Algebraic operations on Pentapartitioned Neutrosophic Vague Sets

Radhika V R [1], Mohana K [2]

[1] Research Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore, India

e-mail: radhikaramakrishnan2805@gmail.com

[2] Assistant Professor, Department of Mathematics, Nirmala College for Women, Coimbatore, India

e-mail: riyaraju1116@gmail.com

*Corresponding author : Radhika V R; radhikaramakrishnan2805@gmail.com

Abstract

In this paper, we present some algebraic operations for Pentapartitioned Neutrosophic Vague Set as an extension of single valued Neutrosophic vague sets. Specifically, we introduce addition, multiplication, scalar multiplication and power for the Pentapartitioned Neutrosophic Vague Sets also the compact spaces with suitable examples and a theorem with proofs.

Keywords: *Neutrosophic Set , Vague Set, Neutrosophic Vague Set, Pentapartitioned Neutrosophic Vague Set, Compact space.*

1. *Introduction*:

In order to solve uncertain**ty** in decision making problems, some theories like *fuzzy set theory* [10] by Lofti A. *Zadeh* (1965), *Intuitionistic fuzzy set theory* [10] by Atanassov (1986), vague set theory [2] by Gau & Beuhrer (1993) were developed.
In fuzzy set theory, *the uncertainty of a set is defined by membership function* that fix *values* called membership degrees *to the universal set elements within the* range$[0, 1]$. As an extension, in *Intuitionistic fuzzy set theory*, non – membership degree is also taken into account. Later, to solve the problems involving incomplete, *indeterminate* and *inconsistent information, Smarandache* developed a new theory called *Neutrosophic set theory* [7] as it includes new membership called *indeterminacy − membership. In this paper, We introduced* a *new concept called Pentapartitioned Neutrosophic Vague Set*(*PNVS*) by which includes four

membership functions namely *truth membership*, *contradiction membership*, *ignorance membership*, unknown membership *and false membership functions*.

In 2010 Wang et al., [9] developed *single valued neutrosophic set* (*SVNS*) and he defined some basic operations like subset, equality, complement, union and intersection on *SVNS*. Meanwhile Liu and Wang [2] introduced relations such as *addition, multiplication, scalar multiplication and power between two SVNSs*.

Many extensions of *Neutrosophic set* were developed and areas like used in decision making, aggregation operators, image processing etc.,[5].

Following these developments, we present some algebraic operations for *Pentapartitioned Neutrosophic Vague Set* and defined some algebraic operations on *PNVSs* with examples.

2. *Preliminaries*

Definition 2.1 [7]: *Let U be a universe. A **Neutrosophic set** A on X can be defined as follows:*

$$A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X\}$$

$$where\ T, I, F: X \rightarrow [0,1]\ and\ 0 \leq TA(x) + IA(x) + FA(x) \leq 3.$$

Here $T_A(x)$ *is the degree of membership*, (x) *is the degree of indeterminacy* and $F(x)$ *is the degree of non − membership*.

For application in real scientific and engineering areas, Wang et al., proposed the concept of a single valued neutrosophic set as follows:

Operations between two SVN − numbers are defined by [6]. *It is recalled as follows*:

Let $x = (T_1, I_1, F_1)$ and $y = (T_2, I_2, F_2)$ be two *SVNS*, *then the operations are defined as* follows:

a) $x \oplus y = (T_1 + T_2 - T_1 T_2, I_1 I_2, F_1 F_2)$

b) $x \otimes y = (T_1 T_2, I_1 + I_2 - I_1 I_2, F_1 + F_2 - F_1 F_2)$

c) $\lambda x = \left(\left(1 - (1 - T_1)^\lambda\right), I_1^\lambda, F_1^\lambda \right)$

d) $x^\lambda = \left(T_1^\lambda, \left(1 - (1 - I_1)^\lambda\right), \left(1 - (1 - F_1)^\lambda\right) \right)$

Definition 2.2 [8]: A ***Neutrosophic vague set*** *ANV* (*NVS*) on *X* written as

$A = \{\langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)\rangle; x \in X\},$, *whose truth membership, indeterminacy membership and false membership functions is defined as*:

$$\hat{T}_A(x) = [T^-, T^+], \quad \hat{I}_A(x)[I^-, I^+], \quad \hat{F}_A(x) = [F^-, F^+],$$

Where, (1) $T^+ = 1 - F^-$ (2) $F^+ = 1 - T^-$ and (3) $0 \le T^- + I^- + F^- \le 2^+$ *when X is continuous, a NVS A can be written as*

$$A = \int \langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)\rangle /x, x \in X$$

When X is discrete, a NVS A can be written as

$$A = \sum_{i=1}^{n} \langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)\rangle /x_i, \quad x_i \in X$$

Definition 2.3 [6]: A *Pentapartitioned Neutrosophic Vague Set* A_{PNV} on the universe of discourse written as

$$A_{PNV} = \{\langle x; \hat{T}_{A_{PNV}}(x); \hat{C}_{A_{PNV}}(x); \hat{G}_{A_{PNV}}(x); \hat{U}_{A_{PNV}}(x); \hat{F}_{A_{PNV}}(x)\rangle; x \in X\}$$

whose truth membership, contradiction membership, ignorance membership, unknown membership, and false membership function is defined as:

$$\hat{T}_{A_{PNV}}(x) = [T^-, T^+], \quad \hat{C}_{A_{PNV}}(x) = [C^-, C^+], \quad \hat{G}_{A_{PNV}}(x) = [G^-, G^+],$$

$$\hat{U}_{A_{PNV}}(x) = [U^-, U^+], \quad \hat{F}_{A_{PNV}}(x) = [F^-, F^+],$$

Where (1) $T^+ = 1 - F^-$ (2) $F^+ = 1 - T^-$ (3) $C^+ = 1 - U^-$ (4) $U^+ = 1 - C^-$

(5) $0 \le T^- + C^- + G^- + U^- + F^- \le 4^+$

Definition 2.4 [6]: *Let* A_{PNV} *and* B_{PNV} *be two of the universe U. If* $\forall x \in U,$
$\hat{T}_{A_{PNV}}(x) \le \hat{T}_{B_{PNV}}(x); \hat{C}_{A_{PNV}}(x) \le \hat{C}_{B_{PNV}}(x);$

$\hat{G}_{A_{PNV}}(x) \ge \hat{G}_{B_{PNV}}(x); \hat{U}_{A_{PNV}}(x) \ge \hat{U}_{B_{PNV}}(x); \hat{F}_{A_{PNV}}(x) \ge \hat{F}_{B_{PNV}}(x)$ *then the PNVS* A_{PNV} is included by B_{PNV}, denoted by $A_{PNV} \subseteq B_{PNV}$.

Definition 2.5 [6]: The complement of **PNVS** A_{PNV} is denoted by A^C_{PNV} and is defined by $\hat{T}^C_{A_{PNV}}(x) = [1 - T^+, 1 - T^-]$, $\hat{C}^C_{A_{PNV}}(x) = [1 - C^+, 1 - C^-]$, $\hat{G}^C_{A_{PNV}}(x) = [1 - G^+, 1 - G^-]$, $\hat{U}^C_{A_{PNV}}(x) = [1 - U^+, 1 - U^-]$, $\hat{F}^C_{A_{PNV}}(x) = [1 - F^+, 1 - F^-]$.

Definition 2.6 [6]: Let A_{PNV} be a *PNVS on the universe of discourse U where* $\forall x \in U$, $\hat{T}_{A_{PNV}}(x) = [1,1]$, $\hat{C}_{A_{PNV}}(x) = [1,1]$,

$\hat{G}_{A_{PNV}}(x) = [0,0]$, $\hat{U}_{A_{PNV}}(x) = [0,0]$, $\hat{F}_{A_{PNV}}(x) = [0,0]$. Then A_{PNV} is *called unit PNVS*(1_{PNV} in *short*).

Definition 2.7 [6]: Let A_{PNV} be a *PNVS on the universe of discourse U where* $\forall x \in U$,

$\hat{T}_{A_{PNV}}(x) = [0,0]$, $\hat{C}_{A_{PNV}}(x) = [0,0]$, $\hat{G}_{A_{PNV}}(x) = [1,1]$,
$\hat{U}_{A_{PNV}}(x) = [1,1]$, $\hat{F}_{A_{PNV}}(x) = [1,1]$

Then A_{PNV} *is called zero PNVS* (0_{PNV} in *short*).

Definition 2.8 [6]: *Let* A_{PNV} *and* B_{PNV} *be two PNVS of the universe U*. *If* $\forall\ x \in U$,

$\hat{T}_{A_{PNV}}(x) = \hat{T}_{B_{PNV}}(x)$; $\hat{C}_{A_{PNV}}(x) = \hat{C}_{B_{PNV}}(x)$; $\hat{G}_{A_{PNV}}(x) = \hat{G}_{B_{PNV}}(x)$; $\hat{U}_{A_{PNV}}(x) = \hat{U}_{B_{PNV}}(x)$; $\hat{F}_{A_{PNV}}(x) = \hat{F}_{B_{PNV}}(x)$ *then the PNVS* A_{PNV} *and* B_{PNV} *are equal*.

Definition 2.9 [6]: *The union of PNVSs two* A_{PNV} *and* B_{PNV} *is a PNVS* K_{PNV} *written as* $K_{PNV} = A_{PNV} \cup B_{PNV}$ *whose* $truth - membership, contradiction - membership, ignorance\ membership, unknown\ membership\ and\ false\ membership\ functions\ are\ related\ to\ those\ of\ A_{PNV}\ and\ B_{PNV}\ by$

$$\hat{T}_{K_{PNV}}(x) = \left[max\left(\hat{T}^-_{A_{PNV}}, \hat{T}^-_{B_{PNV}}\right), max\left(\hat{T}^+_{A_{PNV}}, \hat{T}^+_{B_{PNV}}\right)\right]$$

$$\hat{C}_{K_{PNV}}(x) = \left[max\left(\hat{C}^-_{A_{PNV}}, \hat{C}^-_{B_{PNV}}\right), max\left(\hat{C}^+_{A_{PNV}}, \hat{C}^+_{B_{PNV}}\right)\right]$$

$$\hat{G}_{K_{PNV}}(x) = \left[min\left(\hat{G}^-_{A_{PNV}}, \hat{G}^-_{B_{PNV}}\right), min\left(\hat{G}^+_{A_{PNV}}, \hat{G}^+_{B_{PNV}}\right)\right]$$

$$\hat{U}_{K_{PNV}}(x) = \left[min\left(\hat{U}^-_{A_{PNV}}, \hat{U}^-_{B_{PNV}}\right), min\left(\hat{U}^+_{A_{PNV}}, U^+_{B_{PNV}}\right)\right]$$

$$\hat{F}_{K_{PNV}}(x) = \left[min\left(\hat{F}^-_{A_{PNV}}, \hat{F}^-_{B_{PNV}}\right), min\left(\hat{F}^+_{A_{PNV}}, F^+_{B_{PNV}}\right)\right]$$

Definition 2.10 [6]: *The intersection of two PNVSs* A_{PNV} *and* B_{PNV} *is a PNVS* H_{PNV} , written as $H_{PNV} = A_{PNV} \cap B_{PNV}$ whose $truth - membership, contradiction -$

$membership, ignorance\ membership, unknown\ membership\ and\ false-$
$membership\ functions\ are\ related\ to\ those\ of\ A_{PNV}\ and\ B_{PNV}\ by$

$$\hat{T}_{H_{PNV}}(x) = \left[min(\hat{T}^{-}_{A_{PNV}}, \hat{T}^{-}_{B_{PNV}}), min(\hat{T}^{+}_{A_{PNV}}, \hat{T}^{+}_{B_{PNV}})\right]$$

$$\hat{C}_{H_{PNV}}(x) = \left[min(\hat{C}^{-}_{A_{PNV}}, \hat{C}^{-}_{B_{PNV}}), min(\hat{C}^{+}_{A_{PNV}}, \hat{C}^{+}_{B_{PNV}})\right]$$

$$\hat{G}_{H_{PNV}}(x) = \left[max(\hat{G}^{-}_{A_{PNV}}, \hat{G}^{-}_{B_{PNV}}), max\left(\hat{G}^{+}_{A_{PNV}}, \hat{G}^{+}_{B_{PNV}}\right)\right]$$

$$\hat{U}_{H_{PNV}}(x) = \left[max(\hat{U}^{-}_{A_{PNV}}, \hat{U}^{-}_{B_{PNV}}), max(\hat{U}^{+}_{A_{PNV}}, U^{+}_{B_{PNV}})\right]$$

$$\hat{F}_{H_{PNV}}(x) = \left[max\left(\hat{F}^{-}_{A_{PNV}}, \hat{F}^{-}_{B_{PNV}}\right), max\left(\hat{F}^{+}_{A_{PNV}}, F^{+}_{B_{PNV}}\right)\right]$$

Definition 2.11 [6]: Let $\left\{A_{i_{PNV}} : i \in J\right\}$ be an arbitrary family of $PNVSs$. Then

$$\bigcup A_{i_{PNV}}$$
$$= \left\{ \begin{matrix} x; \left(\max_{i \in J}\left(\hat{T}^{-}_{A_{i_{PNV}}}\right), \max_{i \in J}\left(\hat{T}^{+}_{A_{i_{PNV}}}\right)\right), \left(\max_{i \in J}\left(\hat{C}^{-}_{A_{i_{PNV}}}\right), \max_{i \in J}\left(\hat{C}^{+}_{A_{i_{PNV}}}\right)\right), \left(\min_{i \in J}\left(\hat{G}^{-}_{A_{i_{PNV}}}\right), \min_{i \in J}(\hat{G}^{+}_{A_{PNV}})\right) \\ \left(\min_{i \in J}\left(\hat{U}^{-}_{A_{i_{PNV}}}\right), \min_{i \in J}\left(\hat{U}^{+}_{A_{i_{PNV}}}\right)\right), \left(\min_{i \in J}\left(\hat{F}^{-}_{A_{i_{PNV}}}\right), \min_{i \in J}\left(\hat{F}^{+}_{A_{i_{PNV}}}\right)\right) \end{matrix} \right\rangle ; x$$
$$\in X \Big\}$$

$$\bigcap A_{i_{PNV}}$$
$$= \left\{ \begin{matrix} x; \left(\max_{i \in J}\left(\hat{T}^{-}_{A_{i_{PNV}}}\right), \max_{i \in J}\left(\hat{T}^{+}_{A_{i_{PNV}}}\right)\right), \left(\max_{i \in J}\left(\hat{C}^{-}_{A_{i_{PNV}}}\right), \max_{i \in J}\left(\hat{C}^{+}_{A_{i_{PNV}}}\right)\right), \left(\min_{i \in J}\left(\hat{G}^{-}_{A_{i_{PNV}}}\right), \min_{i \in J}\left(\hat{G}^{+}_{A_{i_{PNV}}}\right)\right) \\ \left(\min_{i \in J}\left(\hat{U}^{-}_{A_{i_{PNV}}}\right), \min_{i \in J}\left(\hat{U}^{+}_{A_{i_{PNV}}}\right)\right), \left(\min_{i \in J}\left(\hat{F}^{-}_{A_{i_{PNV}}}\right), \min_{i \in J}\left(\hat{F}^{+}_{A_{i_{PNV}}}\right)\right) \end{matrix} \right\rangle ; x$$
$$\in X \Big\}$$

3. *Operations on Pentapartitioned Neutrosophic Vague Sets*:

This section introduces various algebraic operations on PNVS based on SVNS operations

The following are the definitions of addition, multiplication, scalar multiplication, and power operations:

Definition 3.1:

Let

$A_{PNV} =$

$$\left\{ \left\langle \begin{array}{c} x;\ \left[\hat{T}^{-}{}_{A_{PNV}}(x), \hat{T}^{+}{}_{A_{PNV}}(x)\right]; \left[\hat{C}^{-}{}_{A_{PNV}}(x), \hat{C}^{+}{}_{A_{PNV}}(x)\right]; \left[\hat{G}^{-}{}_{A_{PNV}}(x), \hat{G}^{+}{}_{A_{PNV}}(x)\right]; \\ \left[\hat{U}^{-}{}_{A_{PNV}}(x), \hat{U}^{+}{}_{A_{PNV}}(x)\right]; \left[\hat{F}^{-}{}_{A_{PNV}}(x), \hat{F}^{+}{}_{A_{PNV}}(x)\right] \end{array} \right\rangle ; x \in X \right\}$$

$B_{PNV} =$

$$\left\{ \left\langle \begin{array}{c} x;\ \left[\hat{T}^{-}{}_{B_{PNV}}(x), \hat{T}^{+}{}_{B_{PNV}}(x)\right]; \left[\hat{C}^{-}{}_{B_{PNV}}(x), \hat{C}^{+}{}_{B_{PNV}}(x)\right]; \left[\hat{G}^{-}{}_{B_{PNV}}(x), \hat{G}^{+}{}_{B_{PNV}}(x)\right]; \\ \left[\hat{U}^{-}{}_{B_{PNV}}(x), \hat{U}^{+}{}_{B_{PNV}}(x)\right]; \left[\hat{F}^{-}{}_{B_{PNV}}(x), \hat{F}^{+}{}_{B_{PNV}}(x)\right] \end{array} \right\rangle ; x \in X \right\}$$

be two *PNVSS*. Then the addition of A_{PNV} and B_{PNV} denoted as $(A \oplus B)_{PNV}$ defined by

$(A \oplus B)_{PNV}$

$$= \left\{ \left\langle \begin{array}{c} x; \left[\hat{T}^{-}{}_{A_{PNV}}(x) + \hat{T}^{-}{}_{B_{PNV}}(x) - \hat{T}^{-}{}_{A_{PNV}}(x)\hat{T}^{-}{}_{B_{PNV}}(x), \hat{T}^{+}{}_{A_{PNV}}(x) + \hat{T}^{+}{}_{B_{PNV}}(x) - \hat{T}^{+}{}_{A_{PNV}}(x)\hat{T}^{+}{}_{B_{PNV}}(x)\right]; \\ \left[\hat{C}^{-}{}_{A_{PNV}}(x) + \hat{C}^{-}{}_{B_{PNV}}(x) - \hat{C}^{-}{}_{A_{PNV}}(x)\hat{C}^{-}{}_{B_{PNV}}(x), \hat{C}^{+}{}_{A_{PNV}}(x) + \hat{C}^{+}{}_{B_{PNV}}(x) - \hat{C}^{+}{}_{A_{PNV}}(x)\hat{C}^{+}{}_{B_{PNV}}(x)\right]; \\ \left[\hat{G}^{-}{}_{A_{PNV}}(x)\hat{G}^{-}{}_{B_{PNV}}(x), \hat{G}^{+}{}_{A_{PNV}}(x)\hat{G}^{+}{}_{B_{PNV}}(x)\right]; \left[\hat{U}^{-}{}_{A_{PNV}}(x)\hat{U}^{-}{}_{B_{PNV}}(x), \hat{U}^{+}{}_{A_{PNV}}(x)\hat{U}^{+}{}_{B_{PNV}}(x)\right]; \\ \left[\hat{F}^{-}{}_{A_{PNV}}(x)\hat{F}^{-}{}_{B_{PNV}}(x), \hat{F}^{+}{}_{A_{PNV}}(x)\hat{F}^{+}{}_{B_{PNV}}(x)\right] \end{array} \right\rangle \right\}$$

Example 3.2:

Let $U = \{m, n\}$ be a set of universe and let

$$A_{PNV} = \left\{ \frac{m}{\{[0.3,0.55],[0.45,0.5],[0.2,0.75],[0.55,0.78],[0.65,0.7]\}}, \frac{n}{\{[0.4,0.85],[0.35,0.6],[0.45,0.8],[0.15,0.76],[0.37,0.9]\}} \right\},$$

$$B_{PNV} = \left\{ \frac{m}{\{[0.43,0.65],[0.56,0.7],[0.37,0.68],[0.59,0.8],[0.4,0.65]\}}, \frac{n}{\{[0.32,0.65],[0.5,0.86],[0.25,0.5],[0.4,0.65],[0.6,0.95]\}} \right\}$$

Then $(A \oplus B)_{PNV}$

$$= \left\{ \frac{m}{\{[0.601, 0.8425], [0.758, 0.85], [0.074, 0.51], [0.3245, 0.624], [0.26, 0.455]\}}, \frac{n}{\{[0.592, 0.9475], [0.675, 0.944], [0.1125, 0.4], [0.06, 0.494], [0.18, 0.855]\}} \right\}$$

which is again a *PNVS*.

Definition 3.3:

Let

$$A_{PNV} =$$

$$\left\{ \left\langle \begin{array}{c} x; \ [\hat{T}^-_{A_{PNV}}(x), \hat{T}^+_{A_{PNV}}(x)]; [\hat{C}^-_{A_{PNV}}(x), \hat{C}^+_{A_{PNV}}(x)]; [\hat{G}^-_{A_{PNV}}(x), \hat{G}^+_{A_{PNV}}(x)]; \\ [\hat{U}^-_{A_{PNV}}(x), \hat{U}^+_{A_{PNV}}(x)]; [\hat{F}^-_{A_{PNV}}(x), \hat{F}^+_{A_{PNV}}(x)] \end{array} \right\rangle ; x \in X \right\}$$

$$B_{PNV} =$$

$$\left\{ \left\langle \begin{array}{c} x; \ [\hat{T}^-_{B_{PNV}}(x), \hat{T}^+_{B_{PNV}}(x)]; [\hat{C}^-_{B_{PNV}}(x), \hat{C}^+_{B_{PNV}}(x)]; [\hat{G}^-_{B_{PNV}}(x), \hat{G}^+_{B_{PNV}}(x)]; \\ [\hat{U}^-_{B_{PNV}}(x), \hat{U}^+_{B_{PNV}}(x)]; [\hat{F}^-_{B_{PNV}}(x), \hat{F}^+_{B_{PNV}}(x)] \end{array} \right\rangle ; x \in X \right\}$$

be two PNVSS. Then the addition of A_{PNV} *and* B_{PNV} *denoted as* $(A \otimes B)_{PNV}$
defined by

$(A \otimes B)_{PNV}$

$$= \left\{ \left\langle \begin{array}{c} x; [\hat{T}^-_{A_{PNV}}(x)\hat{T}^-_{B_{PNV}}(x), \hat{T}^+_{A_{PNV}}(x)\hat{T}^+_{B_{PNV}}(x)]; [\hat{C}^-_{A_{PNV}}(x)\hat{C}^-_{B_{PNV}}(x), \hat{C}^+_{A_{PNV}}(x)\hat{C}^+_{B_{PNV}}(x)]; \\ [\hat{G}^-_{A_{PNV}}(x) + \hat{G}^-_{B_{PNV}}(x) - \hat{G}^-_{A_{PNV}}(x)\hat{G}^-_{B_{PNV}}(x), \hat{G}^+_{A_{PNV}}(x) + \hat{G}^+_{B_{PNV}}(x) - \hat{G}^+_{A_{PNV}}(x)\hat{G}^+_{B_{PNV}}(x)]; \\ [\hat{U}^-_{A_{PNV}}(x) + \hat{U}^-_{B_{PNV}}(x) - \hat{U}^-_{A_{PNV}}(x)\hat{U}^-_{A_{PNV}}(x), \hat{U}^+_{A_{PNV}}(x) + \hat{U}^+_{B_{PNV}}(x) - \hat{U}^+_{A_{PNV}}(x)\hat{U}^-_{B_{PNV}}(x)]; \\ [\hat{F}^-_{A_{PNV}}(x) + \hat{F}^-_{B_{PNV}}(x) - \hat{F}^-_{A_{PNV}}(x)\hat{F}^-_{A_{PNV}}(x), \hat{F}^+_{A_{PNV}}(x) + \hat{F}^+_{B_{PNV}}(x) - \hat{F}^+_{A_{PNV}}(x)\hat{F}^-_{B_{PNV}}(x)] \end{array} \right\rangle \right\}$$

Example 3.4:

Let $U = \{m, n\}$ *be a set of universe and let*

$$A_{PNV} = \left\{ \frac{m}{\{[0.25, 0.43], [0.36, 0.48], [0.35, 0.62], [0.47, 0.6], [0.45, 0.55]\}}, \frac{n}{\{[0.53, 0.55], [0.4, 0.85], [0.26, 0.64], [0.5, 0.75], [0.45, 0.76]\}} \right\},$$

$$B_{PNV} = \left\{ \frac{m}{\{[0.35,0.59], [0.43,0.55], [0.15,0.4], [0.29,0.65], [0.52,0.71]\}}, \frac{n}{\{[0.45,0.8], [0.37,0.9], [0.25,0.5], [0.25,0.4], [0.47,0.5]\}} \right\}$$

Then $(A \otimes B)_{PNV}$

$$= \left\{ \frac{m}{\{[0.0875,0.2537], [0.1548,0.264], [0.4475,0.772], [0.6237,0.86], [0.736,0.8695]\}}, \frac{n}{\{[0.2385,0.44], [0.148,0.765], [0.445,0.82], [0.625,0.85], [0.7085,0.88]\}} \right\}$$

Definition 3.5:

Let

$A_{PNV} =$

$$\left\{ \left\langle \begin{array}{c} x; \; [\hat{T}^-{}_{A_{PNV}}(x), \hat{T}^+{}_{A_{PNV}}(x)]; [\hat{C}^-{}_{A_{PNV}}(x), \hat{C}^+{}_{A_{PNV}}(x)]; [\hat{G}^-{}_{A_{PNV}}(x), \hat{G}^+{}_{A_{PNV}}(x)]; \\ [\hat{U}^-{}_{A_{PNV}}(x), \hat{U}^+{}_{A_{PNV}}(x)]; [\hat{F}^-{}_{A_{PNV}}(x), \hat{F}^+{}_{A_{PNV}}(x)] \end{array} \right\rangle ; x \in X \right\}$$

be a $PNVS$. Then the scalar (λ) multiplication of A_{PNV} denoted as λA_{PNV} defined by

$\lambda A_{PNV} =$

$$\left\{ \left\langle \begin{array}{c} x; \left[1 - \left(1 - \hat{T}^-{}_{A_{PNV}}(x) \right)^\lambda, 1 - \left(1 - \hat{T}^+{}_{A_{PNV}}(x) \right)^\lambda \right]; \\ \left[1 - \left(1 - \hat{C}^-{}_{A_{PNV}}(x) \right)^\lambda, 1 - \left(1 - \hat{C}^+{}_{A_{PNV}}(x) \right)^\lambda \right]; \\ \left[\left(\hat{G}^-{}_{A_{PNV}}(x) \right)^\lambda, \left(\hat{G}^+{}_{A_{PNV}}(x) \right)^\lambda \right]; \\ \left[\left(\hat{U}^-{}_{A_{PNV}}(x) \right)^\lambda, \left(\hat{U}^+{}_{A_{PNV}}(x) \right)^\lambda \right]; \\ \left[\left(\hat{F}^-{}_{A_{PNV}}(x) \right)^\lambda, \left(\hat{F}^+{}_{A_{PNV}}(x) \right)^\lambda \right] \end{array} \right\rangle ; x \in X \right\}$$

Example 3.6:

Let $U = \{m, n\}$ be a set of universe, let

A_{PNV}

$$= \left\{ \frac{m}{\{[0.3,0.5], [0.4,0.6], [0.2,0.7], [0.3,0.6], [0.5,0.8]\}}, \frac{n}{\{[0.1,0.6], [0.6,0.7], [0.3,0.4], [0.5,0.7], [0.6,0.9]\}} \right\}$$

be a $PNVS$ and if $\lambda = 3$, then

$$\lambda A_{PNV}$$

$$= \left\{ \frac{m}{\{[1-(1-0.3)^3, 1-(1-0.5)^3], [1-(1-0.4)^3, 1-(1-0.6)^3], [(0.2)^3, (0.7)^3], [(0.3)^3, (0.6)^3], [(0.5)^3, (0.8)^3]\}}, \frac{n}{\{[1-(1-0.1)^3, 1-(1-0.6)^3], [1-(1-0.6)^3, 1-(1-0.7)^3], [(0.3)^3, (0.4)^3], [(0.5)^3, (0.7)^3], [(0.6)^3, (0.9)^3]\}} \right\}$$

$$\lambda A_{PNV} = \left\{ \frac{m}{\{[0.657,0.875], [0.784,0.936], [0.008,0.343], [0.027,0.216], [0.125,0.512]\}}, \frac{n}{\{[0.271,0.936], [0.936,0.973], [0.027,0.064], [0.125,0.343], [0.216,0.729]\}} \right\}$$

Definition 3.7:

Let

$$A_{PNV} =$$

$$\left\{ \left\langle x; \begin{array}{c} [\hat{T}^-_{A_{PNV}}(x), \hat{T}^+_{A_{PNV}}(x)]; [\hat{C}^-_{A_{PNV}}(x), \hat{C}^+_{A_{PNV}}(x)]; [\hat{G}^-_{A_{PNV}}(x), \hat{G}^+_{A_{PNV}}(x)]; \\ [\hat{U}^-_{A_{PNV}}(x), \hat{U}^+_{A_{PNV}}(x)]; [\hat{F}^-_{A_{PNV}}(x), \hat{F}^+_{A_{PNV}}(x)] \end{array} \right\rangle ; x \in X \right\}$$

be a $PNVS$.

Then, $PNVS$ power denoted by $A_{PNV}{}^\lambda$ is defined by

$$A_{PNV}{}^\lambda = \left\{ \left\langle \begin{array}{c} x; \left[\left(\hat{T}^-_{A_{PNV}}(x)\right)^\lambda, \left(\hat{T}^+_{A_{PNV}}(x)\right)^\lambda \right]; \\ \left[\left(\hat{C}^-_{A_{PNV}}(x)\right)^\lambda, \left(\hat{C}^+_{A_{PNV}}(x)\right)^\lambda \right]; \\ \left[1 - \left(1 - \hat{G}^-_{A_{PNV}}(x)\right)^\lambda, 1 - \left(1 - \hat{G}^+_{A_{PNV}}(x)\right)^\lambda \right]; \\ \left[1 - \left(1 - \hat{U}^-_{A_{PNV}}(x)\right)^\lambda, 1 - \left(1 - \hat{U}^+_{A_{PNV}}(x)\right)^\lambda \right]; \\ \left[1 - \left(1 - \hat{F}^-_{A_{PNV}}(x)\right)^\lambda, 1 - \left(1 - \hat{F}^+_{A_{PNV}}(x)\right)^\lambda \right] \end{array} \right\rangle ; x \in X \right\}$$

Example 3.8:

Let $U = \{m, n\}$ be a set of universe, let

$$A_{PNV}$$

$$= \left\{ \frac{m}{\{[0.3,0.5], [0.4,0.6], [0.2,0.7], [0.3,0.6], [0.5,0.8]\}}, \frac{n}{\{[0.1,0.6], [0.6,0.7], [0.3,0.4], [0.5,0.7], [0.6,0.9]\}} \right\}$$

be a $PNVS$ and if $\lambda = 2$, then

$A_{PNV}{}^{\lambda}$

$$= \left\{ \frac{m}{\{[(0.3)^2, (0.5)^2], [(0.4)^2, (0.6)^2], [[1-(1-0.2)^2, 1-(1-0.7)^2]], [[1-(1-0.3)^2, 1-(1-0.6)^2]], [[1-(1-0.5)^2, 1-(1-0.8)^2]]\}}{\{[(0.1)^2, (0.6)^2], [(0.6)^2, (0.7)^2], [[1-(1-0.3)^2, 1-(1-0.4)^2]], [[1-(1-0.5)^2, 1-(1-0.7)^2]], [[1-(1-0.6)^2, 1-(1-0.9)^2]]\}} } \right\}$$

$A_{PNV}{}^{\lambda}$

$$= \left\{ \frac{m}{\{[0.09, 0.25], [0.16, 0.36], [0.36, 0.91], [0.51, 0.84], [0.75, 0.96]\}}{\{[0.01, 0.36], [0.36, 0.49], [0.51, 0.64], [0.75, 0.91], [0.84, 0.99]\}} } \right\}$$

Theorem 3.9:

Let A_{PNV} *and* B_{PNV} *be two PNVS and* $\lambda, \lambda_1, \lambda_2 > 0$. *Then*

i) $(A \oplus B)_{PNV} = (B \oplus A)_{PNV}$

ii) $(A \otimes B)_{PNV} = (B \otimes A)_{PNV}$

iii) $\lambda(A \oplus B)_{PNV} = \lambda A_{PNV} \oplus \lambda B_{PNV}$

iv) $\lambda_1 A_{PNV} \oplus \lambda_2 A_{PNV} = (\lambda_1 \oplus \lambda_2) A_{PNV}$

v) $A_{PNV}{}^{\lambda_1} \otimes A_{PNV}{}^{\lambda_2} = A_{PNV}{}^{\lambda_1 + \lambda_2}$

vi) $A_{PNV}{}^{\lambda} \otimes B_{PNV}{}^{\lambda} = (A \otimes B)_{PNV}{}^{\lambda}$

Proof:

The results $(i), (ii), (iv)$ *and* (vi) *are obvious, thus we prove the others.*

Proof of (iii)

By definition, we have

$(A \oplus B)_{PNV} =$

$$= \left\{ \left\langle \begin{array}{c} x; \left[\hat{T}^{-}{}_{A_{PNV}}(x) + \hat{T}^{-}{}_{B_{PNV}}(x) - \hat{T}^{-}{}_{A_{PNV}}(x)\hat{T}^{-}{}_{B_{PNV}}(x), \hat{T}^{+}{}_{A_{PNV}}(x) + \hat{T}^{+}{}_{B_{PNV}}(x) - \hat{T}^{+}{}_{A_{PNV}}(x)\hat{T}^{+}{}_{B_{PNV}}(x)\right]; \\ \left[\hat{C}^{-}{}_{A_{PNV}}(x) + \hat{C}^{-}{}_{B_{PNV}}(x) - \hat{C}^{-}{}_{A_{PNV}}(x)\hat{C}^{-}{}_{B_{PNV}}(x), \hat{C}^{+}{}_{A_{PNV}}(x) + \hat{C}^{+}{}_{B_{PNV}}(x) - \hat{C}^{+}{}_{A_{PNV}}(x)\hat{C}^{+}{}_{B_{PNV}}(x)\right]; \\ \left[\hat{G}^{-}{}_{A_{PNV}}(x)\hat{G}^{-}{}_{B_{PNV}}(x), \hat{G}^{+}{}_{A_{PNV}}(x)\hat{G}^{+}{}_{B_{PNV}}(x)\right]; \left[\hat{U}^{-}{}_{A_{PNV}}(x)\hat{U}^{-}{}_{B_{PNV}}(x), \hat{U}^{+}{}_{A_{PNV}}(x)\hat{U}^{+}{}_{B_{PNV}}(x)\right]; \\ \left[\hat{F}^{-}{}_{A_{PNV}}(x)\hat{F}^{-}{}_{B_{PNV}}(x), \hat{F}^{+}{}_{A_{PNV}}(x)\hat{F}^{+}{}_{B_{PNV}}(x)\right] \end{array} \right\rangle \right\}$$

$\lambda(A \oplus B)_{PNV}$

$$= \left\{ \left\langle x; \begin{array}{l} \left[1 - \left(1 - \left(\hat{T}^-{}_{A_{PNV}}(x) + \hat{T}^-{}_{B_{PNV}}(x) - \hat{T}^-{}_{A_{PNV}}(x)\hat{T}^-{}_{B_{PNV}}(x) \right) \right)^\lambda, 1 - \left(1 - \left(\hat{T}^+{}_{A_{PNV}}(x) + \hat{T}^+{}_{B_{PNV}}(x) - \hat{T}^+{}_{A_{PNV}}(x)\hat{T}^+{}_{B_{PNV}}(x) \right) \right)^\lambda \right]; \\[2mm] \left[1 - \left(1 - \left(\hat{C}^-{}_{A_{PNV}}(x) + \hat{C}^-{}_{B_{PNV}}(x) - \hat{C}^-{}_{A_{PNV}}(x)\hat{C}^-{}_{B_{PNV}}(x) \right) \right)^\lambda, 1 - \left(1 - \left(\hat{C}^+{}_{A_{PNV}}(x) + \hat{C}^+{}_{B_{PNV}}(x) - \hat{C}^+{}_{A_{PNV}}(x)\hat{C}^+{}_{B_{PNV}}(x) \right) \right)^\lambda \right]; \\[2mm] \left[\left(\hat{G}^-_{A_{PNV}}(x)\hat{G}^-_{B_{PNV}}(x) \right)^\lambda, \left(\hat{G}^+_{A_{PNV}}(x)\hat{G}^+_{B_{PNV}}(x) \right)^\lambda \right]; \\[2mm] \left[\left(\hat{U}^-_{A_{PNV}}(x)\hat{U}^-_{B_{PNV}}(x) \right)^\lambda, \left(\hat{U}^+_{A_{PNV}}(x)\hat{U}^+_{B_{PNV}}(x) \right)^\lambda \right]; \\[2mm] \left[\left(\hat{F}^-_{A_{PNV}}(x)\hat{F}^-_{B_{PNV}}(x) \right)^\lambda, \left(\hat{F}^+_{A_{PNV}}(x)\hat{F}^+_{B_{PNV}}(x) \right)^\lambda \right] \end{array} \right\rangle; x \in X \right\}$$

We solve truth membership functions

$$= 1 - \left(1 - \left(\hat{T}^-{}_{A_{PNV}}(x) + \hat{T}^-{}_{B_{PNV}}(x) - \hat{T}^-{}_{A_{PNV}}(x)\hat{T}^-{}_{B_{PNV}}(x) \right) \right)^\lambda$$

$$= 1 - \left(1 - \hat{T}^-_{A_{PNV}}(x) - \hat{T}^-_{B_{PNV}}(x) + \hat{T}^-_{A_{PNV}}(x)\hat{T}^-_{B_{PNV}}(x) \right)^\lambda$$

$$= 1 - \left(\left(1 - \hat{T}^-_{A_{PNV}}(x) \right) - \hat{T}^-_{B_{PNV}}(x)\left(1 - \hat{T}^-_{A_{PNV}}(x) \right) \right)^\lambda$$

$$= 1 - \left(\left(1 - \hat{T}^-_{A_{PNV}}(x) \right)\left(1 - \hat{T}^-_{B_{PNV}}(x) \right) \right)^\lambda$$

By similar calculation, we get

$\lambda(A \oplus B)_{PNV} =$

$$\left\{ \left\langle x; \begin{array}{l} \left[1 - \left(\left(1 - \hat{T}^-_{A_{PNV}}(x) \right)\left(1 - \hat{T}^-_{B_{PNV}}(x) \right) \right)^\lambda, 1 - \left(\left(1 - \hat{T}^+_{A_{PNV}}(x) \right)\left(1 - \hat{T}^+_{B_{PNV}}(x) \right) \right)^\lambda \right]; \\[2mm] \left[1 - \left(\left(1 - \hat{T}^-_{A_{PNV}}(x) \right)\left(1 - \hat{T}^-_{B_{PNV}}(x) \right) \right)^\lambda, 1 - \left(\left(1 - \hat{T}^+_{A_{PNV}}(x) \right)\left(1 - \hat{T}^+_{B_{PNV}}(x) \right) \right)^\lambda \right]; \\[2mm] \left[\left(\hat{G}^-_{A_{PNV}}(x)\hat{G}^-_{B_{PNV}}(x) \right)^\lambda, \left(\hat{G}^+_{A_{PNV}}(x)\hat{G}^+_{B_{PNV}}(x) \right)^\lambda \right]; \\[2mm] \left[\left(\hat{U}^-_{A_{PNV}}(x)\hat{U}^-_{B_{PNV}}(x) \right)^\lambda, \left(\hat{U}^+_{A_{PNV}}(x)\hat{U}^+_{B_{PNV}}(x) \right)^\lambda \right]; \\[2mm] \left[\left(\hat{F}^-_{A_{PNV}}(x)\hat{F}^-_{B_{PNV}}(x) \right)^\lambda, \left(\hat{F}^+_{A_{PNV}}(x)\hat{F}^+_{B_{PNV}}(x) \right)^\lambda \right] \end{array} \right\rangle x \in X \right\} \rightarrow (1)$$

Now

$$\lambda A_{PNV} \oplus \lambda B_{PNV} =$$

$$\left\{ \left\langle \begin{array}{c} x; \left[\begin{array}{l} \left[\left(\left(1-\left(1-\hat{T}^-_{A_{PNV}}(x)\right)^\lambda\right) + \left(1-\left(1-\hat{T}^-_{B_{PNV}}(x)\right)^\lambda\right) - \left(1-\left(1-\hat{T}^-_{A_{PNV}}(x)\right)^\lambda\right)\left(1-\left(1-\hat{T}^-_{B_{PNV}}(x)\right)^\lambda\right) \right), \\ \left[\left(\left(1-\left(1-\hat{T}^+_{A_{PNV}}(x)\right)^\lambda\right) + \left(1-\left(1-\hat{T}^+_{B_{PNV}}(x)\right)^\lambda\right) - \left(1-\left(1-\hat{T}^+_{A_{PNV}}(x)\right)^\lambda\right)\left(1-\left(1-\hat{T}^+_{B_{PNV}}(x)\right)^\lambda\right) \right) \end{array} \right]; \\ \left[\begin{array}{l} \left[\left(\left(1-\left(1-\hat{C}^-_{A_{PNV}}(x)\right)^\lambda\right) + \left(1-\left(1-\hat{C}^-_{B_{PNV}}(x)\right)^\lambda\right) - \left(1-\left(1-\hat{C}^-_{A_{PNV}}(x)\right)^\lambda\right)\left(1-\left(1-\hat{C}^-_{B_{PNV}}(x)\right)^\lambda\right) \right), \\ \left[\left(\left(1-\left(1-\hat{C}^+_{A_{PNV}}(x)\right)^\lambda\right) + \left(1-\left(1-\hat{C}^+_{B_{PNV}}(x)\right)^\lambda\right) - \left(1-\left(1-\hat{C}^+_{A_{PNV}}(x)\right)^\lambda\right)\left(1-\left(1-\hat{C}^+_{B_{PNV}}(x)\right)^\lambda\right) \right) \end{array} \right]; \\ \left[\left(\hat{G}^-_{A_{PNV}}(x)\right)^\lambda \left(\hat{G}^-_{B_{PNV}}(x)\right)^\lambda, \left(\hat{G}^+_{A_{PNV}}(x)\hat{G}^+_{B_{PNV}}(x)\right)^\lambda \right]; \left[\left(\hat{U}^-_{A_{PNV}}(x)\right)^\lambda \left(\hat{U}^-_{B_{PNV}}(x)\right)^\lambda, \left(\hat{U}^+_{A_{PNV}}(x)\hat{U}^+_{B_{PNV}}(x)\right)^\lambda \right]; \\ \left[\left(\hat{F}^-_{A_{PNV}}(x)\hat{F}^-_{B_{PNV}}(x)\right)^\lambda, \left(\hat{F}^+_{A_{PNV}}(x)\hat{F}^+_{B_{PNV}}(x)\right)^\lambda \right] \end{array} \right\rangle; x \in X \right\}$$

Consider the truth membership function

$$= \left(1-\left(1-\hat{T}^-_{A_{PNV}}(x)\right)^\lambda\right) + \left(1-\left(1-\hat{T}^-_{B_{PNV}}(x)\right)^\lambda\right) - \left(1-\left(1-\hat{T}^-_{A_{PNV}}(x)\right)^\lambda\right)\left(1-\left(1-\hat{T}^-_{B_{PNV}}(x)\right)^\lambda\right)$$

$$= 2 - \left[\left(1-\hat{T}^-_{A_{PNV}}(x)\right)^\lambda + \left(1-\hat{T}^-_{B_{PNV}}(x)\right)^\lambda \right] - \left[1 - \left(1-\hat{T}^-_{A_{PNV}}(x)\right)^\lambda - \left(1-\hat{T}^-_{B_{PNV}}(x)\right)^\lambda + \left(1- \hat{T}^-_{A_{PNV}}(x)\right)^\lambda \left(1-\hat{T}^-_{B_{PNV}}(x)\right)^\lambda \right]$$

$$= 2 - \left(1-\hat{T}^-_{A_{PNV}}(x)\right)^\lambda - \left(1-\hat{T}^-_{B_{PNV}}(x)\right)^\lambda - 1 + \left(1-\hat{T}^-_{A_{PNV}}(x)\right)^\lambda + \left(1-\hat{T}^-_{B_{PNV}}(x)\right)^\lambda - \left(1-\hat{T}^-_{A_{PNV}}(x)\right)^\lambda \left(1-\hat{T}^-_{B_{PNV}}(x)\right)^\lambda$$

$$= 1 - \left(\left(1-\hat{T}^-_{A_{PNV}}(x)\right)^\lambda \left(1-\hat{T}^-_{B_{PNV}}(x)\right)^\lambda \right)$$

$$= 1 - \left[\left(1-\hat{T}^-_{A_{PNV}}(x)\right)\left(1-\hat{T}^-_{B_{PNV}}(x)\right) \right]^\lambda$$

In the similar manner, we have

$$\lambda A_{PNV} \oplus \lambda B_{PNV} =$$

$$\left\{ \left\langle \begin{array}{c} x; \left[1-\left(\left(1-\hat{T}^-_{A_{PNV}}(x)\right)\left(1-\hat{T}^-_{B_{PNV}}(x)\right) \right)^\lambda, 1-\left(\left(1-\hat{T}^+_{A_{PNV}}(x)\right)\left(1-\hat{T}^+_{B_{PNV}}(x)\right) \right)^\lambda \right]; \\ \left[1-\left(\left(1-\hat{C}^-_{A_{PNV}}(x)\right)\left(1-\hat{C}^-_{B_{PNV}}(x)\right) \right)^\lambda, 1-\left(\left(1-\hat{C}^+_{A_{PNV}}(x)\right)\left(1-\hat{C}^+_{B_{PNV}}(x)\right) \right)^\lambda \right]; \\ \left[\left(\hat{G}^-_{A_{PNV}}(x)\hat{G}^-_{B_{PNV}}(x)\right)^\lambda, \left(\hat{G}^+_{A_{PNV}}(x)\hat{G}^+_{B_{PNV}}(x)\right)^\lambda \right]; \left[\left(\hat{U}^-_{A_{PNV}}(x)\hat{U}^-_{B_{PNV}}(x)\right)^\lambda, \left(\hat{U}^+_{A_{PNV}}(x)\hat{U}^+_{B_{PNV}}(x)\right)^\lambda \right]; \\ \left[\left(\hat{F}^-_{A_{PNV}}(x)\hat{F}^-_{B_{PNV}}(x)\right)^\lambda, \left(\hat{F}^+_{A_{PNV}}(x)\hat{F}^+_{B_{PNV}}(x)\right)^\lambda \right] \end{array} \right\rangle; x \in X \right\} \rightarrow (2)$$

From (1) and (2), we have $\lambda(A \oplus B)_{PNV} = \lambda A_{PNV} \oplus \lambda B_{PNV}$

Proof of (v):

The proof is similar as the proof of (***iii***).

4. *Pentapartitioned Neutrosophic Vague Compact Space*

Definition 4.1:

Let (X_{PNV}, τ_{PNV}) be PNVTS.

   i)   If a family $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle; i \in J\}$ of PNVOS in X satisfies the
        condition $\cup \{\langle x, T_A, C_A, G_A, U_A, F_A \rangle; i \in J\} = 1_{PNV}$, then it is
        called Pentapartitioned Neutrosophic vague open cover of X. A finite
        subfamily of Pentapartitioned neutrosophic vague open cover
        $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle: i \in J\}$ of X, which is also a pentapartitioned
        neutrosophic vague cover of X, is called Pentapartitioned
        neutrosophic vague finite subcover of $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle: i \in J\}$.
   ii) A family $\{\langle x, T_B, C_B, G_B, U_B, F_B \rangle; i \in J\}$ of PNVCS in X
        satisfies finite intersection property iff every finite subfamily
        $\{\langle x, T_B, C_B, G_B, U_B, F_B \rangle; i = 1, 2, \ldots \ldots n\}$ of the family satisfies the
        condition $\cap_{i=1}^{n}\{\langle x, T_{B_i}, C_{B_i}, G_{B_i}, U_{B_i}, F_{B_i} \rangle\} \neq 0_{QNV}$.

Definition 4.2: A PNVTS (X_{PNV}, τ_{PNV}) is Pentapartitioned neutrosophic
vague compact iff every Pentapartitioned neutrosophic vague open cover of X has a
Pentapartitioned neutrosophic vague finite subcover.

Corollary 4.3: A PNVTS (X_{PNV}, τ_{PNV}) is Pentapartitioned neutrosophic vague
compact iff every family $\{\langle x, T_B, C_B, G_B, U_B, F_B \rangle: i \in J\}$ of PNVCSs in X having the FIP
has nonempty intersection.

Corollary 4.4: Let (X_{PNV}, τ_{PNV}), (Y_{PNV}, σ_{PNV}) be PNVTSs and $f: (X_{PNV}, \tau_{PNV}) \rightarrow$
(Y_{PNV}, σ_{PNV}) a Pentapartitioned neutrosophic vague continuous surjection. If
(X_{PNV}, τ_{PNV}) is Pentapartitioned neutrosophic vague compact, then so is (Y_{PNV}, σ_{PNV}).

Definition 4.5: Let (X_{PNV}, τ_{PNV}) be PNVTS and A_{PNV} a PNVS in X.

i) If a family $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle: i \in J\}$ of $PNVOS$ in X $satisfies$ the $condition$ $A_{PNV} \subseteq \cup\{\langle\langle x, T_A, C_A, G_A, U_A, F_A \rangle: i \in J\rangle\}$, $then$ it is $called$ $Pentapartitioned$ $neutrosophic$ $vague$ $open$ $cover$ of A_{PNV}. A $finite$ subfamily of Pentapartitioned $neutrosophic$ $vague$ $open$ cover $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle: i \in J\}$ of A_{PNV}, which is also a pentapartitioned $neutrosophic$ $vague$ $cove$r of A_{PNV}, is called Pentapartitioned $neutrosophic$ $vague$ $finite$ $subcover$ of $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle: i \in J\}$.

ii) A $PNVS$ in a $PNVTS$ (X_{PNV}, τ_{PNV}) is called Pentapartitioned n$eutrosophic$ $vague$ compact iff every Pentapartitioned $neutrosophic$ $vague$ $cover$ A_{PNV} of has finite subcover.

Corollary 4.6: Let (X_{PNV}, τ_{PNV}), (Y_{PNV}, σ_{PNV}) be $PNVTSs$ and $f: (X_{PNV}, \tau_{PNV}) \rightarrow$ (Y_{PNV}, σ_{PNV}) a Pentapartitioned $neutrosophic$ $vague$ continuous function. If A_{PNV} is Pentapartitioned $neutrosophic$ $vague$ $compact$ in (X_{PNV}, τ_{PNV}), then so if $f(A_{PNV})$ in (Y_{PNV}, σ_{PNV}).

Conclusion

In this paper, we defined some new operations on Pentapartitioned Neutrosophic Vague Set under neutrosophic environment. The basic algebraic operations on Pentapartitioned neutrosophic vague sets namely addition, multiplication, scalar multiplication and power also compact space along with illustrative examples were presented. Subsequently, the basic properties of these operations such as commutative law and relevant laws are mathematically proven. This new extension will broaden the fundamental knowledge of existing set theories and subsequently could be applied to real life experiments where truthness, indeterminacy and falsity could be dealth with.

References

[1]. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 1986; 20, pp. 87–96.

[2]. W.-L. Gau and D. J. Buehrer, "Vague sets," *IEEE Trans. Syst. Man. Cybern.*, vol. 23, no. 2, pp. 610–614, 1993.

[3]. Hazwani Hashim, Lazim Abdullah and Ashraf Al-Quran, "Algebraic operations on new interval neutrosophic vague sets, ICoAIMS 2019 , IOP Publishing ,Journal of Physics: Conference Series **1366** (2019) 012062".

[4]. P. Liu and Y. Wang, "Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean," *Neural Comput. Appl.*, vol. 25, no. 7–8, pp. 2001–2010, Dec. 2014.

[5]. X. Peng and J. Dai, "A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017," *Artificial Intelligence Review*, Springer Netherlands, pp. 1–57, 18-Aug-2018.

[6]. V. R. Radhika , K. Mohana. Pentapartitioned Neutrosophic Vague Sets and its

Topological spaces(Communicated)

[7]. F. Smarandache, *Neutrosophic set, a generalisation of the intuitionistic fuzzy sets*, in "Inter. J. Pure Appl. Math.", 24 (2005), 287-297.

[8] Shawkat Alkhazaleh. Neutrosophic vague set theory, Critical Review. 2015; 10, pp. 29–39.

[9]. H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, "Single valued neutrosophic set," in *Multispace and Multistructure*, vol. IV, 2010, pp. 410–413.

[10]. L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965.