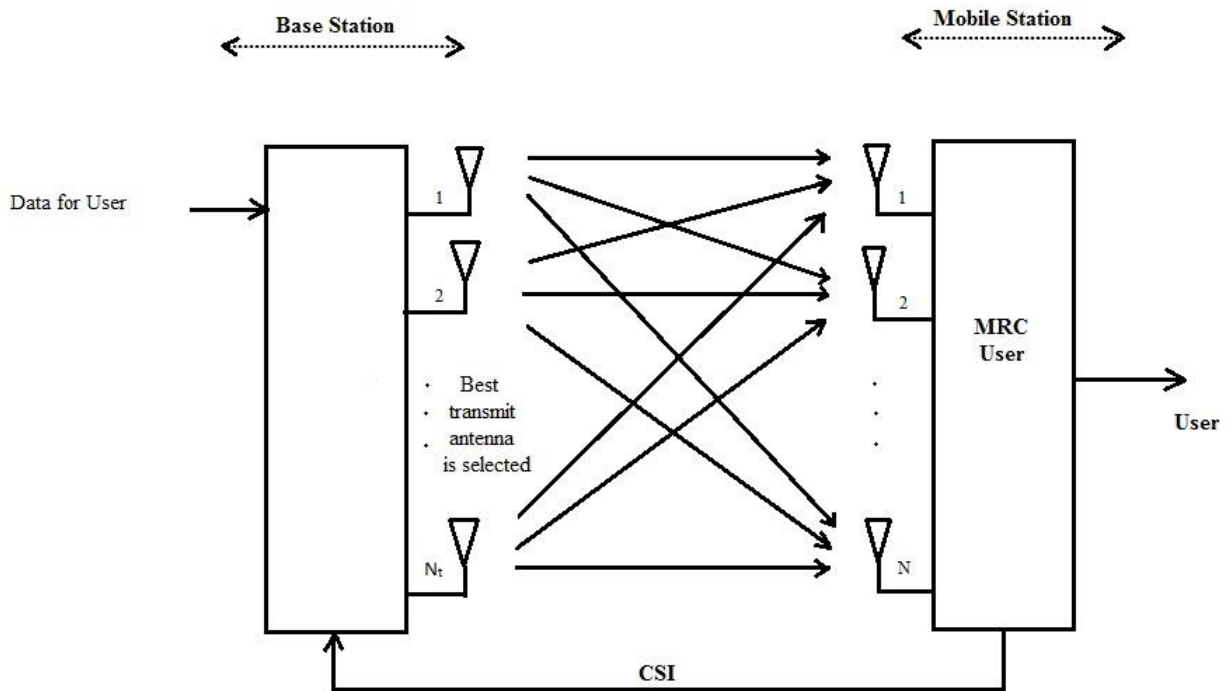


Figure1: TAS with MRC receiver

It was verified that both a single  $K_G$  RV and the sum of independent  $K_G$  RVs can be closely approximated by a single Gamma RV [9]. Denoting  $\gamma_i$  as a Gamma distributed RV with a shape parameter  $\rho$  and a scale parameter  $\theta$ , the probability density function (PDF) of the instantaneous SNR  $\gamma_i$  is given as [10][11]

$$f_{\gamma_i}(\gamma) = \frac{\theta^{-\rho}}{\Gamma(\rho)} \gamma^{\rho-1} \exp\left(-\frac{\gamma}{\theta}\right). \quad (1)$$

Where,  $\theta = (AF - \varepsilon) \bar{\gamma}$  and  $\rho = \frac{N_r}{AF - \varepsilon}$ . If  $AF$  denotes the amount of fading,  $AF = \frac{1}{m} + \frac{1}{k} + \frac{1}{mk}$ .

$\varepsilon$  is the adjustment parameter.  $m$  and  $k$  are the fading parameters.

Therefore the CDF of the instantaneous SNR over a  $K_G$  fading channel is given as

$$F_{\gamma_i}(\gamma) = \frac{\theta^{-\rho}}{\Gamma(\rho)} \int_0^{\gamma} \gamma^{\rho-1} \exp\left(-\frac{\gamma}{\theta}\right) d\gamma. \quad (2)$$

Simplifying using [7, (3.381.1)],

$$F_{\gamma_i}(\gamma) = \left[ \frac{1}{\Gamma(\rho)} g\left(\rho, \frac{\gamma}{\theta}\right) \right], \quad (3)$$

where,  $g(.,.)$  is the lower incomplete Gamma function [7].

In TAS with MRC at receiver, the best  $\gamma$  has been selected from  $N_t N_r$  number of RVs. The CDF of which can be obtained as

$$F_{\gamma}(\gamma) = \frac{1}{[\Gamma(\rho)]^{N_t}} \left[ g\left(\rho, \frac{\gamma}{\theta}\right) \right]^{N_t}. \quad (4)$$

### III. OUTAGE PROBABILITY ANALYSIS

It is denoted by  $P_{out}$  and is defined as the probability that the instantaneous error probability exceeds a specified value or equivalently, the probability that the output SNR  $\gamma$ , falls below a certain specified threshold,  $\gamma_{th}$  [8]. For the TAS with MRC receiver system an expression for the outage probability can be obtained from (4) by putting  $\gamma = \gamma_{th}$  as

$$P_{out} = \left[ \frac{1}{[\Gamma(\rho)]} g\left(\rho, \frac{\gamma_{th}}{\theta}\right) \right]^{N_t}. \quad (5)$$

Defining  $\bar{\gamma}_N \triangleq \frac{\bar{\gamma}}{\gamma_{th}}$  as the normalized average branch SNR, the expression of outage probability can be given as

$$P_{out} = \left[ \frac{1}{[\Gamma(\rho)]} g\left(\rho, \frac{1}{(AF - \varepsilon)\bar{\gamma}_N}\right) \right]^{N_t}. \quad (6)$$

#### IV. AVERAGE BIT ERROR RATE ANALYSIS

The ABER depends on the fading distribution and modulation technique. The ABER can be calculated by averaging the conditional error probability (CEP), i.e., the error rate under AWGN, over the output SNR. It can be given as [1],

$$\bar{P}_e = -\int_0^{\infty} P_e'(\gamma) F_\gamma(\gamma) d\gamma, \quad (7)$$

where,  $P_e'(\gamma)$  is the derivative of the CEP, which can be written as [1]

$$P_e'(\gamma) = -\frac{\varpi^\delta \gamma^{\delta-1} e^{-\varpi\gamma}}{2\Gamma(\delta)}. \quad (8)$$

Where,  $\Gamma(\delta)$  is the gamma function and the value of the constants  $\varpi$  and  $\delta$  are [1]:  $(\varpi, \delta) = (1, 0.5)$  for BPSK,  $(\varpi, \delta) = (0.5, 0.5)$  for BFSK. Putting the value of  $F_\gamma(\gamma)$  and  $P_e'(\gamma)$  from (4) and (8), respectively ABER can be given as

$$\bar{P}_e = \frac{\varpi^\delta}{2\Gamma(\delta)[\Gamma(\rho)]^{N_t}} \int_0^{\infty} \gamma^{\delta-1} e^{-\varpi\gamma} \left[ g\left(\rho, \frac{\gamma}{\theta}\right) \right]^{N_t} d\gamma. \quad (9)$$

Using [13, (1.7)] in (9) and after simplification, the ABER can be obtained as

$$\bar{P}_e = \frac{\varpi^\delta}{2\Gamma(\delta)[\Gamma(\rho)]^{N_t}} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \dots \sum_{n_{N_t}=0}^{\infty} \frac{\left(\frac{1}{\theta}\right)^{\left(N_t\rho + \sum_{i=1}^{N_t} n_i\right)}}{\prod_{i=1}^{N_t} (\rho)_{n_i+1}} \int_0^{\infty} \gamma^{\left(N_t\rho + \sum_{i=1}^{N_t} n_i + \delta\right)-1} e^{-\gamma\left(\varpi + \frac{N_t}{\theta}\right)} d\gamma. \quad (10)$$

Solving the integral using [7, (3.381.4)] and simplifying, the expression of ABER can be given as

$$\bar{P}_e = \frac{\varpi^\delta}{2\Gamma(\delta)[\Gamma(\rho)]^{N_t}} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \dots \sum_{n_{N_t}=0}^{\infty} \frac{\left(\frac{1}{\theta}\right)^{\left(N_t\rho + \sum_{i=1}^{N_t} n_i\right)} \Gamma\left(N_t\rho + \sum_{i=1}^{N_t} n_i + \delta\right)}{\left(\varpi + \frac{N_t}{\theta}\right)^{\left(N_t\rho + \sum_{i=1}^{N_t} n_i + \delta\right)} \prod_{i=1}^{N_t} (\rho)_{n_i+1}}. \quad (11)$$

## V. NUMERICAL RESULTS AND DISCUSSIONS

Numerically evaluated results for outage probability and ABER with BPSK modulation scheme have been presented in this section.

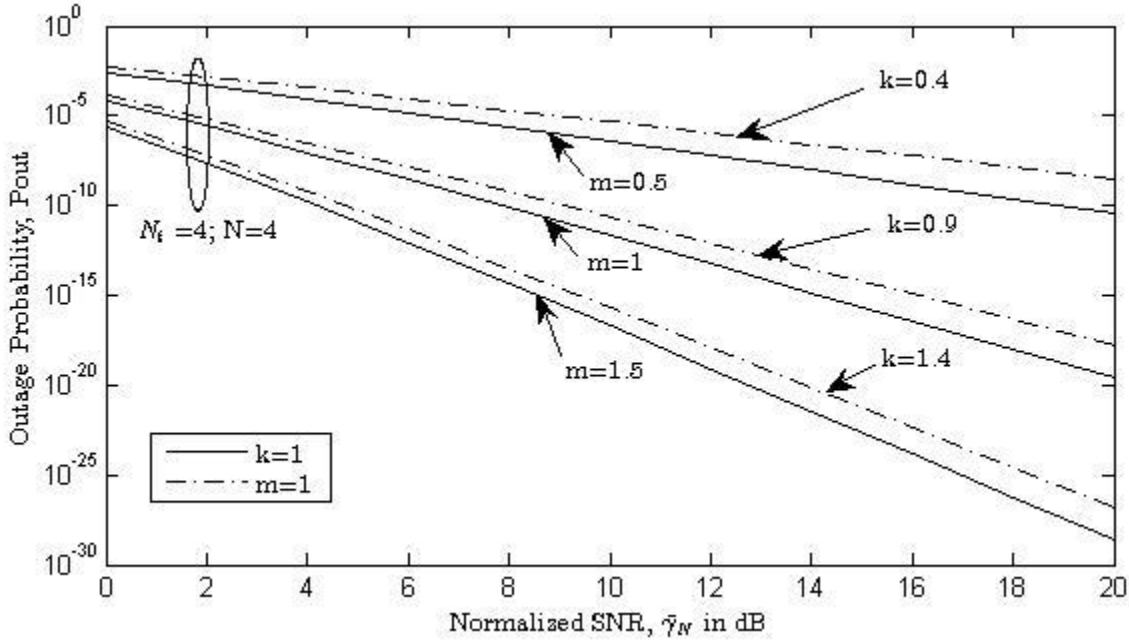


Figure 2: Outage Probability vs. Normalized SNR in dB for different values of fading parameters and  $N_t = 4, N_r = 4$ .

Outage probability vs. normalized SNR per branch (in dB), has been plotted in Figure 2, considering  $N_t = 4$  and  $N_r = 4$ . From the Figure 2, it is observed that for a constant value of  $k$  ( $k = 1$ ), the outage performance improves as the value of  $m$  increases, corresponding to that the fast fading becomes less severe. Similarly, it has been observed that for a fixed value of  $m$  ( $m = 1$ ) the outage performance improves as the value of  $k$  increases, implying that the channel becomes less shadowing. It is considered that the adjustment parameter  $\varepsilon = 1$ . The performance of the system gets better with the increase of the fading parameters. This is because the smaller is the fading parameter, the severe is the channel fading, and the worse is the system performance.

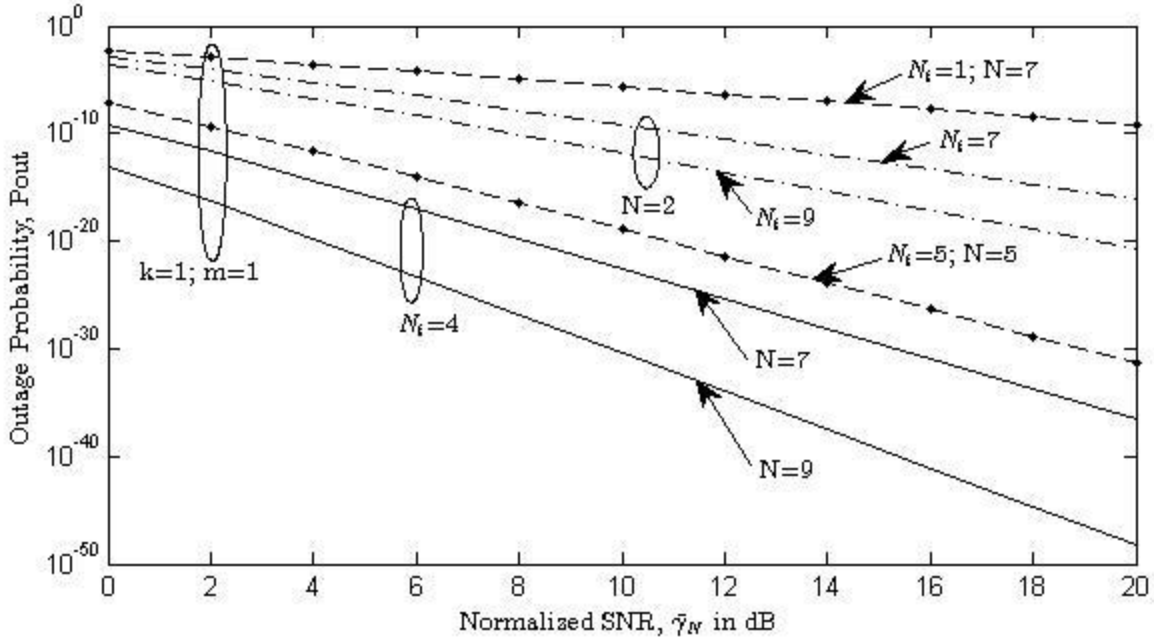


Figure 3: Outage Probability vs. Normalized SNR for a different number of transmit antenna ( $N_t$ ) and received antenna ( $N_r$ ) with  $k = 1, m = 1$ .

In Figure 3, the outage probability is plotted against normalized SNR with different diversity order and fading parameters are considered as  $k = 1, m = 1$ . It is considered that  $\varepsilon = 1$  for analysis. The outage probability improves with the increase in normalized SNR. From the Figure 3, it is observed that outage performance improves with the increase in the number of transmitting antennas at the base station, for a fixed value of diversity order ( $N_r = 2$ ). Similar observations can be made by increasing the number of receiving antennas at the user terminal for a constant value of transmitting antenna ( $N_t = 4$ ) of the system.

In Figure 4, Average Bit Error Rate vs. Average SNR per branch  $\bar{\gamma}$  (in dB), has been plotted for BPSK modulation scheme with  $N_t = 3, N_r = 3$ . The ABER performance improves with an increase in fading parameter  $m$  for a constant value of fading parameter  $k$ . Similarly, one can observe that with an increase in fading parameter  $k$  for a constant value of  $m$  the ABER performance improves.

In Figure 5, the ABER performance of TAS configuration with BPSK modulation and for different number of transmit antenna  $N_t$  and receive antenna  $N_r$ , is shown. From the Figure 5, one can observe that when the number of transmit antennas become larger for a fixed number of receive antennas, the ABER performance of the system improves. Similar observations can be made by increasing the number of receive antennas for a fixed number of transmit antennas. From the figures, it is observed that with the increase in selection gain, the ABER performance of the TAS with MRC receiver system has improved. The results obtained numerically by means of the derived expressions have been validated by Monte Carlo simulations.

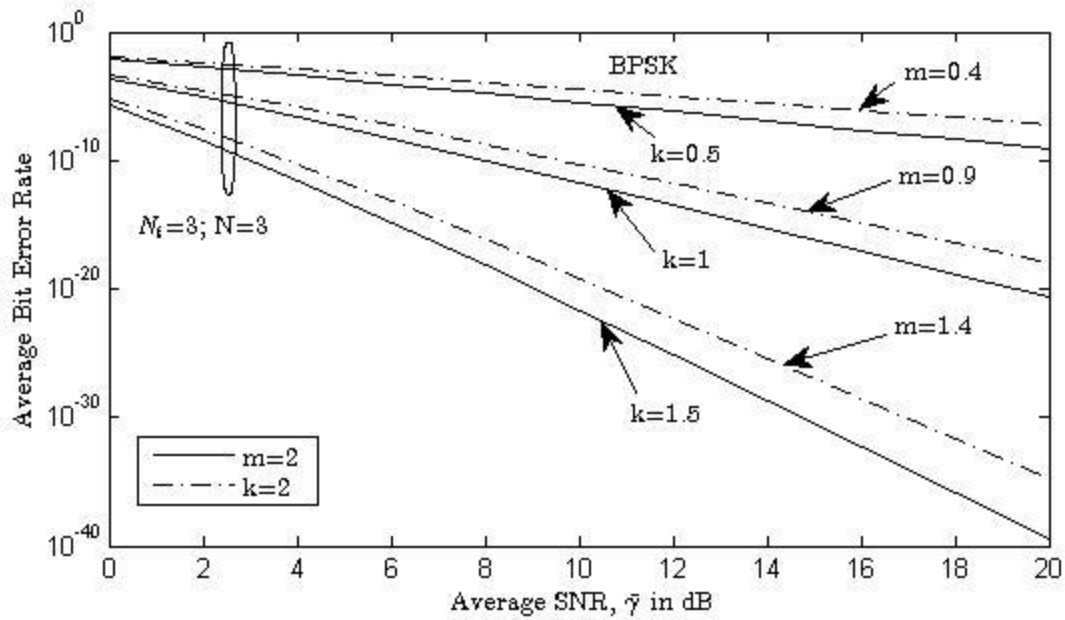


Figure 4: Average Bit Error Rate vs. Average SNR for different values of fading parameters and  $N_t = 3, N_r = 3$ .

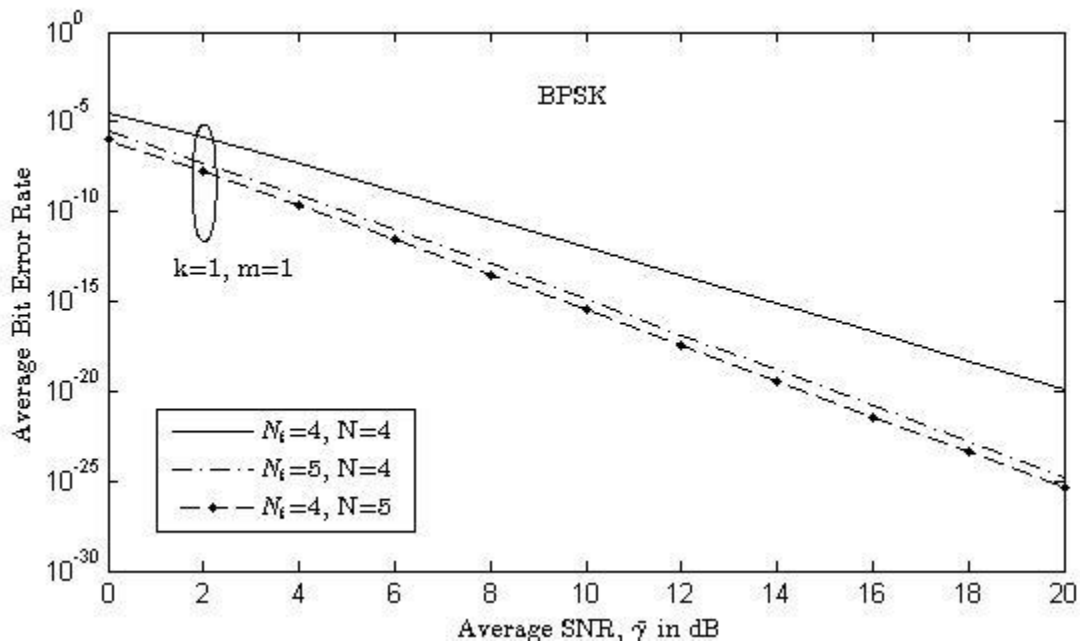


Figure 5: Average Bit Error Rate vs. Average SNR for a different number of transmit antenna ( $N_t$ ) and received antenna ( $N_r$ ) with  $k = 1, m = 1$ .

## VII. CONCLUSIONS

The outage probability and ABER over the  $K_G$  fading channels have been investigated. In this work, the TAS and MRC receiver scenario is considered. The expressions of outage probability and the ABER have also been derived in terms of incomplete Gamma function. The arbitrary number of transmit, receive antennas and fading parameters are considered for the analysis. Finally, computer simulations have been provided to assess the accuracy of the proposed analytical framework.

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