The relativistic surface gravity of spinning black holes

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Abstract

The basis of research work is mainly concerned with discussion of black holes based on theory of relativity like relativistic surface gravity, relativistic frequency and relativistic gravitational force of black holes. A black hole deforms space time caused by a compact mass. It traps all the bodies, messages, information, sounds, radiations etc. towards itself passing through the event horizon. It reflects nothing, so it looks like a black body and hence named as black hole. The mass and density of a black hole is so high that its gravitational attraction becomes abnormally high. . Initially the basics of research work like the frames of references, special theory of relativity with mass variation, mass energy equivalence relation, Schwarzschild radius of spinning black holes and non-spinning black holes are studied. Secondly, the relativistic surface gravity of spinning black holes in terms of spin parameter are studied for XRBs & AGN. Thirdly the model for the relativistic study of the frequency of Hawking radiation and fourthly the model for relativistic force of attraction of black holes on the light particles are studied for the same scenario and concluded that The relativistic surface gravity is higher for the lower masses and smaller for heavier masses of black holes. The relativistic surface gravity of black holes in AGN is very-very smaller than to that of black holes in XRBs.

1. INTRODUCTION

Stellar evolution is the process by which a star undergoes a sequence of radical changes during its lifetime. Depending on the mass of the star, this lifetime ranges from only a few million years for the most massive to trillions of years for the least massive, which is considerably longer than the age of the universe. The table shows the lifetimes of stars as a function of their masses. All stars are born from collapsing clouds of gas and dust, often called nebulae or molecular clouds. Over the course of millions of years, these proto-stars settle down into a state of equilibrium, becoming what is known as a main-sequence star[1].

When stars are born, they form from existing gas dust. This is called interstellar matter. When cloud of interstellar matter crosses the spiral arm of the galaxy, it begins to form clumps. The gravitational forces within the clumps cause them to contract, forming protostar. When the centre of a protostar reach at a temperature of a several million of degree Celsius due to collision of hydrogen and helium atoms, the fusion reaction begins and releasing a large amount of energy radiation which prevents the protostar to contract. Any stellar structure in our universe is in equilibrium under the influence of two opposing forces- the gravitational force and the pressure of gas & radiation [2].

The formation of a star begins with gravitational instability within a molecular cloud due to some reasons. As the cloud collapses, the gravitational energy is converted into heat energy and its central temperature rises. When the temperature raises sufficiently high, hydrogen burning sets at the core of the star and generates sufficient radiation pressure to stop the contraction. Gradually hydrogen is converted into helium at the core and star again begins to contract. Hydrogen burning becomes restricted in a shell-layer surrounding the core. Eventually the core is compressed enough to start helium fusion. Depletion of helium at the core gives rise to a carbon core. The core contracts until the temperature and pressure are sufficient to fuse carbon. This process continues, with the successive stages being fuelled by neon, oxygen, and silicon. Near the end of the star's life, fusion can occur along a series of onion-layer shells within the star. Each shell fuses a different element with the outermost shell fusing hydrogen; the next shell fusing helium, and so forth[3].

The final stage is reached when the star begins producing iron. The iron is the element having highest binding energy and no energy will be released due to its ignition and the star will continue to collapse until some pressure is generated at the centre to counterbalance the collapse.

When the exhaustion of all nuclear fuel takes place within the star, it meets the fate of death and becomes white dwarf, neutron star, and black hole depending on its initial mass of the star. Due to gradual contraction of the star the density at the centre of the star will go on increasing till a state of density is reached when electron degeneracy pressure generates within the stellar object. This electron degeneracy pressure is what supports a white dwarf against gravitational collapse[4,5].

2. GRAVITATIONAL COLLAPSE;

Gravitational collapse is the inward fall of a body due to the influence of its own gravity. In any stable body, this gravitational force is counterbalanced by the internal pressure of the body, in the opposite direction to the force of gravity. Gravitational collapse is usually associated with very massive bodies, such as neutron stars, quasars and massive collections of stars such as globular clusters and galaxies[6].

Gravitational collapse is at the heart of structure formation in the universe. For example, a star is born through the gradual gravitational collapse of a cloud of interstellar matter. The compression caused by the collapse raises the temperature until nuclear fuel reignites in the centre of the star and the collapse comes to a halt. The thermal pressure gradient compensates the gravity and a star is in dynamical equilibrium between these two forces. After a star has exhausted its nuclear fuel, it can no longer remain in equilibrium and must ultimately undergo gravitational collapse[7].

The importance of gravitational collapse processes in relativistic astrophysics was realized when Datt and Oppenheimer and Synder used general relativity to study the dynamical collapse of a homogeneous Spherical dust cloud under its own gravity. This model gave rise to concept of a black hole. A black hole is a region of space time from which no light or matter can escape away to far away external observers. In order to create a black hole as the final state of gravitational collapse of the star, an event horizon must develop in the space time earlier than the time when the final space time singularity forms[8].

If the event horizon is developed prior to the formation of the singularity , neither the singularity nor the collapsing matter that has fallen within it would be observable to an external observer, and a black hole is said to have formed as the final end state of the collapsing star. The **event horizon** is where light loses the ability to escape from the black hole[1].

3. NAKED SINGULARITY:

In general relativity, a naked singularity is a gravitational singularity, without an event horizon. In a black hole, there is a region around the singularity, the event horizon, where the gravitational force of the singularity is strong enough so that light cannot escape. Hence, the singularity cannot be directly observed. A naked singularity, by contrast, is observable from the outside. A radiating stellar model has been proposed by Banerjee et al. in which the horizon is never encountered.

The theoretical existence of naked singularities is important because their existence would mean that it would be possible to observe the collapse of an object to infinite density. A naked singularity could allow scientists to observe an infinitely dense material, which would under normal circumstances be impossible by the cosmic censorship hypothesis[7]. The cosmic censorship hypothesis says that a naked singularity cannot arise in our universe from realistic initial conditions. Singularities that arise in the solutions of Einstein's equations are typically hidden within event horizons, and therefore cannot be seen from the rest of space time. Singularities which are not so hidden are called naked. According to cosmic censorship hypothesis conceived by Roger Penrose in 1969 no naked singularities other than the Big-Bang singularity exist in the universe. Cosmic Censorship Hypothesis states that the universe contains no singularities that can be visible to a very far away observer[8].

There are some compact stellar objects like white dwarf, neutron star, quasar, pulsar and black hole have been discussed as:

4. WHITE DWARF:

A white dwarf is a small star composed mostly of electron-degenerate weather after contraction of core part of stars leaving the matter of outer part. A white dwarf's mass is comparable to that of the Sun and its radius is comparable to that of the earth hence they are very dense. The average density of white dwarf lays in the range 10^5- 10^6 gm cm^{-3}. Its faint luminosity comes from the emission of stored thermal energy. For stellar masses less than about 1.44 solar masses, the energy from the gravitational collapse is not sufficient to produce the neutrons of a neutron star, so the collapse is halted by electron degeneracy to form white dwarfs. If electron degeneracy pressure is not able to balance the force of gravity, then it would collapse into a denser object such as neutron star. This maximum mass for a white dwarf is called the Chandrasekhar limit. As the star contracts, all the lowest electron energy levels are filled and the electrons are forced into higher and higher energy levels, filling the lowest unoccupied energy levels. This creates an effective pressure called electron degeneracy pressure. This electron degeneracy pressure supports a white dwarf against gravitational collapse. When a medium sized star nears the end of its life and has used up all of its available hydrogen, it will slowly expand into a red giant which fuses helium into carbon and oxygen. Once this process has completed, the star will throw off its outer layers to form a planetary nebula. The core that remains will be a white dwarf composed of carbon and oxygen nuclei compressed by gravity and stripped of their electrons. This extremely dense matter makes up a stellar remnant (white dwarf). Due to its very high density, the classical equation of state for a perfect gas do not apply for white dwarf , instead the pressure is given by the equation of state for degenerate matter that is dense, cold matter[3,9].

5. CHANDRASEKHAR LIMIT:

The Chandrasekhar limit (M_{ch}) is defined as the 1.4 times of the solar mass i.e., 1 $M_{ch}=1.4M_{\Theta}$. When the mass of dying star is greater than the Chandrasekhar limit (M_{ch}), then the neutron star is formed which finally becomes black hole. But when the mass of dying star is less than the Chandrasekhar limits (M_{ch}), white dwarf is formed[10]. In other words, we can say that if the mass of the star exceeds the Chandrasekhar limit the gravitational contraction can not be counter balanced by electron degeneracy pressure and consequently the star continues to contract until some new pressure develops at the centre of the stellar object to counter balance the gravitational contraction. The neutron degeneracy pressure will bring the stellar object again into a new equilibrium state known as neutron star[1].

If this pressure also fails in preventing the gravitational contraction then the contraction will continue forever and no force in the universe can prevent the collapse to a point singularity and the concept of black hole comes into picture.

6. NEUTRON STAR:

Neutron stars are the product of supernovae, gravitational collapse events in which the core of a massive star reaches nuclear densities and stabilizes against further collapse. Neutron stars are one of the possible dead states of a star. They result from massive stars which have mass greater than 4 to 8 times that of our sun. After these stars have finished burning their nuclear fuel, they undergo a supernova explosion. This explosion blows off the outer layers of a star into a supernova remnant. The central region of the star collapses under gravity. It collapse so much that protons and electrons combine to form neutrons through the reaction[5]

$$p+e^{-}=n+\nu \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The neutrinos escape the star. Enough electrons and protons must remain so that the Pauli principle prevents neutron beta decay,

$$n\rightarrow p+e^{-}+\nu^{-} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The condition for the neutrons to be stable against beta decay is that the electron.

The neutron degeneracy pressure within the star counter balances the gravitational collapse paving the way for the formation of neutron star. Neutron star contains matter in one of the densest forms found in the universe.

A neutron star is about 20 km in diameter and has mass of about 1.4 times that of our sun. Because of its small size and high density, a neutron star possesses a surface gravitational field about 2×10^{11} times that of the earth. A Neutron Star is made up of cold catalyzed matter i.e., matter which has reached the end point of stellar evolution. The magnetic field of the neutron star is of the order of ~ 10^{12} Gauss.

The temperature inside a newly formed neutron star is from around 10^{11} to 10^{12} Kelvin. However, the huge number of neutrinos it carries away so much energy that the temperature falls within a few years to around 1 million Kelvin. Neutron stars are known to have rotation periods between about 1.4 milliseconds to 30 seconds. A neutron star's structure is very simple, and it has three main layers: Neutron stars also have a very tiny (a few centimeters - about an inch) atmosphere, but this is not very important in the functioning of the star.

An approximation of neutron star dimension is that it has thick metallic surface layer, below which there is about 1 km thick solid layer of material of density 10^5-10^{14} gm cm^{-3}. [5].

The maximum mass limit for neutron star has been discussed by many authors drawing different conclusions. Arnett and Bowers have estimated the upper mass limit as 1-3 M_Θ . Rhodes and Ruffini have estimated the upper mass limit as 3.2 M_Θ. Brecher and Caporasso suggest this limit as 4.8 M_Θ.[1].

7. PULSAR:

The spinning neutron star emitting sharp pulse of exactly spaced interval of time is known as Pulsar. Pulsar is a highly magnetized, rapidly rotating neutron star. Neutron stars are very dense. They have short, regular rotational periods. These produce a very precise interval, between pulses that range from roughly milliseconds to seconds for an individual pulsar. Extremely short periods of pulsars, suggests that pulsars are rotating neutron stars possessing a super high magnetic field.

Neutron stars have very intense magnetic fields, as compared to Earth's magnetic field. However, the axis of the magnetic field is not aligned with the neutron star's rotation axis. The magnetic axis of the pulsar determines the direction of the electromagnetic beam. Millisecond pulsars have provided us with best working ground for testing of general relativity. The suggestion that pulsars were rotating neutron stars was put forth independently by Thomas Gold and Franco Pacini in 1968, and was soon proven by the discovery of a pulsar with a very short (33-millisecond) pulse period in the crab nebula. Pulsars were discovered in late 1967 by graduate student Jocelyn Bell Burnell as radio sources that blink on and off at a constant frequency. The discovery of a pulsar at the centre of crab-nebula, where the astronomers had predicted Neutron Star, established the oneness of Neutron Star and pulsars. T75.360x10^{14}he close observation of the pulsars can help to elucidate the interior properties of neutron stars and can provide us with windows into the interiors of neutron stars. The discovery of pulsars allowed astronomers to be acquainted with the conditions of an intense gravitational field.

8. QUASAR;

The word quasar is derived from Quasi Stellar Object. Quasars are the most luminous, powerful, and energetic objects known in the universe. They are the most distant known objects in the universe. The quasars, with total luminosity hundred times greater than that of giant galaxies, are extremely unusual in their properties. The most luminous quasars radiate at a rate that can exceed the output of average galaxies. Quasars have large red shifts relative to normal stars and galaxies. The accretion of material into super massive black holes in the nuclei of distant galaxies is believed to be one of the main causes of energy content of quasar. All observed quasar spectra have red shifts between 0.056 and 7.085.

If the lines in the spectrum of the light from a star or galaxy appear at a lower frequency, the object exhibits positive red shift. Cosmological red shift is seen due to the expansion of the universe. Gravitational red shift is a relativistic effect observed in electromagnetic radiation of very compact objects. Red shift is simply the time dilation in the presence of gravitational field. Quasars are very high red shift objects indicating that they are very far away from us. The brightest quasar is 3C273, two billion light years away from us. The red shift of 3C 273 is z=0.158, meaning that the wavelengths of its spectral lines are stretched by 15.8%**.** Models of quasars have been proposed by Durgapal and Gehlot and it has been shown that the maximum surface red-shift can be as large as 4.828. The red shift has been also obtained for these models[13,14].

9. BLACK HOLES-NON-SPINNING AND SPINNING:

Four types of black holes have discussed in the field equations on the basis of general theory of relativity. Two of these like the Kerr and Kerr-Newman black holes rotate and possesses spin angular momentum and usually known as Spinning Black holes and rest two with zero spin angular momentum are known as non-spinning black holes.

events outside it.

The black hole possesses an event horizon (a one-way membrane) that casually isolates the inside of the black hole from the rest of the universe. The radius of the event horizon of a non-spinning BH is given by the Schwarzschild radius can be obtained as:[11,12].

$$R_s = \frac{2GM}{c^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

But when spinning property is included, then the radius of the event horizon of a Spinning Black hole(Schwarzschild black hole) can be obtained by multiplying 1/2 to equation(2.4) and hence[17].

$$R_s = \frac{GM}{c^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

10. Special Theory of Relativity:

The failure of the Michelson Morely experiments to determine the velocity of the earth relative to the hypothetical medium called ether became the basis of the foundation of the theory of relativity. The formulation of Lorentz transformation is based on the requirement that the laws of electromagnetic remain invariant, since the electromagnetic disturbance is propagated with speed $3x10^8$ m/s in free space, independent of the choice of reference system and no signal can travel as faster as the speed greater than that of the speed of light c.

There are two postulates of Einstein's theory of relativity for the inertial frame of reference[17].

Ist Postulate

The fundamental laws of Physics have the same form for every inertial frame of references. This postulate is drawn from Newtonian mechanics.

IInd Postulate

The velocity of light in vacuum is maximum and independent of the source and observers. This postulate is responsible to differentiate the classical theory of relativity and special theory of relativity.

11. Variation of Mass with Velocity:

Let us consider two frames of reference S and S' where S' is moving with velocity v relative to S along the common x-axis. Suppose there be two equal particles in the system S' travelling with equal and opposite velocities i.e. $u_x^{'}$ and $(-u_x^{'})$ along positive direction of x-axis. Also suppose that these two particle collide and after impact collapse in to one particle. According to the principle of conservation of momentum, the collapsed particle comes to the rest and by the principle of conservation of mass, their combined mass is 2m' in the system S', after impact. Let there be an observer in the system S to observe this impact when m_1 and m_2 are the masses of the two particles as seen from system S and when the velocities of two particles according to the law of composition of velocity are given by u_1 and u_2 in the system S, then for the particles moving with velocity $u_x^{'}$, we have[10].

$$u_1 = \frac{u_x^{'} + v}{1 + \frac{u_x^{'}.v}{c^2}} \qquad \text{......................(5)}$$

and for the particles moving with $-u_x^{'}$,

$$u_2 = \frac{-u_x^{'} + v}{1 - \frac{u_x^{'}.v}{c^2}} \qquad \text{.............. (6)}$$

As the particles travel with different velocities in system S, their masses will also be different.

Consider that m_1 and m_2 be the masses of the particles moving with velocities u_1 and u_2 respectively, then by the principle of conservation of momentum as viewed from S, we have

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v \qquad \text{.........(7)}$$

Since, the particles after impact in S’ are traveling with velocity v relative to the system S, as they are at rest in the system S’. We get from eq^{n} (1.53)

$$m_1u_1 + m_2u_2 = m_1v + m_2v$$

$$m_1(u_1 - v) = m_2(v - u_2)$$

$$\frac{m_1}{m_2} = \frac{v - u_2}{u_1 - v}$$

Putting values of u_1 and u_2 from equation (5) and (6) in the equation(7), and solving we get,

$$\frac{m_1}{m_2} = \frac{1 + \frac{u_x^{'}v}{c^2}}{1 - \frac{u_x^{'}v}{c^2}} \qquad \text{.............(8)}$$

On rearranging equations, we have

$$u_1^2\left(1 + \frac{u_x^{'}v}{c^2}\right)^2 = (u_x^{'} + v)^2 = u_x^{'2} + v^2 + 2u_x^{'}v \qquad \text{............(9)}$$

$$u_2^2\left(1-\frac{u_x'v}{c^2}\right)^2 = -u_x'+v\ {}^{2} = u_x'^{\,2}+v^2-2u_x'v \text{ ....(10)}$$

eq^n .(10) is subtracted from eq^n .(9)

$$u_1^2\left(1+\frac{u_x'v}{c^2}\right)^2-u_2^2\left(1-\frac{u'v}{c^2}\right)^2=4u_x'v$$

Solving above eq^n finally, we get

$$\frac{m_1}{m_2}=\frac{\sqrt{\left(1-\dfrac{u_2^2}{c^2}\right)}}{\sqrt{\left(1-\dfrac{u_1^2}{c^2}\right)}} \qquad \text{......... (11)}$$

If the particles of mass m_2 is moving with zero velocity, then m_2 is the mass of the particle at rest, which is usually denoted by m_0 and m_1 becomes the mass of the particles whose velocity is u_1. Hence putting $u_2 = 0$ and $m_2 = m_0$ we have.

$$m_1=\frac{m_0}{\sqrt{1-\dfrac{u_1^2}{c^2}}}$$

In usual notation, by putting m_1=m and u_1 = v, we get.

$$m=\frac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}} \qquad \text{......... (12)}$$

Thus m_0 is mass of the particle at rest, and this is generally known as rest mass. It is clear that the mass of a body increases with increase of velocity.

12. Mass Energy Equivalence:

By applying a force F, the increase in energy of any particle may be defined in terms of work which is the product of force and displacement[17].

Hence, when a particle is displaced by a distance dr by the application of the force F, the increase in kinetic energy is given by

$$\Delta T = F.dr \qquad \text{....(13)}$$

Force is defined as the rate of change of momentum, so

$$F = \frac{d\ \ mv}{dt}$$

$$F = m\frac{dv}{dt} + v\frac{dm}{dt}$$

From eq^{n} (13), we have,

$$\Delta T = \left[m\frac{dv}{dt} + v\frac{dm}{dt} \right].dr$$

According to theory of relativity, mass of a particle varies with the variation of velocity, thus

$$\Delta T = m\frac{dr}{dt}.dv + v\frac{dr}{dt}.dm \qquad \text{......(14)}$$

Again from variation of mass with velocity, we have,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m\sqrt{1 - \frac{v^2}{c^2}} = m_0$$

$$m^2\left(1 - \frac{v^2}{c^2}\right) = m_0^2$$

$$m^2\left(\frac{c^2 - v^2}{c^2}\right) = m_0^2$$

$$m^2c^2 - m^2v^2 = m_0^2c^2$$

Differentiating the above eq^{n} , we have

$$c^2.2mdm - v^2.2mdm - m^2.2vdv = 0$$

$$c^2dm = mvdv + v^2dm \qquad \text{......(15)}$$

Eq^{n} (2.36) is integrated to get total kinetic energy

$$\int_{m_o}^{m} dm = c^2 \;\; m \,{}_{m_o}^{m}$$

or $$K.E = \;\; m - m_0 \;\; c^2 \qquad \text{…… (16)}$$

Hence the total energy of a moving particle is given by

$$E = K.E + P.E$$

$$E = \;\; m - m_0 \;\; c^2 + mc^2$$

$$E = mc^2 - m_0c^2 + m_0c^2$$

$$E = mc^2 \qquad \text{…….(17)}$$

The equation(17) is known as mass energy equivalence principle.

13. Zeroth Law of Black Hole Mechanics:

In thermal physics, the zeroth law states that the temperature T of body at thermal equilibrium is constant throughout the body. Otherwise heat will flow from hot spots to the cold spots. Correspondingly for stationary black holes one can show that surface gravity κ is constant on the event horizon. This is obvious for spherically symmetric horizons but is true also more generally for non-spherical horizons of spinning black holese. The surface gravity (κ) can be thought of roughly as the acceleration at horizon of black hole. The surface gravity tends to zero, when the magnitude of charge of a black hole becomes equal to the mass of black hole[12].

14. First Law of Black Hole Mechanics:

The general form of the first law takes the form is given by[15].

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\delta J + v\delta Q \qquad \text{…………….... (18)}$$

where M be mass, J be spin, Q be the charge, v be the chemical potential and where Ω is the angular velocity of black holes.

The first law of black hole mechanics indicates the change in the mass of the black holes corresponding change in the area of event horizon, total angular momentum (spin) and charge of the black holes.

The term $\kappa/2\pi$ truly is the physical temperature of a black hole, not merely a quantity playing a role mathematically analogous to temperature in the laws of black hole mechanics.

15. Second Law of Black Hole Mechanics:

Black holes are found to be closely related to ordinary thermodynamics in the sense that their mechanical laws are the same to the laws of the thermodynamics. In 1971 Stephen Hawking stated that the area, A of the event horizon of a black hole can never decrease in any process:

$$\Delta A \geq 0 \qquad \text{..................} \; (19)$$

The area of the event horizon increases when (i) mass increases and (ii) spin decreases. It was later noted by Bekenstein that this result is analogous to the statement of the ordinary second law of thermodynamics, namely that the total entropy(S) of a closed system never decreases in any process:

$$\Delta S \geq 0 \qquad \text{..................} \; (20)$$

The above relation establishes the laws of black hole mechanics in parallel to the laws of ordinary thermodynamics by using parameters of the black hole as follows:[12].

16. Third Law of Black Hole Mechanics:

The limit $\kappa = 0$ cannot be reached within a finite time, in other words it is not possible how many processes we do, we will never reach the limit $\kappa = 0$. However, the external black holes, for example the Kerr black hole in which $a/M = 1$, do have $\kappa = 0$ thus zero temperature (absolute zero) but non-zero entropy. To actually reduce the surface gravity to zero is merely an idealized case because it is forbidden by the cosmic censorship conjecture. $T = 0$ cannot be reached and $\kappa = 0$ cannot be reached[15].

17. Surface gravity of spinning black holes;

The surface gravity is the acceleration needed to keep an object at the horizon as exerted at infinity and given by the following equation[14,16].

$$\kappa = \frac{\sqrt{M^2 - Q^2 - J^2 / M^2}}{2M^2 - Q^2 + 2M\sqrt{M^2 - Q^2 - J^2 / M^2}} \qquad (21)$$

Where M= Mass of black holes,

Q= Electric charge on black holes,

And J= Angular momentum of black holes.

CONCLUSIONS

1. The velocity of black holes is mainly responsible for either increasing or decreasing the surface gravity of black holes.

2. The relativistic surface gravity is higher for the lower masses and smaller for heavier masses of black holes.

3. The relativistic surface gravity of black holes in AGN is very-very smaller than to that of black holes in XRBs.

4. The relativistic change in frequency is lesser than to that of non-relativistic frequency and this change is lower for heavier mass and also does not depend on mass and speed.

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