Triaxial Star

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**Abstract:** The idea of a “Triaxial Star” first proposed by Chandrasekhar in 1969. More than 50 years passed, the detection of triaxial star remains unreachable. Detection of gravitational waves would be a probe to the astronomers to investigate the properties of the compact objects in a new direction in the light of gravitational waves. Recent discovery of cosmic baby , i.e., swift J1818.0-1607 with age ~ 300 years, offers the astronomers an opportunity , through continuous observations, to increase our knowledge about the physics of the evolution of a magnetar from its new born phase to end stage, generation of ultra-strong magnetic field in its interior, source of continuous gravitational waves, physics of coupling between magnetic field decay and cooling for keeping the internal temperature ~ 108 K for a period of ~ 104 years, , etc.

**Key Words:** Ellipsoid, Triaxial Star, Gravitational Waves, Neutron Star, Magnetar

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11. **Compact Star and Professor Chandrasekhar**

A compact star is a system inside which a struggle is continuing between gravity and degenerate pressure created by its constituents. Gravity tries to crash the star’s material by pulling towards the center of star, while degenerate pressure, created among the material particles due compactness, tries to counteract and overcome the gravity pull. In his book “An Introduction to the study of Stellar structure (Dover, 1967)“ Professor Chandrasekhar first gave a detailed picture of “what is a star, what is going on inside it, etc” in the form of mathematical language.

Before the discovery of neutrons in 1932, electrons were the latest discovered particles on that time. Professor Chandrasekhar thus considered the latest star, i.e., white dwarf. He calculated the maximum mass of a white dwarf full of electrons = 1.4 M⊙ ( M⊙ being the solar mass) which is known as Chandrasekhar limit. In 1932 Chadwick discovered neutrons. Then T. Gold proposed the possible existence of a new compact star beyond white dwarf, called Neutron Star which is made full of neutrons.

In the year 1969 Professor Chandrasekhar theoretically proposed another new compact star, called “Triaxial Star”. Note that, in 1967 Jocelyn Bell, a Ph.D. scholar of Cambridge University, UK, detected a peculiar type signal but its rhythm was very accurate. This star later identified as neutron star. However, the triaxial star was confined in theoretical works only although neutron stars ( isolated , rotating neutron stars or pulsars, and magnetars) became a gold mine to the astronomers and scientists. In searching the gravitational wave sources, isolated neutron stars, millisecond pulsars became the efficient gravitational wave sources. Many theoretical studies suggested that the triaxial neutron stars, i.e. triaxially deformed neutron stars, are the most significant gravitational wave sources . So, triaxially deformation in star thus gave an impetus to the astronomers, scientists to search for a triaxial star.

1. **What is Triaxiality in a Star**

More than 50 years passed after the proposed concept of triaxial star but till date it is remained undetectable. Several theoretical works suggested that star with fast rotation can be deformed and would become a triaxial star . Classically, the figures of equilibrium of uniformly, rotating homogeneous masses pertain various sequences of ellipsoidal figures. The Maclaurin sequence is a sequence of oblate *spheroid*s along which the eccentricity (“e” ) of the meridional sections *increases from zero to one* (Chandrasekhar and Lebovitz 1964 ). Initially it was though that the sequence of the square of the angular velocity of rotation ( Ω2) is not a parameter of unrestricted range. This means that for each value of Ω2 , (except less than a certain determinate maximum, there are two permissible spheroidal figures of equilibrium. But Jacobi first showed that a sequence of genuine triaxial ellipsoids of equilibrium diverges from the Maclaurin sequence. This means that there is a bifurcation point which clearly distinguish the permissible sequences of figures of equilibrium from Maclaurin spheroidal sequence to Jacobi ellipsoidal sequence.

Let us consider a situation of general ellipsoid ( also called a triaxial ellipsoid, see fig.1) whose quadratic surface can be expressed in Cartesian co-ordinates by

 ( x2/ a2 ) + (y2 / b2) + (z2 / c2) = 1 (1)

where the semi – axes are of lengths a, b, c.

Now,

1. If all the three are same, i.e., a = b = c then it is a sphere.
2. If the length of two axes of an ellipsoid are the same, then the figure is called a spheroid , i.e., an oblate spheroid or a prolate spheroid depending on whether c < a or c > a, respectively.

In spherical coordinate system (r, θ, ϕ ) the eqn. (1) becomes

 (2)

Geodesic form

 (a)
 (b)

**Fig. 1 :** (a) Ellipsoid in Cartesian Coordinates (b) Ellipsoid in Geodesic Coordinates (Panou 2013)

In this case we consider a triaxial ellipsoid where Cartesian coordinates are ( x, y, z ) but three semi-axes are a = ax, b = ay, and c = b then eqn.(1) takes the form

 (3)

The linear eccentricities can be calculated as

  (4)

and he2  = hx2 – hy2 .

This triaxial ellipsoid can be parameterized as

  (5)

where  Ellipsoidal latitude and ellipsoidal longitude can therefore be interpreted with the help of this parameter. Thus, in this parameterization the first fundamental coefficients E, F, G can be expressed as (Panou 2013)

 (6)

 (7)

This implies that

1. F = 0 indicates that the β-curves and λ-curves are orthogonal.
2. B $\ne $ 0 and Λ $\ne $ 0 for all points and E = G = 0 means umbrella shape.
3. For an orthogonal parameterization the line element “ds” on the triaxial ellipsoid is

ds2 = E dβ2 + G dλ2 (8)

1. **Deformation in Star due Rotation**

A stable star can be deformed if it rotates fast. The classical solution of Maclaurin spheroids and Jacobian ellipsoids for self-gravitating and uniformly rotating, incompressible fluids in equilibrium provides two models of rapidly rotating stars. Bifurcation of these two models, i.e., the sequence of triaxial Jacobi ellipsoids diverges from that of the axisymmetric Maclaurin spheroids in the case of increasing in rotation of an equilibrium, appears when the ratio of kinetic energy (T) to gravitational energy (W) reaches T / |W| ~ 0.14 (Bonazzola et al 1998) . This means the configurations are no longer a precise ellipsoid in relativistic gravity or for compressible fluids, the triaxially deformed rotating compact star (or simply “Triaxial Star” ) are rather than ‘ellipsoids’. The importance of this triaxial model in relativistic astrophysics is that it includes fluid compressibility for modeling the realistic neutron star as an axisymmetric and uniformly rotating configuration associated with the equation of state (EoS) of high density nuclear matter (Friedman and Stergioulas 2013 ; Straumann 2013).

Let us consider a triaxial configurations possessing the tri-planar symmetry w.r.t. three orthogonal x, y, z planes for relatively stiff (piecewise) polytropic equation of state (EoS). Uryū et al (2016) found supra-massive triaxial solutions the masses of which exceed the maximum mass of the spherical solution but are always lower in comparison to those of axisymmetric equilibrium. The facts they obtained are :

1. The difference in the maximum masses of triaxial and axisymmetric equilibrium solutions depends sensitively on equation of states (EoSs) ;
2. In the case when this difference turns out to be only 10%, then it will be treated as a strong evidence that the EoS of high density matter in the core of neutron stars becomes substantially softer (2016 ).

Another important fact regarding the criteria for the star collapsing to black hole (Piro and Ott 2011) is that the supernova fall back accretion. It may spin up a newly born neutron star having a strong magnetic field (B) $\leq $ 5 x 1014 G as fast as T / |W| ~ 0.14 for 50 – 200 s until the star collapses to a black hole. This means there is a possibility that a triaxially deformed compact stars, like above mentioned newly neutron star, may be formed transiently from massive stellar core collapse. Once such triaxial star is formed, emission of enormous amount of gravitational waves then provides us an opportunity to extract properties of high density nuclear matter. Considering the typical value ( Lai and Shapiro 1995) of the amplitude of gravitational waves produced from triaxial stars and using a realistic excess cross-power search algorithm

 h ~ 9.1 x 10-21 (30 Mpc / D ) ( M / 1.4 Mo )3/4  ( R / 10Km )1/4 $f$ - 1/5  (9)

where D, M, R and f are the distance to source, the source mass, the mean radius, and the wave frequency in Hz , respectively. Piro and Thrane ( 2012) estimated the detectability of gravitational waves produced by triaxially compact stars under the fall back accretion scenario for Advanced LIGO detector (Harry 2010) ~ 17 Mpc.

1. **Deformation of a star due strong magnetic fields**

A neutron star or magnetar (special type of neutron star with internal strong magnetic field) can be deformed into a triaxial compact star by its intrinsic ultra-strong core magnetic field (Haskell et al 2008; Lambibi et al 2015; Rather et al. 2021 ) . Investigating the influence of a magnetic field on an ellipsoidal figure of equilibrium, taking into account the effect nonlinear in the deformation, Tsvetkov (1983) found that near the bifurcation point the magnetic field is the main factor controlling the form of an ellipsoidal figure.

1. **Magnetar and Triaxiality**

The concept of a magnetar was first proposed in 1987 and used to successfully explain soft gamma repeaters (SGR) in 1992. A magnetar is a neutron star with an ultra-strong magnetic field. At ~1015 G which is thousand trillion times stronger than the Earth’s, and between 100 and 1,000 times stronger than that of a radio pulsar, making them the most magnetic objects known.

Like other neutron stars, magnetars are around 20 kilometres in diameter, and have a mass of about 1.4 M⊙ . They are formed by the collapse of a star with a mass 10–25 M⊙ . The density of the interior of a magnetar is such that a tablespoon of its substance would have a mass of over 100 million tons. They are differentiated from other neutron stars by having even stronger magnetic fields, and by rotating more slowly in comparison. Most observed magnetars rotate once every 2 – 12 s (Kaspi 2010 ) , whereas typical neutron stars, observed as radio pulsars, rotate one to ten times per second ( Condon and Ransom 2021). The important properties of a magnetar's magnetic field gives rise to very strong and characteristic bursts of X-rays and gamma rays. However, the active life of a magnetar is short compared to other celestial bodies. Their strong magnetic fields decay after about 10,000 years, after which activity and strong X-ray emission cease.

Regarding the formation of the magnetar it is believed that they are formed in the same way as all neutron stars, through the core-collapse of a massive star in a supernova explosion. It is not entirely clear what conditions cause a magnetar to be created instead of an ordinary neutron star or pulsar, but in order to achieve such strong magnetic fields, some theories suggest the neutron star must initially rotate between 100 and 1,000 times per second.

The dominant theory of the strong fields of magnetars is that it results from a magneto-hydrodynamic dynamo process in the turbulent, extremely dense conducting fluid that exists before the neutron star settles into its equilibrium configuration (Thompson and Duncan 1993). These fields then persist due to persistent currents in a proton-superconductor phase of matter that exists at an intermediate depth within the neutron star (where neutrons predominate by mass). A similar magnetohydrodynamic dynamo process produces even more intense transient fields during coalescence of pairs of neutron stars (Price and Rosswog 2006). But another theory is that they simply result from the collapse of stars with unusually strong magnetic fields( Zhou et al 2019).

Note that the measurement of the spin-down rate of a SGR from observed pulsations suggested that it was a neutron star with a magnetic field strength of 8 × 1014 G and since that time, both SGRs and anomalous X-ray pulsars have been explained successfully by the magnetar model which is based on the decay of the magnetic field powering the emission of X-rays and gamma rays. However, it appears that magnetars are only X-ray bright for a short period of time since their pulse periods are clustered between 6 and 12 seconds. If they remained active for an extended period of time, we should also see magnetars with pulse periods of tens of seconds or longer.

1. **Recent detected Magnetar, the Cosmic Baby**

On 12th March 2020 at 21:16:49 UT the Swift Burst Alert Telescope (BAT) on board the Neil Gehrels Swift Observatory ( Gehrels et al 2004 ) detected a typical characteristics of short burst from magnetar ( Evans et al 2020 ). 64 seconds afterwards observation by the Swift X-ray Telescope (XRT) finally detected a new uncatalogued x-ray source, so called Swift 1818.0 – 1607 which is presently known as Cosmic Baby. The important parameters of this cosmic baby , obtained from the timing analysis of early observations at the time of discovery, are shown in table I.

 **Table I :** Various early observed / measured parameters of Swift J1818.0 – 1607 (Parui 2023,2023a)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Instruments | Observation based on | Typical properties | Value  | Reference |
| BAT Radio Telescope | 9 ms hard X-ray burst and long-lived outbursts | Characteristic age(shortest known ) | ~ 240 years | Esposito et al (2020 ) |
| Surface magnetic field | ~ 2.7 x 1014 G |
| Dipolar magnetic field at poles | $≈$ 7 x 1014 G |
| Spin down Luminosity ( Ėrot) | ~ 1.4 x 1036 erg.s-1 |
| XMM Newton Telescope |  | Luminosity | ~ 8 x 1034 erg.s-1 |
| Coherent periodicity of x-ray signal | 1.36 s | Enoto et al (2020 ) |
| Sardini Radio Telescope | Radio observation | Spin period derivative | ~ 8.2 x 10-11 s.s-1 | Champion et al(2020) |
| Period derivative | ~ 9 x 10-11 s.s-1 |
| Spin Period | 0.7333920 s |

After a series of observations through various telescopes (such as TMRT, NICER ) at several wavelengths till July 27, 2020 some confirmed parameters of the cosmic baby magnetars were available. For example: Rotation / spin period derivative is ~ 3.74 x 10-11 s-2, surface dipole magnetic field ~ 3 x 1014 G, spin down luminosity ~ 1.1 x 1036 erg. S-1, etc. Although Chandra Observatory ( Blumer and Safi-Harb 2020 ) began its observation to swift J1818.0 – 1607 less than a month after the discovery of cosmic baby but its observations gave the astronomers the first high resolution view of the cosmic baby in x-rays. Still there is a lot of variation almost in all parameters of the cosmic baby properties.

1. **Ellipticity of Cosmic Baby Magnetar Swift J1818.0-1607**

Robert Duncan and Christopher Thompson (1992 ) first proposed the existence of Magnetars in 1992 to explain the properties of transient sources of gamma ray i.e. Soft Gamma Repeaters (SGRs). Magnetars are isolated young Neutron Stars having intense magnetic fields which power a wide variety of high energy-emission from giant flares to fast radio bursts. The basic idea regarding these peculiar high energy sources could be the ultra-strong magnetized neutron stars, magnetars with surface (dipole) fields in the range 1014 – 1016 G and internal magnetic fields >~1016 G ( at least one order of magnitude stronger ) (Paczynski 1992, Thompson and Duncan 1995 ). As the density of the neutron star core is beyond the density ~ 1015 g.cm-3, it creates an anisotropic nature of the compact object ( Ruderman 1972 ) i.e. internal pressure can be decomposed into two parts: the radial pressure (pr) and the transverse pressure (pt) where pt is the orthogonal to pr . This pressure anisotropy affects the physical properties, stability and structure of the stellar matter ( Dev and Gleiser 2002) . The ultra-strong internal magnetic field of the stellar matter can also create anisotropic pressure i.e. the deformation of spheroidal shape of rotating neutron star and emission from it due deformation.

In searching the strong Gravitational Wave (GW) emission Cutler ( 2002 ) first pointed out the internal field structure of a neutron star becoming an effective, efficient source of GW emission. According to him a magnetically distorted neutron star, i.e. a neutron star with a strong internal toroidal field, turn into a prolate shape that offer the best chance of being a strong GW emitter. Thus, the ellipticity of the magnetic deformation (εB) in the shape of stellar object has crucial role in the strong GW source. In other word, the triaxiality of the stellar body —its dependence on the internal strong magnetic field strength, decay of the field strength, dynamical shape transition to a triaxial ellipsoid configuration.

A magnetar is a slowly rotating isolated neutron star having very strong magnetic field ranging 1016 – 1018 G and even more up to 1020 G ( Lai and Shapiro 1991) in its interior. In general, a magnetar is a triaxial stellar body (Melatos 1999) , particularly in new born phase. Its internal ultra-strong magnetic field (i.e. the toroidal magnetic field is greater than at least one order of its surface dipolar magnetic field) is estimated by inferring from the geometric distortion of the neutron star (or magnetar) caused by strong toroidal magnetic field. Therefore, limits on the ellipticities of magnetars can be used to constrain the toroidal magnetic field as a period or duration of GW emission. So, if the field is dipolar, then hydro-magnetic stresses, arising from non-radial gradients of the super-strong internal magnetic field, deform the magnetar (i.e. between the magnetic poles and the equator ) and the fractional difference (ε) between the principal moments of intertia can be expressed as ( Melatos 1999; Goldreich 1970; deCampli 1980)

 ε ~ δp R5 / I1 $≈$ 2 x 10-9 (Bint / 1010 T )2 (10)

where δp = induced matter-density perturbation

 ~ Bint 2 / μoCs2 ,

 R = the stellar radius,

 Cs = the isothermal sound speed ( = 3-1/2 c, c being the velocity of light),

 Bint = the characteristic magnitude of the internal magnetic field

1. ≈ Bo, if the internal magnetic field is confined to the stellar crust,
2. >~ Bo, if it is generated deep inside the star (i.e. convective dynamo model ( Thopson and Duncan 1993),
3. Bint <~ 109 T in the case of rotation powered pulsars

According to Melatos (1999) a) the hydrodynamic deformation in a magnetar is much larger in comparison to the elastic deformation arising from shear stresses in the crystalline stellar crust of a rotating neutron star; b) the principal axes of inertia in a rotating neutron star are oriented arbitrarily w.r.t.the magnetic axis of the external magnetic dipole field where as in the case of magnetar the magnetic axis is approximately parallel to one of the principal axes of inertia (say **e**3 ) i.e. the alignment of magnetic axis is not exact to inertia axis due to the complicated structure of the internal field near its generation site (i.e. in other word, a non-axisymmetry state remains).

1. **Origin and decay of core magnetic field of Swift J1818.0-1607**

Idealizing the magnetar as a neutron star with the shape of slightly deformed, homogeneous ellipsoid and having a small ellipticity

 $ϵ$ = ( I1 –I2 ) / I3 (11)

where I1, I2, I3 are the principal moments of inertia of the neutron star such that I3 is assumed to be aligned with the spin axis then using eqn. (10) we obtain the following constraint on the magnetar ellipticity (Moriya and Tauris 2016 ) as

| $ϵ$ | < (5 / 3G )1/2 { C R3 Po Bdipole / 24 $π $I }

 $≅$ 3 x 10 – 4 ( Bdipole  / 1014 G ) (Po / 1ms) (12 )

with fiducially standard neutron star properties of I=1045 g.cm2, R = 10 km, Po = initial spin period and the angle between the spin axis and the principal axis of the neutron star distortion = $π$/2 ( Cutler and Jones 2001).

If we assume that magnetar’s internal toroidal magnetic field component (Btoroidal ) is the main cause of neutron star deformation then we can constrain the average value of this component by the relation (Cutler 2002 )

 | $ϵ$ | ~ 1.6 x 10 – 4  (Btoroidal  / 1016 G ) 2 (13),

and Btoroidal  as (Moriya 2016)

 Btoroidal  < ~ 1.4 x 1016 G ( Bdipole / 1014 G) ½  (Po / 1 ms ) 1/2  (14)

Considering the observed dipolar magnetic field strength Bdipole = 7 x 1014 G and spin period Po = 1.36s of the Swift J1818.0-1607 we can estimate the ultra-strong internal toroidal field strength Btoroidal <~ 1018 G. This value is consistent with the value 1017 – 1018 G in the case of newly born proto-neutron stars (Del Zanna et al 2018; Ciolfi et al 2019; Franceschetti and Del Zanna 2020 ) and also supports the model proposed by Dall”Osso et al (2012) that the internal magnetic field must be a very large initial value ( >~ 1016 G) for the internal magnetic field decay.

*The decay of Core Magnetic field*

Studies (Dall’Osso et al 2012; Thompson and Duncan 1996) of magnetic field decay in neutron star core hint three separate processes — ohmic dissipation, ambipolar diffusion and Hall drift are involved which affect the evolution and dissipation of magnetic fields in magnetar interior .In particular, ohmic dissipation and ambipolar diffusion are directly active in dissipation while Hall drift is active indirectly. Further studies (Goldreich and Reisenegger 1992; Pons and Geppert 2007) also indicate the conservation of total energy remains almost same after the Hall drift i.e. due to Hall diffusion a new equilibrium configuration with smaller total energy will appear in the magnetar interior. But initial stable magneto-hydrodynamic configurations remain close to the new equilibrium configuration as per results found in their studies. Even, the ambipolar diffusion in the neutron star core is expected to be dominant mode of interior field decay as long as early phase evolution ( i.e. ages much less than ~104 yrs) of magnetar is concerned.

Another important result was obtained in ref (Goldreich and Reisenegger 1992; Pons and Geppert 2007) regarding ambipolar diffusion at high temperature in neutron star ( i.e. magnetar) core. Ambipolar diffusion actually drives a slow motion of charged particles (situated among neutrons in the neutron star core) which is opposed

by both particle friction and chemical potential gradients in the stable neutron star medium. Depending on their effect on chemical composition two modes of ambipolar diffusion are active inside the core : a) solenoid mode —counteracting only by particle friction without perturbing chemical equilibrium b) irrotational mode — perturbing the chemical equilibrium but can not evolve on time scales shorter than the β-reaction time-scale. As the neutron star (i.e. magnetar) core magnetic field <~ 1018 G (which is >~ 1016G) the temperature in the magnetar core material will be > 109 K. In this case the field decay is not frozen. This means an equilibrium condition between heating and cooling in the high-Temperature regime may appear.

Using the relation for heating rate per unit volume through field decay

dU+ /dt = (B2 / 4$π$ tdecay (early)  $≈$ 3.69 x 1019 { B16 4 / T9 2 (ρ15)2/3 } erg.cm-3. s-1 (15)

and the relation for cooling rate per unit volume through modified URCA reaction

dU — /dt $ ≅ $ 9.6 x 1020 .( T9 )8 . (ρ15)2/3 erg.cm-3.s-1 (16)

and then equating the two rates we found (Dall’Osso et al 2012) found the equilibrium temperature as

Teq $≅$ 6.6 x 108 . ( B / 1016 G)2/5 . (ρ15 / 0.7)2/3. (L / 2 km)-1/5 K (17)

where B16, T9 , ρ15 are in their usual notations, ‘L’ and ‘a’ are the characteristic scale of variation of the Lorentz force and the chemical potential, respectively, with (L/a) $≈$ 9.6 T9 4 (ρ15) -1/3 ( L / 2 km) (Goldreich et al 1992). Comparing the obtained Teq with the transition time Ttr $≈$ 5.73 x 108 (ρ15 / 0.7 ) 1/12  K ( i.e. when β-reaction are very efficient in deleting the chemical equilibrium imbalance ) it is found Teq is higher than Ttr ( i.e. Teq > Ttr ) at magnetar interior core field environment where field decay occurs on the same time scale in both active modes. This means that magnetar core fields larger than that would be able to i) dissipate enough energy as well as ii) to balance neutrino cooling in the early phase under the effective solenoidal and irrotational modes are still degenerate. It can be said that the field decay is negligible as long as the temperature is high enough ( e.g. > T9 ) when the time scale of field decay occurs on the same time scale in both two modes. The significance of ambipolar diffusion is that it becomes active soon after the formation of magnetar and can prevent the cooling of magnetar core below a temperature ~ 109 K for a period of thousands yrs ( at least 103 yrs ) ( Zhou et al 2018). This means the decay of an internal magnetic field >~ 1016 G couples with the magnetar cooling at the early stage.

1. **Ellipticity and Triaxiality of Swift J1818.0-1607**

Theoretically, a rotating neutron star will break its axial symmetry spontaneously when rotational kinetic energy to gravitational binding energy ratio i.e. T / |W| exceeds the critical

value. A newly born rotating compact star can also achieve higher value of T / |W| when it is born from core collapse supernova. In the case of triaxial neutron star the ratio T / |W| is essentially constant along with the triaxial sequence for higher compactness (Zhou et al 2018 ). Till date thirty magnetars have been discovered excluding this swift J1818.0-1607. For these 30 magnetars their spin / rotational periods range 2 – 10 s and surface dipolar fields (calculated from the periods, period derivatives ) are 1013 – 1015 G (Kouveliotou et al 1999) . But Jawor and Tauris ( 2022) showed that initial period of magnetar must be less than 2s. Analysis of observed data indicates magnetars are young and most of them having characteristic spin down ages of less than 104 years (White etal 2022). Since they are slow rotator, so spin down energy losses can not power their emission. Alternate source is believed to be the dissipation and re-arrangement of their magnetic energy. Magnetar interior structure specially equation of state (EoS) and the cooling process in the simultaneous presence of high density, strong gravity, and strong magnetic field is important in determining the deformation in shape as well as triaxial value of the magnetar ( Yakovlev et al 2005; Chamel and Haensel 2005).

 It is seen from numerical simulations that a newly born magnetar will born with a strong magnetic field and a rapid rotation (Lindblom et al 1998; Doneva et al 2015 ) which lead to the stellar deformation. As a result a magnetar can emit observable gravitational waves i.e. a baby magnetar will spin down due to a magnetic dipole torque as well as gravitational wave quadrupole radiation. Spin down evolution of the magnetar also produces electro-magnetic radiation. In other words, the dynamic evolution of magnetar spin down provides a relation to the theoretical breaking index (n) such that

1. n = 3 when magnetic dipole radiation (i.e. electromagnetic phenomena ) dominates the spin down of the magnetar ;
2. n = 5 when the GW radiation dominates the magnetar spin down.

On the other hand, the comparison between the observed spin down light curves and their related models provide us an opportunity to constrain the initial spin period (Po), dipole magnetic fields (Bdipole) and the ellipticity ( $ϵ$ ) of the neutron star (i.e. magnetar). For example, magnetars with initial period Po ~ 1 ms and surface dipole magnetic field Bdipole ~ 1014 – 1015 G usually have the ellipticity $ϵ$ ~ 10 -3 . But theoretical value of the minimum rotation period of the magnetar is ~ 0.3 – 0.5 ms ( Koranda et al 1997) . The best fitting relations (Xie et al 2022) are

 log $ϵ$ = 3.79+0.52-0.43  + ( 2.19 +0.17-0.15 ) log Po (18)

and log $ϵ$ = - 22.50+2.15-2.22 + (1.29 +0.15-0.14 ) log Bdipole (19)

suggest that

1. magmetar having a stronger magnetic field and / a slower spin period corresponds to the longer ellipticity;
2. a longer rotation period corresponds to possession of a stronger magnetic field;
3. the neutron star deformation is related to its surface dipole magnetic field to some extent.

But it is argued (Majid et al 2022; Rizaldy et al 2018 ) instead of dipole magnetic field neutron star deformation may be induced by a strong internal magnetic field (Bint ) in the stellar core through the relation

 $ϵ $ $≈$ 10 – 8  (Bint / 1012 G) (20)

This relation hints the possession of a very strong internal magnetic field (i.e. Bint ~ 1016 – 1017 G) is needed in order to obtain the ellipticity $ϵ$ ~ 10 – 3 – 10 – 4  and also the required strength of the internal core magnetic field which should be at least 1 – 2 order of magnitude greater than the surface (i.e. external ) magnetic field ( Bdipole  ~ 1015 G ).

***The Triaxiality of Swift J1818.0-1607***

At the time of its discovery on 12th March 2020 the Swift J1818.0 – 1607 was appeared to the astronomers as a new un-catalogued x-ray source. Presently it is a confirmed magnetar (Majid et al 2022 ). The follow up observations suggest the following properties :

1. rotational period = 1.36 s
2. surface dipolar magnetic field = 3 x 1014 G
3. surface magnetic field at poles = 7 x 1014 G
4. spin down luminosity ~ 1.1 x 1036 erg.s -1
5. characteristic age ~ 300 years

Using equations (12), (13), (14) and (20) and with the above parameters as input we calculate the ellipticity, internal core magnetic field of Swift J1818.0-1607 and found ~ 9 x 10 -3 and 8.9424 x 10 17 G, respectively.

 Numerical study (Rizaldy and Sulaksono 2018) of magnetized deformation of neutron stars shows an interesting consequence for neutron stars with low masses i.e. the effect of magnetic field is more prominent for internal magnetic field Bint > 4 x 10 18 G. According to Rizaldy and Sulakseno (2018) the balance between the gravity and magnetic field is significantly different for different directions in the case of small mass rather than massive neutron star. Even the gravity pull of magnetic field on the z-axis is significantly more than for the other axes resulting which oblate-shape appears in the low mass neutron stars. In the case of massive neutron star this oblate shape is very less in comparison to that of less massive. In other words, we can say internal toroidal magnetic field is more effective than the poloidal field. The deformation associated to the poloidal field ( Bp $≈$ 1014 and 1015  G) and the corresponding correction in ellipticity (i.e. ~ 10 - 4 – 10 – 2 , respectively) is negligible ( Morasi et al 2011).

 Although the recent view of magnetized deformation of neutron stars (i.e. magnetars ) is due to the effect of both the toroidal and the poloidal magnetic fields i.e. a mixed magnetic field but we shall consider the effect of toroidal magnetic field only. Because our main purpose is to check the ellipticity of the deformed neutron star i.e. magnetar Swift 1818.0 – 1607 and its stability. In a comparative analysis pulsars and magnetars Heras (2012) found the initial magnetic fields in the interior of nascent neutron stars lie in the range 1014 – 1016 G in realistic case. As ambipolar is active that prevents both the decay of interior magnetic fields and cooling of the neutron star i.e. magnetar (as the effect is same and applicable for magnetars as well as neutron stars also ) such that magnetar core temperature stays higher than several times 108 K for a period of few thousands of years ( at least 103 years) . This ellipticity of new born magnetar might not be changed too much during the period of thousand years. As the characteristic age of the Swift J1818.0 – 1607 is only ~ 300 years i.e. the baby phase still compare to thousands years it will definitely exhibit triaxility i.e. triaxial behavior at least up to its age 1000 years.

The estimated ellipticity of this magnetar lies within the range for triaxiality and its ellipticity will remain on that value for exhibiting triaxial nature for several thousands years. Thus, we can consider the Swift J1818.0 – 1607 as a triaxial magnetar or simply a triaxial star.

1. **Conclusion : What do we want from Triaxial Cosmic Baby**

Newly born rotating neutron star can break their axial symmetry spontaneously if the ratio T / |W| exceeds some threshold value. Magnetars are classified as a rare class of relatively slow rotating neutron stars that possess very strong magnetic fields. As a non-dissipative mechanism a magnetic field with a component parallel to the rotation axis breaks circular conservation and introduces symmetry breaking spontaneously. The swift J1818.0-1607 is a baby magnetar of characteristic age of ~ 300 years, having very strong internal core magnetic field ~ 8.9424 x 1017G. It is the fastest among the detected 31 magnetars having spin or rotational period ~1.36 s. Its magnetic fields are not aligned with the rotational axes. The above properties of the swift J1818.0-1607 indicate that it is an ideal triaxial magnetar (compact object) where detection of a triaxial star and its bizarre properties can be tested / studied. Its internal core magnetic field, though too much strong, yet have ( at least) a slow decay mode through ambipolar diffusion which becomes active soon after formation (birth). As this process can prevent the cooling of the magnetar core below a temperature of few times of 108 K (i.e. < 109 K) for thousands of years, continuous observation of Swift J1818.0 – 1607 will thus offer us an opportunity for understanding our knowledge of the evolution of magnetic fields of magnetars.

 As the rotational period of this magnetar is 1.36s i.e. within the range 1 – 10s, the frequency of the emitted continuous gravitational waves would be very low (Sieniawska and Bejger 2019; Ibrahim et al 2023 ) this author encourages the Gravitational Wave Community to observe this triaxial baby magnetar (i.e. the swift J1818.0-1607) continuously during their observation of compact objects through electromagnetic counterparts. It is hoped that future observed data definitely help us for better understanding the physics of the evolution of the Magnetars.

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