**Mathematics for Forecasting**

# INTRODUCTION

⏺ Forecasting involves predicting the future.

⏺ Forecasting allows logistics managers to be proactive rather than reactive.

⏺ The forecasting process affects every area of logistics in some way or the other.

⏺ A major purpose of forecasting is to give us choice of alternative plan, design, and backup resources.

⏺ It estimates the quantity of a product or service that consumers will purchase.

⏺ Demand forecasting involves techniques including both informal methods, such as qualitative and quantitative methods.

⏺ Demand forecasting may be used in making pricing decisions, whether to enter a new market etc.

⏺ A major component of demand management is forecasting the amount of product that will be purchased by customers or end users.

⏺ Logistics function assumes responsibility for scheduling production on a short-term basis in order to co-ordinate demand for finished product with the timing and availability of needed production inputs.

# REASONS FOR FORECASTING

1. Increasing customer satisfaction

2. Reducing stock outs

3. Lowering safety stock needs

4. Scheduling production efficiently

5. Reducing product obsolescence costs

6. Managing pricing and promotion in a better way

7. Improving shipping

|  |
| --- |
| **Quantitative Methods**  |

A statistical technique projection about the future which uses numerical facts is known as the quantitative method.

**a) Graphical Method**

 ⏺ Plotting information in a graphical form. It is relatively easy to convert a spreadsheet into a graph that conveys the information in a visual manner.

 ⏺ Trends & patterns are easier to spot & extrapolation of previous demand can be used to predict future demands.

**b) Econometric Modellingg**

 ⏺ A set of equations intended to be used simultaneously to capture the way in which dependent and independent variables are interrelated.

**c)** **Moving Average Techniques**

 ⏺ Moving average forecasting uses an average of the most recent period's sales.

 ⏺ The average may contain any number of previous time periods, although one, three, four, and twelve-period averages are common

 ⏺ A one-period moving average results in next period's forecast being projected by last period's sales.

 ⏺ A twelve-period moving average, such as monthly, uses the average of the last twelve periods.

 ⏺ Each time a new period of actual data becomes available, it replaces the oldest time period's data. Thus the number of time periods included in the average is held constant.

 E.g. of moving average:

|  |  |  |
| --- | --- | --- |
| Years | Demand (Units) | **3 yearly moving average** |
| 1 | 120 | - |
| 2 | 90 | - |
| 3 | 150 | - |
| 4 | 180 | 120 |
| 5 | 210 | 140 |
| 6 | ? | 180 |

The forecast for the 4th year as per the demand based on the previous years is calculated as follows:

 4th year forecast = (120+90+150)/3 = 120 units

**d)Exponential Smoothing Analysis/Curve:** This is another time series forecasting technique where the forecast for the next period is calculated as 'weighted average' of all previous values. It is based on the principle that the most recent values are the most important for predicting the future value. Also, it presumes that values prior to the current value are also relevant but in a declining importance as we go back in time. We have the equation as follows:

 Y1t+1= ayt + (1 - a) Y1t

Where,

 Y1t+1=new demand forecast

 Y1t=old demand based on moving average

 yt=old demand (actual value)

 a = Smoothing constant, such that 0 < a < 1 (“a” lies between 0 and 1), generally

 a = 0.2 ( if Not Given)

 e.g.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Year | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 |
| Demand (Units) | 213 | 201 | 198 | 207 | 220 | 232 | ? |

 Using exponential smoothing analysis,

 t + 1 = 7 ( as we need to forecast for the 7th Year)

 Therefore, t = 6

 Substituting in the above formula we get,

 Y17=ay6+ (1 - a) Y16

 = 0.2 232 + (1 - 0.2) 212

 = 46.4 + 0.8 ` 212

 = 46.4 + 169.6

 = 216 units

 Calculation of Y16= = 212 units

**e) Regression Analysis:** Regression is derived from the word 'to regress', which means to go in past. Statistics meaning of regression analysis is to collect the past available data and analyze them for the future strategies. The general regression model is given by

 Y = a + bx + ui

###### Where,

 Y = dependent variable

 x = independent variable

 ui = un-explanatory terms

 For simple regression analysis we assume ui = 0

 Therefore, the model becomes;

 Y = a + bx ; r2

###### Where,

 r2= explanatory power of the model. It explains whether the model is good or bad.

 Significance of Y = a + bx ; r2

 ⏺ If sign of b is positive, which implies that y is directly proportional to x

 ⏺ If sign of b is negative then it implies that y is inversely proportional to x

 ⏺ Value of r2lies between 0 and 1, i.e. 0 < r2< 1

 e.g.:

 If r2= 0.9 = 90% which implies that 90% of variations in Y is explained by x and the remaining 10% is due to ui

**Note:**

 ⏺ If r2is greater than and equal to 0.8 then the model is a good one

 ⏺ If r2is greater than and equal to 0.6 and is less than 0.8 then the model is an average one.

 ⏺ If r2is less than 0.6 then the model is a poor one.

 **Multiple regression analysis:** A multiple regression model in 'n' variables is given by:

 **Y = a + b1x1 + b2x2 + ……….bn-1 xn-1**

###### Where,

 a = constant

 b1, b2……bn-1= regression co-efficient

 x1, x2……xn-1= independent variable

 Y = dependent variable.

#  SOLVED PROBLEMS

1. For the following data calculate a 3 period and 5 period moving average?

 Also Forecast demand for 11th month.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Demand (Units) | 110 | 120 | 135 | 142 | 154 | 160 | 173 | 180 | 190 | 210 | ? |

**Solution:**

|  |  |  |  |
| --- | --- | --- | --- |
| Period | Demand (limits) | 3 period average | 5 period average |
| 1 | 110 | – | – |
| 2 | 120 | – | – |
| 3 | 135 | – | – |
| 4 | 142 | 121.67 | – |
| 5 | 154 | 132.33 | – |
| 6 | 160 | 143.67 | 132.2 |
| 7 | 173 | 152 | 142.2 |
| 8 | 180 | 162.33 | 152.8 |
| 9 | 190 | 171 | 161.8 |
| 10 | 210 | 181 | 171.4 |
| 11 | ? | 193.33 | 182.6 |

 ∴ As 3 period moving average forecasted value for 11th period is 193.33 units and as per 5 period moving average forecasted value is 182.60 units.

2. You are given the following information about demand of an item:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Demand(units) | 220 | 228 | 217 | 219 | 258 | 241 | 239 | 244 | 256 | 260 | 265 |

Calculate forecasted values using (i) three – monthly moving averages, (ii) five – monthly moving averages.

**Solution:**

The required values are given in third to fifth columns of Table 1. The three monthly values are obtained as (220 + 228 + 217) / 3 = 221.67, (228 + 217 + 219) / 3 = 221.33 and so on. Similarly, five–monthly values are obtained by considering five monthly data.

**Table 1 : Calculation of Forecasted Demand**

|  |  |  |  |
| --- | --- | --- | --- |
| **Month** | **Demand Y** | **3 – monthly****Moving Average** | **5 – monthly****Moving Average** |
| 1 | 220 |  |  |
| 2 | 228 |  |  |
| 3 | 217 |  |  |
| 4 | 219 | 221.67 |  |
| 5 | 258 | 221.33 |  |
| 6 | 241 | 231.33 | 228.40 |
| 7 | 239 | 239.33 | 232.60 |
| 8 | 244 | 246.00 | 234.80 |
| 9 | 256 | 241.33 | 240.20 |
| 10 | 260 | 246.33 | 247.60 |
| 11 | 265 | 253.33 | 248.00 |
| 12 |  | 260.33 | 252.80 |

3. Obtain the profit forecasts using (i) four–yearly moving averages, and (ii) five yearly moving averages, from the following data relating to sales (‘000’)

 Year 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015

 Sales(`) 48 53 55 56 58 63 68 60 61 68 58 63 70 76 83 88

**Solution:**

 The given data and the required moving averages are shown in Table 2.

|  |  |  |  |
| --- | --- | --- | --- |
| **Year** | **Sales** | **4 – Yearly****Moving Average** | **5 – Yearly****Moving Average** |
| 2000 | 48 |  |  |
| 2001 | 53 |  |  |
| 2002 | 55 |  |  |
| 2003 | 56 |  |  |
| 2004 | 58 | 53.00 |  |
| 2005 | 63 | 55.50 | 54.00 |
| 2006 | 68 | 58.00 | 57.00 |
| 2007 | 60 | 61.25 | 60.00 |
| 2008 | 61 | 62.25 | 61.00 |
| 2009 | 68 | 63.00 | 62.00 |
| 2010 | 58 | 64.25 | 64.00 |
| 2011 | 63 | 61.75 | 63.00 |
| 2012 | 70 | 62.50 | 62.00 |
| 2013 | 76 | 64.75 | 64.00 |
| 2014 | 83 | 66.75 | 67.00 |
| 2015 | 88 | 73.00 | 70.00 |
| 2016 |  | 79.25 | 76.00 |

4. Find the “forecast” for year 7th using weighted “Three years” Moving Average.

|  |  |  |  |
| --- | --- | --- | --- |
| Year **(`)** | Sales (1000 `) | Weighted | Weighted moving Averages |
| 1 | 10 | 2 | - |
| 2 | 15 | 3 | - |
| 3 | 12 | 4 | - |
| 4 | 14 | 5 | [2(10)+3(15)+4(12)] / 2+3+4 = 12.56 |
| 5 | 16 | 6 | [3(15)+4(12)+5(14)] / 3+4+5 =13.58 |
| 6 | 18 | 7 | [4(12)+5(14)+6(16)] / 4+5+6 =14.27 |

 Thus for 7th year for casted value = = 16.22

5. Calculate weighted 5 period moving average for the following data. Also find forecasted value for 10th period.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Demand (Units) | 10 | 15 | 5 | 25 | 40 | 30 | 50 | 45 | 60 |

**Solution**

|  |  |  |
| --- | --- | --- |
| Period | Demand (Units) | 5-period moving weighted average |
| 1 | 10 | – |
| 2 | 15 | – |
| 3 | 5 | – |
| 4 | 25 | – |
| 5 | 40 | – |
| 6 | 30 |  = 23.67 |
| 7 | 50 |  = 27.34 |
| 8 | 45 |  = 36.34 |
| 9 | 60 |  = 41.34 |
| 10 | ? |  = 48.67 |

**Practice Questions**

1. Find weighted 5 period moving averages

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Period | 1 | 2 | 3 | 4 | 5 | 6 |
| NAV (Cr) | 12 | 15 | 21 | 30 | 33 | 36 |

# Weightage are considered in order of increasing order, 3(2)13

# Hint: 3(2)13 means weights start from 3 till 13 with difference of 2 in weights.

2. For the following data calculate a 3 period and 5 period moving average? Forecast demand for the 11th month.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Demand in units  | 110 | 120 | 135 | 142 | 154 | 160 | 173 | 180 | 190 | 210 | ? |