Nuclei and Radioactivity

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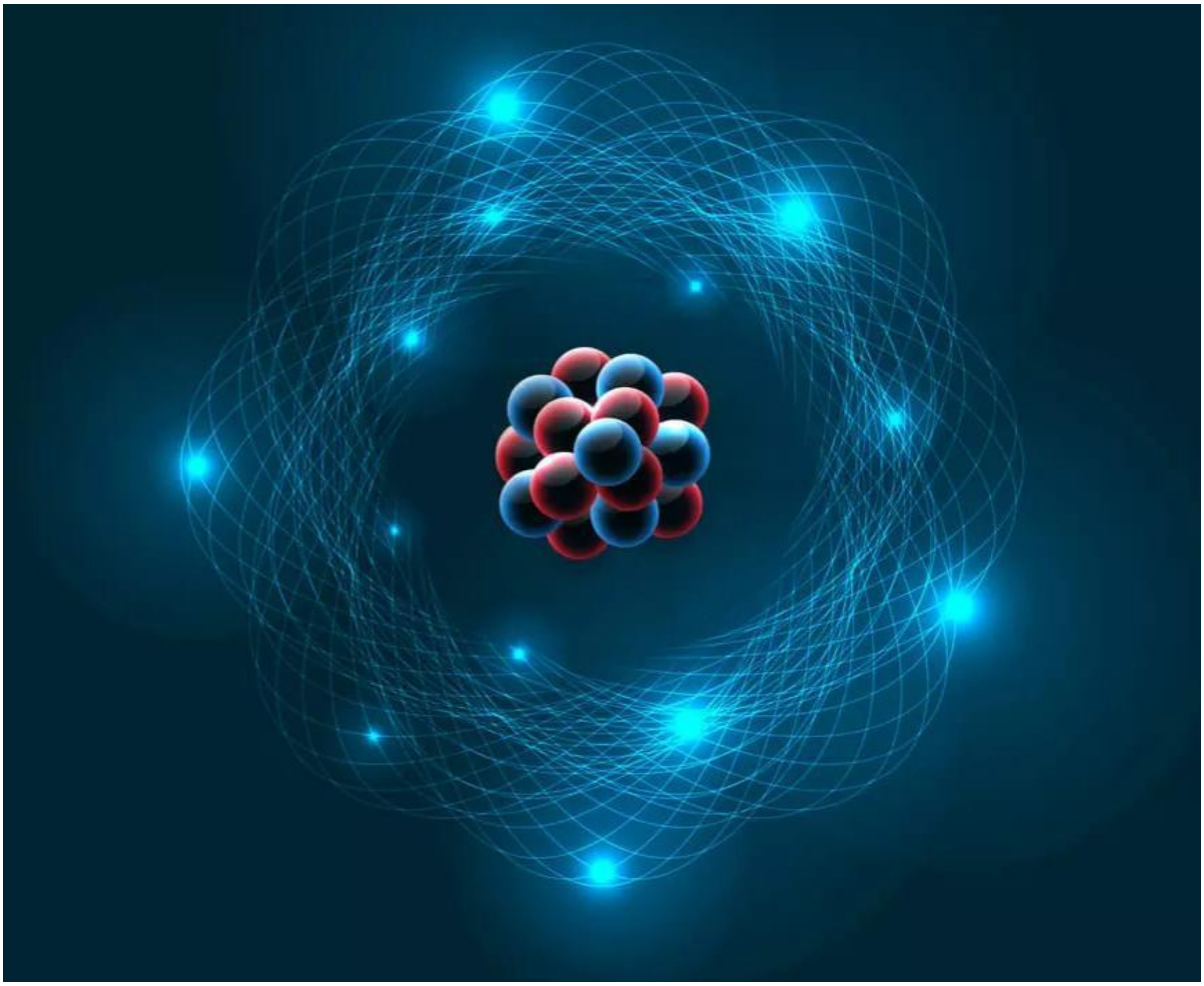


Introduction

A nucleus inside an atom is made up of protons and neutrons which are together called as nucleons. Both protons and neutrons are subatomic particles. Protons have a charge of +1e(+1.6 $\times 10^{-19}$ C) and mass 1.672 $\times 10^{-27}$ kg(~1 amu). Neutrons do not carry any charge on them, ie, 0 C and have a mass of 1.674 $\times 10^{-27}$ kg(~1 amu).

Some terminologies

Atomic number (Z): Atomic number refers to the number of protons in the nucleus of an atom. In case of a neutral atom the number of protons equals the number of electrons. The properties of elements are a periodic function of their atomic number.

Mass number(A): Mass number refers to the number of nucleons in the nucleus of an atom. Thus, it accounts for both the number of protons as well as the number of neutrons.

Atomic mass: It refers to the mass of one atom of an element.

So, atomic mass = total mass of protons + total mass of neutrons + total mass of electrons

= no. of protons × mass of one proton + no. of neutrons × mass of one neutron + 0

[Since mass of electrons is negligible as compared to mass of neutron and mass of protons]

= no. of protons × 1 amu + no. of neutrons × 1 amu

= (no. of protons + no. of neutrons) × 1 amu

= Mass number × 1 amu = A amu

Eg, 1. Mass number of carbon-12 isotope is 12 but its atomic mass is 12 amu.

2. Mass number of helium is 4 but its atomic mass is 4 amu.

Representation: Let us suppose an atom X whose mass number is A and atomic number is Z, then it is represented as $_{Z}\mathbf{X}^{A}$.

Classification of nuclei

1. **Isotopes:** Isotopes are some of an element whose nuclei have same number of protons but different number of neutrons, i.e., atomic number (Z) is same but mass number(A) is different. E.g. $^{1}H_{1}$, $^{2}H_{1}$, $^{3}H_{1}$, etc.
2. **Isobars:** Isobars are atoms of different elements whose nuclei have same number of nucleons but different number of protons, i.e. , atomic number(Z) is different but mass number (A) is same. E.g. $^{40}Ca_{20}$ and $^{40}Ar_{18}$, etc.
3. **Isotones:** Isotones are atoms of different elements whose nuclei have same number of neutrons.

Nuclear shape

Shape of nucleus is assumed to be approximately (90%) spherical.

Nuclear size

The volume of nucleus is directly proportional to the number of nucleons inside the nucleus, i.e. , more the nucleons more is the volume of nucleus. So, the volume of nucleus directly depends on the mass number of the nucleus. The radius of the nucleus is related to the mass number by **$R = R^* \times (A)^{1/3}$, where $R^* = 1.3 \times 10^{-15}$** m.

Nuclear density

The density of nucleus is approximately 2.3×10^{17} kg/m³. Nuclear density is independent of mass number A. It is same for nuclei of all atoms. Also, nuclear density is 10^{14} times more than that of water (10^3kg/m³).

Radioactivity

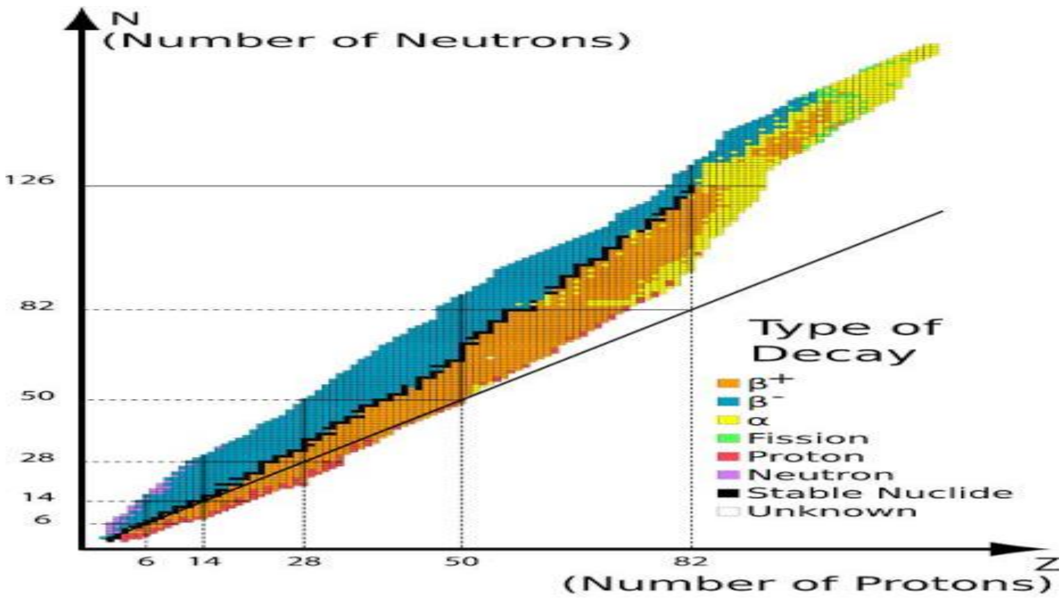
Amongst 1500 nuclides discovered so far, less than 260 are stable. The unstable nuclides gain stability by emitting alpha (α) or beta particle (β) and gamma waves (γ). The spontaneous emission of alpha particle or beta particle and gamma waves from an unstable nuclei is called radioactivity.

The rate of emission of alpha particle, beta particle and gamma waves depends only on the concentration of nuclei.

Radioactivity is a nuclear phenomenon. It is unaffected by change in pressure, temperature and other physical conditions.

Radioactivity was discovered accidently by Henry Becquerel in 1896 when he was investigation Uranium salts.

For those elements whose atomic number is less than 20, those isotopes are stable whose neutron to proton ratio(n/p) is equal to 1. For those elements whose atomic number is more than 20, those isotopes are stable whose neutron to proton ratio(n/p) less between 1 to 1.6 .



Those isotopes whose n/p ratio lies below the stability band tries to increase their n/p ratio and undergoes α and β+ decay. Those isotopes whose n/p ratio lies above the stability band tries to decrease their n/p ratio and undergoes β- decay.

<u>Types of radioactive decay:</u>

1. **Alpha decay(α)**
2. **Beta decay: {a}β+ decay {b}β- decay**
3. **Gamma decay(**γ)

Alpha decay(α)

$^{A}X_{Z} \rightarrow {}^{4}He_{2} + {}^{A-4}Y_{Z-2}$

Here, X is the parent nuclei with atomic number Z and mass number A, He is helium nucleus (α particle) and Y is the daughter nucleus with atomic number Z-2 and mass number A-4.
E.g. $^{238}U_{92} \rightarrow {}^{4}He_{2} + {}^{234}Th_{90}$

Mass defect: In any radioactive decay, although the sum of mass number and sum of atomic number of reactant is equal to the sum of mass number and sum of atomic number of product respectively, but some mass disappear in every spontaneous nuclear reaction, i.e., the mass of product is less than mass of reactant. This is called **mass defect**.

Mass defect (Δm)=mass of reactant-mass of product
Mass defect $(\Delta m)=[m_{x} - (m_{He} + m_{Y})]$

Energy: This mass actually gets converted to energy ($\Delta E/Q=\Delta mc^2$) and this is the energy which gets released in a nuclear reaction.

Momentum conservation: By conservation of linear momentum,

Initial momentum=final momentum
0= final momentum
$|P_{\alpha}|=|P_{Y}|$
$m_{\alpha} v_{\alpha} = m_{y} v_{y}$

Kinetic energy of α particle and daughter nuclei (Y) in terms of Q: The kinetic energy of α particle and daughter nuclei(Y) is obtained from the energy released due to mass defect(Q).

$KE(\alpha) + KE(Y) = Q$
$P^2(\alpha)/2m(\alpha) + P^2(Y)/2m(Y)=Q$
$P^2/2m(\alpha) + P^2/2m(Y)=Q \; [P_{\alpha} = P_{Y}]$
$P^2/m(\alpha) + P^2/m(Y) = 2Q$
$P^2=(2Qm_{\alpha} m_{Y})/(m_{\alpha}+m_{Y})$

So, $KE_{\alpha} = P^2/2m_{\alpha} = (Qm_{Y})/(m_{\alpha}+m_{Y})$
Also, $KE_{Y} = (Qm_{\alpha})/(m_{\alpha}+m_{Y})$

Since , $m_{Y} >> m_{\alpha}$ so **$KE_{\alpha} >> KE_{Y}$. Therefore, almost all energy is taken by α particle.**

β decay

β particle is of two types:

1. β^{-} particle (electron/ $^{0}e_{-1}$ / $^{0}\beta_{-1}$)
2. β^{+} particle (positron/ $^{0}e_{+1}$ / $^{0}\beta_{+1}$)

β^{-} decay: (electron emission)

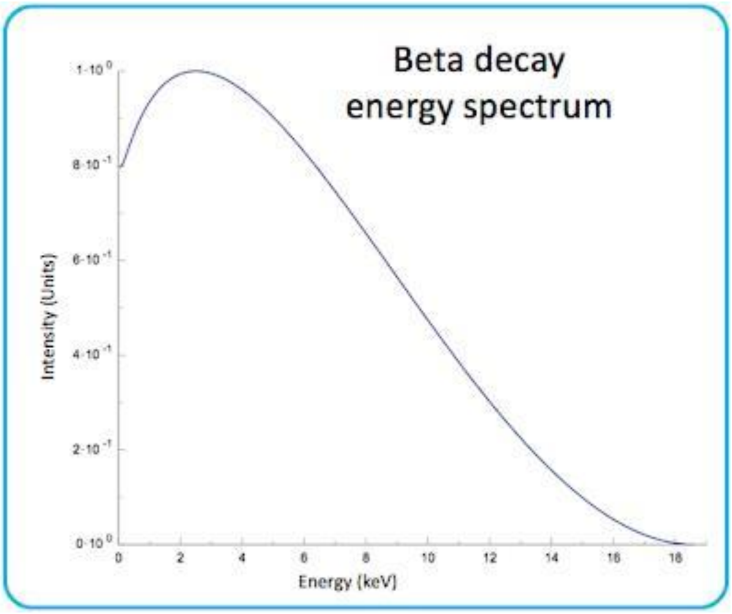
$^{A}X_{Z} \rightarrow {}^{0}\beta_{-1} + {}^{A}Y_{Z+1} + \dot{\upsilon}$ (antineutrino)

Here, X is the parent nuclei with atomic number Z and mass number A, β^- particle is the electron released during reaction and Y is the daughter nucleus with atomic number Z+1 and mass number A.
E.g. $^{14}C_{6} \rightarrow {}^{0}\beta_{-1} + {}^{14}N_{7} +$ ü
The question is, since β^- particle is electron and radioactivity is a nuclear phenomenon(involves only the nucleus) then how an electron is getting emitted during β^- decay ?
The answer is that this electron is not the outer shell electron. It gets produced in the nucleus itself during the course of decay. Inside the nucleus, a neutron gets converted to a proton and an electron(β^- particle), with the simultaneous release of an antineutrino. The reaction is described below:

$^{1}n_{0} \rightarrow {}^{1}p_{1} + {}^{0}e_{-1} +$ ü (antineutrino)

The purpose of emitting the electron(β – particle) is to conserve charge.

Almost all of the energy(Q) of this reaction is taken by β^- particle (electron) and daughter nuclei does not have any considerable KE.
For different nuclei of same sample, the energy(Q) posed by β^- particle should have been same but it is experimentally observed that energy of different β^- particles of same sample is different.



So, it is suggested that antineutrino is emitted to conserve energy as well as spin.

<u>Some features of antineutrino:</u>
1. It is antiparticle of neutrino.
2. It has no charge.
3. It has no rest mass.
4. It has high penetrating power.

<u>β^+ decay:</u> (positron emission)

$^{A}X_{Z} \rightarrow {}^{0}\beta_{+1} + {}^{A}Y_{Z-1} + \upsilon$ (neutrino)
Here, X is the parent nuclei with atomic number Z and mass number A, $\beta+$ particle is the positron released during reaction and Y is the daughter nucleus with atomic number Z-1 and mass number A.
E.g. $^{12}N_{7} \rightarrow {}^{0}\beta_{+1} + {}^{12}C_{6} + \upsilon$

In $\beta+$ decay, a proton is converted to a neutron inside nucleus and a positron is created, with the simultaneous release of neutrino. The reaction is described below:

$^{1}p_{1} \rightarrow {}^{1}n_{0} + {}^{0}e_{+1} + \upsilon$

$Q=[m_p-(m_n+m_e)]c^2<0$ [as $m_n > m_p$]
Therefore, this reaction is non-spontaneous. Energy for this reaction is provided by binding energy of nucleus or external source.

Electron capture

Sometimes when β + decay is difficult to happen, electron capture occurs. A proton in nucleus swallows a k-shell electron and converts to neutron. The reaction is described below:

$^{1}p_{1} + {}^{0}e_{-1} \rightarrow {}^{1}n_{0} + \upsilon$(neutrino)

Thus reaction is more spontaneous than β + decay .

γ decay

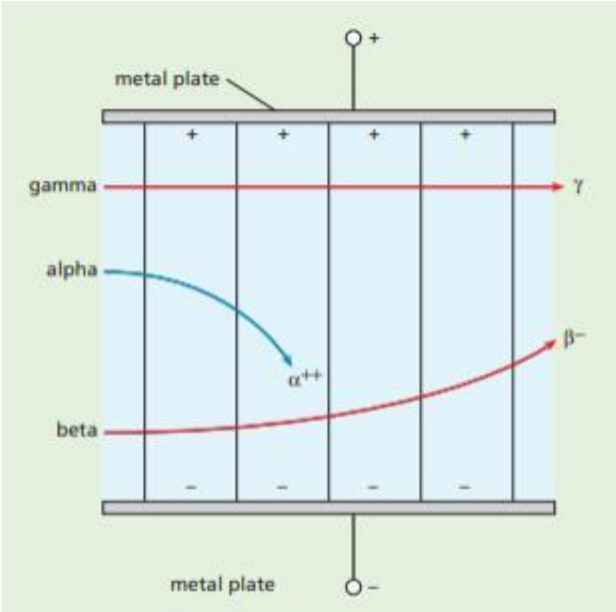
Just like the energy levels of electron, there exist different energy levels of nucleus.
Sometimes after emission of α or β particle, the nucleus reach in excited state(by absorbing part of Q). The nucleus immediately falls back to ground state (in 10^{-10} seconds). In this process, a photon is emitted (γ particle).

$^{A}X^{*}_{Z}$ (excited state)$\rightarrow$ $^{A}X_{Z}$ (ground state)+ $^{0}\gamma_{0}$

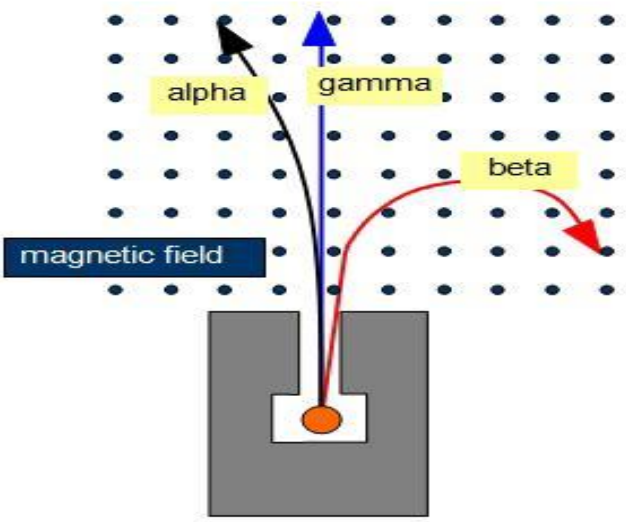
Comparison of α , β and γ particle:

Property	**α particle**	**β particle**	**γ rays**
1. Nature	Helium nuclei	Electron of nuclear origin	High energy electromagnetic radiations
2. Mass	6.67×10^{-27} kg/4 amu	9.11×10^{-31} kg	Rest mass is zero
3. Charge	+2e	-e	0
4. Deflection by electric and magnetic field	Deflected towards negative pole	Deflected towards positive pole	Nil
5. Speed	10^{7} m/s	10^{8} m/s but variable	3×10^{8} m/s
6. Ionizing power	10^{4} times γ rays	10^{2} times γ rays	Minimum
7. Penetrating power	Minimum	10^{2} times α rays	10^{4} times α rays
8. Effect on photographic plate and ZnS phosphor	Strong effect	Less effect	Least effect

1. A component which could hardly pass through 0.1 cm thick aluminum foil called α rays.
2. A component which was stopped by 5mm thick aluminum sheet called β rays.
3. A component which could pass through even 30 cm thickness of an iron piece called γ rays.



Deflection of α, β and γ rays in electric field



Deflection of α, β and γ rays in magnetic field

Law of radioactive process:

Radioactive decay is a random process. It is impossible to predict when a particular nucleus will decay. So, study of individual nucleus decay is not possible. Rutherford and Soddy conducted experimental study of decay of various radioactive nuclei and gave a statistical law:

" When there is a large number of nuclei in a sample, the rate of decay is directly proportional to the number of nuclei that are present (undecayed) in the sample. "

$\mathbf{N = N_0 e^{-\lambda t}}$, where N_0 → initial number of nuclei N → number of nuclei left undecayed
λ → decay constant. t → time

Activity (R) of a radioactive substance:

Activity (R) of a sample is the rate of decay of nuclei in that sample.

$\mathbf{R = R_0 e^{-\lambda t}}$, where R → activity after time 't' and R_0 → initial activity

Units of activity:

1. Becquerel(Bq) : 1 Bq= 1 disintegration per second
2. Rutherford : 1 Rutherford = 10^6 disintegration per second
3. Curie (Ci) : 1 Ci = 3.7×10^{10} Bq = 3.7×10^{10} disintegration per second

Half-life of decay: ($t_{½}$)

Half life is the time in which number of nuclei reduces to half of initial number of nuclei.

OR

Half life is the time in which activity of sample reduces to half of its initial value.

$$t_{½} = \ln 2 / \lambda = 0.693 / \lambda$$

Number of nuclei left after 'n' half lives:

If we have initially N_0 number of nuclei then after 'n' half lives, let the number of nuclei left in the sample be N, then

$\mathbf{N = N_0 / 2^n}$, where $n = t/t_{½}$

Similarly, $\mathbf{R = R_0 / 2^n}$, where R_0 is the initial activity and R is the activity after 'n' half lives.

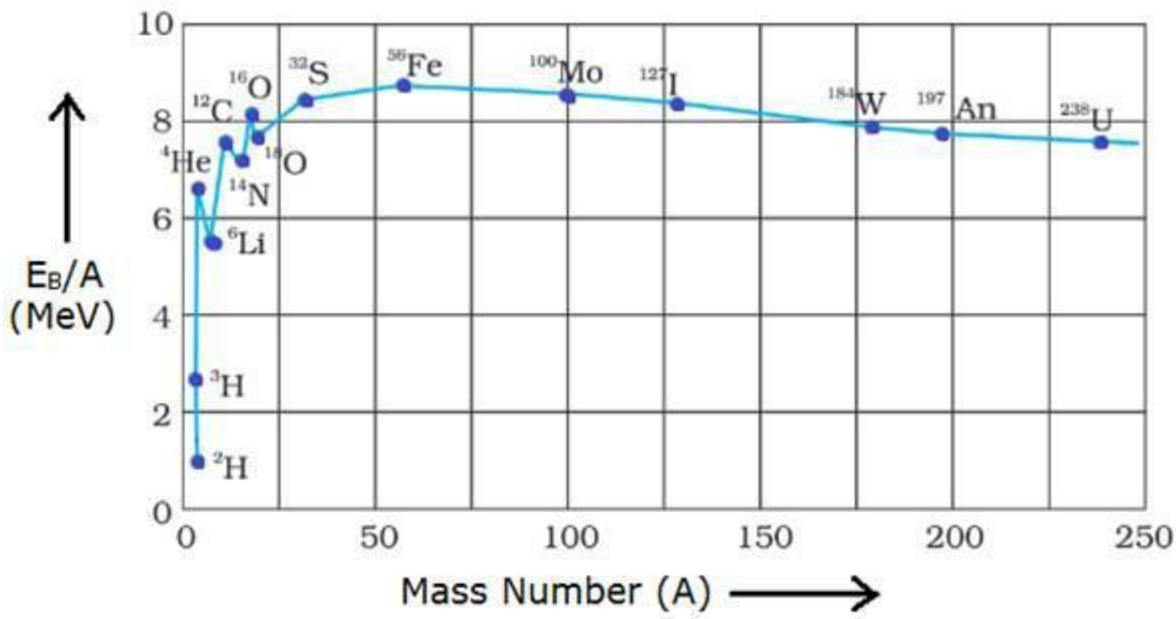
Nuclear stability and binding energy per nucleon

Binding energy is the amount of energy released during the formation of a nucleus.

Or

Binding energy is the amount of energy required to separate all nucleons.

Binding energy increases with increase in mass number(A). Binding energy per nucleon(BE/A) is the binding energy of a nucleus divided by total number of nucleons in that nucleus. The stability of a nucleus is attributed to its binding energy per nucleon. More is the binding energy per nucleon, more is the stability of the nucleus.



1. From A=50 to A=80, BE/A = 8.5 MeV and these nuclei are stable.
2. Fe (A=56) has BE/A = 8.8 MeV and has highest BE/A, hence it has the most stable nuclei.
3. For A>80, the BE/A decreases slowly. For A = 238, it reaches BE/A =7.6 MeV. These heavy nuclei are unstable. They undergo nuclear fission to gain stability.
4. For A<50, the BE/A decreases sharply and for A=2 BE/A=1.1 MeV. These lighter nuclei are unstable. They undergo nuclear fusion to gain stability.

Nuclear fission

Nuclear fission is a nuclear reaction in which a heavy nucleus splits into two or more lighter nuclei. Most of nuclear fission is binary. 4 out of 1000 fission is ternary. Most of the time, the daughter nuclei are of comparable mass. Nuclear fission occurs in nuclear reactors and atom bomb.

E.g. $^{1}n_{0} + {}^{235}U_{92} \rightarrow {}^{144}Ba_{56} + {}^{89}Kr_{36} + 3\,{}^{1}n_{0}$

Energy released per nucleus of $^{235}U_{92}$ =200 MeV

Nuclear fusion

Nuclear fusion is a process in which two or more lighter nuclei combine to form a heavier nucleus along with simultaneous release of large amount of energy.

E.g. $^{1}H_{1} + {}^{1}H_{1} \rightarrow {}^{2}H_{1} + {}^{0}e_{+1} + \upsilon$ (neutrino)

Energy released in this reaction is 0.42 MeV

Two nuclei repels each other due to electrostatic force. Large amount of work is to be done to bring two nuclei close enough (nuclear distance $\rightarrow 10^{-12}$ to 10^{-14} m) such that nuclear forces starts acting them. The nuclear forces is attractive in nature and approximately 100 times greater than electrostatic force.

This can be done by hearing the nuclei at very high temperature so that it's kinetic energy reaches 400 keV . For 400 keV kinetic energy, temperature must be around 3×10^{9} K (fusion starts at 10^{9} K). At such high temperature matter goes from solid to plasma state. Nucleus is surrounded by electron cloud. Nuclear fusion is exothermic process. The heat released is so large that it can carry on the fusion of remaining nuclei and no external heating is further required.

<u>Nuclear fusion on earth and stars:</u> Though nuclear fusion is exothermic and self sufficient but it needs about 10^{9}K temperature to start. Such high temperature is not possible to achieve on earth except one way. Such high temperature can be achieved by uncontrolled nuclear fission (atom bomb). Thus will result in a hydrogen bomb. So, to trigger a H-bomb , we need an atom bomb.

In sun, nuclear fusion occurs which is described as:

$4\,{}^{1}H_{1} + 2\,{}^{0}e_{-1} \rightarrow {}^{4}He_{2} + \upsilon + 6\gamma + 26.7$ MeV

Even though per reaction, nuclear fission releases more energy as compared to nuclear fusion but if we consider per unit mass, then nuclear fusion releases more energy as compared to nuclear fission.

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