Existance and uniqueness of ϖ-contraction mapping using Continuity of soft-t norm under Soft Fuzzy Metric Spaces.

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Abstract. The primary goal of this manuscript to introduce the notion of Banach's contraction principle and ϖ-contraction mapping in to the soft fuzzy metric space ($\mathcal{SFZMS}$). Throughout this manuscript we taken under consideration absolute soft set, soft point as a restriction of our results and we successfully applied Continuity of soft-t-norm to newly developed ϖ-contraction mapping principle which admits existance and uniqueness of soft fiexed pints, Subsequently some illustrations are supplied to support our main contraction mapping principle. Our newly developed results are generalized version of some famous results from literature.

Keywords: Fixed point; Cauchy sequence; Soft-t-norm; Soft fuzzy metric; Completeness; altering distance; Banach's contraction principle.

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1. Introduction

Soft set theory was first constructed by Molodtsov [7] in 1999 for modelling vagueness and uncertainties, which occupied the human mind for centuries. In modern research, we face uncertainty and vagueness in different areas such as Mathematics, economics, engineering, medical science, sociality and environmental sciences. Maji et al.[17] extended results of soft theory and presented an application of soft sets in decision making problems which is completely based on the reduction of parameters to hold optimal choice for an objects. In 2022 T. M. Al-shami given [30] Soft some what open sets which gives Soft separation axioms along with the medical application to nutrition. Also, Das and Samanta [26, 27, 28] introduce the notion of soft metric space which is completely based on soft point of given soft sets. S. rathee et.al. [25] given some contraction fixed point results under soft multipicative metric space, subsequently In 2018 B. sadi et.al. [9] proved some theorem under soft S-metric space and given some example to validate work. R. I. Sabri et. al. [19] studied soft sets which made progress in the soft set theory and analyzed the soft set and it's property under Compactness of soft fuzzy problems. The soft set theory has been successfully applied to many fields (for examples [29, 30, 31, 20, 16, 18, 14, 24]).

Theorem 1.1. [11] *Kramosil and Michalek introduced the idea of a fuzzy distance between two elements using nonempty set for the concepts of fuzzy set along with t-norm in 1975 as follows,*

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(1) Ξ *is continuous,*
(2) Ξ *associative and commutative.*
(3) $\Xi(\alpha, 1) = \alpha$, *where every* $0 \leq \alpha \leq 1$
(4) *For every* $\alpha, \beta, \gamma, \delta \in [0, 1]$, *we write if* $\alpha \leq \delta$ *and* $\beta \leq \gamma$ *then* $\Xi(\alpha, \beta) \leq \Xi(\delta, \gamma)$,

where Ξ *is a mapping* $\Xi : [0, 1] \times [0, 1] \to [0, 1]$.

Under $\mathcal{SFZMS}$ we do not have any generalization of Banach contraction mapping principle so to fulfill this research gap we established ϖ-contraction mapping with ψ-function and shown that our contraction map do hold unique soft fuzzy fixed point. In this manuscript, we prove existence and uniqueness for a soft fixed point in soft fuzzy metric space ($\mathcal{SFZMS}$) and to validate our Main theorem we present some Illustrations. We divided our paper in to following structure: our section first it goes to Preliminaries part then section second we introduce contraction mapping principle and its existance and uniqueness of fuzzy soft fixed point under $\mathcal{SFZMS}$ using Ψ-function, ϖ-contraction map followed by section Third goes to established main result with some validating examples and finally last section Conclusion.

2. Preliminaries

In this part of Preliminaries we provide some properties and definition from literature [8, 15, 17, 19, 27, 32] to establish the main Theorem, Subsequently throughout this manuscript we use assembly of parameter as a φ, the universal set as χ, and the collection of all subsets of set χ represented as $\varphi(\chi)$ resp. For more we prefer to see [10, 11, 12, 13, 21, 22, 23, 28].

Definition 2.1. [17] $\mathcal{W}(\tilde{K}) = \chi$, $\forall\ \tilde{K} \in \varphi$ *then soft set* $(\mathcal{W}, \varphi)$ *over universal set* χ *said to be a absolute soft set. We used* χ_φ *as a absolute soft set over* χ *and parameter* φ.

Definition 2.2. [26] $\mathcal{W}(\tilde{K}) = \{\}$ *for every* $\tilde{K} \in \varphi$ *then soft set* $(\mathcal{W}, \varphi)$ *over universal set* χ *said to be a void soft set or null soft set and it's denoted as* $\tilde{\phi}$.

Definition 2.3. [7] *Pair* $(\mathcal{W}, \varphi)$ *over universal set* χ *said to be a soft set if* $\mathcal{W} : \varphi \to \varphi(\chi)$.

In this paper we suppose $\Re$ be the collectoin of all real numbers and we denote assembly for every non-void bounded subset of $\Re$ with $B(\Re)$.

Definition 2.4. [27] *Universal set* χ *possess a soft set called as a soft point whenever there exist exactly one parameter* $\tilde{K} \in \varphi$, $\mathcal{W}(\tilde{K}) = \zeta$ *where* $\zeta \in \chi$ *and* $\mathcal{W}(\upsilon) = \phi$ *for every* $\upsilon \in \varphi\ \tilde{K}$. *The collection of all soft points of* $(\mathcal{W}, \varphi)$ *is written as* $SP(\mathcal{W}, \varphi)$.

Definition 2.5. [27] *The collection of all soft element* $\mathcal{V}$ $(SS(\mathcal{V}))$ *having function* $\nabla : \varphi \to \chi$ *then* ∇ *said tobe a soft element where* φ *is a parameter and* χ *is a universal set.*

Definition 2.6. [26] *Map* $\mathcal{W} : \varphi \to B(\Re)$ *said to be soft real set* $(\mathcal{W}, \varphi)$. *A soft real set* $(\mathcal{W}, \varphi)$ *is a soft real number if, every* $\tilde{K} \in \varphi$, $\mathcal{W}(\tilde{K})$ *be a singleton number of* $B(\Re)$ *which denoted as* ζ.

Throughout our manuscript we use following abbreviation, collection of soft real numbers and non-negative soft real numbers having parameter set φ represented by $\Re(\varphi)$ and $\Re(\varphi)^*$ resp. Collection of all soft real $[a, b]$ and $[0, \infty)$ is represented as $[a, b](\varphi)$ and $[0, \infty)(\varphi)$ resp.

Definition 2.7. [28] *Consider a mmapping* $\mathfrak{L} : \mathcal{K}_\rho(\tilde{\chi}_\varphi) \times \mathcal{K}_\rho(\tilde{\chi}_\varphi) \to \Re(\varphi)^*$ *and* $\tilde{\chi}_\varphi$ *is a absolute-soft set if we have following condition,*

$(SS)^1$ *for every* $\tilde{w}_{o_i}, \tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_\varphi$, $\mathfrak{L}\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}\right) = \bar{0}$ *if and only if* $\tilde{w}_{o_i} = \tilde{z}_{o_j}$,
$(SS)^2$ *for every* $\tilde{w}_{o_i}, \tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_\varphi$, $\tilde{0} \leqslant \mathfrak{L}\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}\right)$,

$(SS)^3$ *for every* $\tilde{w}_{o_i}, \tilde{z}_{o_j}, \tilde{t}_{p_k} \tilde{\in} \tilde{\chi}_\varphi$, $\mathfrak{L}\left(\tilde{w}_{o_i}, \tilde{t}_{p_k}\right) \tilde{\leq} \mathfrak{L}\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}\right) + \mathfrak{L}\left(\tilde{z}_{o_j}, \tilde{t}_{p_k}\right)$,
$(SS)^4$ *for every* $\tilde{w}_{o_i}, \tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_\varphi$, $\mathfrak{L}\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}\right) = \mathfrak{L}\left(\tilde{z}_{o_j}, \tilde{w}_{o_i}\right)$.

then we say $(\tilde{\chi}_\varphi, \mathfrak{L})$ *or* $(\tilde{\chi}_\varphi, \mathfrak{L}, \varphi)$ *is a soft metric space.*

Definition 2.8. [8] *Suppose mapping* $\tilde{\circledcirc} : [0,1](\varphi) \times [0,1](\varphi) \longrightarrow [0,1](\varphi)$ *then* $\tilde{\circledcirc}$ *called as continuous soft t-norm whenever* $\tilde{\circledcirc}$ *satisfies following listed conditions:*

i. *continuity of* $\tilde{\circledcirc}$,
ii. $\tilde{c} \,\tilde{\circledcirc} \tilde{1} = \tilde{c}$ *for every* $\tilde{c} \in [0,1](\varphi)$,
iii. $\tilde{c} \tilde{\circledcirc} \tilde{d} \leq \tilde{p} \tilde{\circledcirc} \tilde{q}$ *when* $\tilde{c} \leq \tilde{d}$ *and* $\tilde{p} \leq \tilde{q}$ *for* $\tilde{c}, \tilde{d}, \tilde{p}, \tilde{q} \in [0,1](\varphi)$,
iv. $\tilde{\circledcirc}$ *holds associativity and commutativity laws.*

Definition 2.9. [26] *We defined following operation on two soft real number* $\tilde{w}$ *and* $\tilde{v}$,

$$(\tilde{w} \oplus \tilde{v})(\tilde{k}) = \{\tilde{w}(\tilde{k}) + \tilde{v}(\tilde{k}), \tilde{k} \in \varphi\},$$
$$(\tilde{w} \circ \tilde{v})(\tilde{k}) = \{\tilde{w}(\tilde{k}).\tilde{v}(\tilde{k}), \tilde{k} \in \varphi\},$$
$$(\tilde{w} \odot \tilde{v})(\tilde{k}) = \{\tilde{w}(\tilde{k}) - \tilde{v}(\tilde{k}), \tilde{k} \in \varphi\}.$$

Definition 2.10. [15] $(\tilde{\chi}_{\varphi_\mathcal{A}}, \mathfrak{L}, \varphi_\mathcal{A})$ *and* $\left(\tilde{\mathcal{V}}_{\varphi_\mathcal{B}}, \mathfrak{L}, \varphi_\mathcal{B}\right)$ *be two soft metric space with the mapping* $(\digamma, \vartheta) : (\tilde{\chi}_{\varphi_\mathcal{A}}, \mathfrak{L}, \varphi_\mathcal{A}) \to \left(\tilde{\mathcal{V}}_{\varphi_\mathcal{B}}, \mathfrak{L}^\mathcal{L}, \varphi_\mathcal{B}\right)$, *then we say* $(\digamma, \vartheta)$ *is a soft mapping if* $\digamma : \tilde{\chi}_{\varphi_\mathcal{A}} \to \tilde{\mathcal{V}}_{\varphi_\mathcal{B}}$ *and* $\vartheta : \varphi_\mathcal{A} \to \varphi_\mathcal{B}$.

Definition 2.11. [8] *Let's consider mapping* $\Im_\flat : \mathcal{K}_\rho(\tilde{\chi}_\varphi) \times \mathcal{K}_\rho(\tilde{\chi}_\varphi) \times (0,\infty)(\varphi) \longrightarrow [0,1](\varphi)$. *We say* $\Im_\flat$ *is a soft fuzzy metric* $(\mathcal{SFZM})$ *on* $\tilde{\chi}_\varphi$ *if its satisfies following conditions,*

$(\mathcal{SM})^1$ *for every* $\tilde{w}_{o_i}, \tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_\varphi$, $\tilde{\kappa} > \tilde{0} \implies \tilde{0} \leqslant \Im_\flat\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}\right)$,
$(\mathcal{SM})^2$ *for every* $\tilde{w}_{o_i}, \tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_\varphi$, $\tilde{\kappa} > \tilde{0}$, $\Im_\flat\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}\right) = \tilde{1} \iff \tilde{w}_{o_i} = \tilde{z}_{o_j}$,
$(\mathcal{SM})^3$ *for every* $\tilde{w}_{o_i}, \tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_\varphi$, $\tilde{\kappa} > \tilde{0}$ $\Im_\flat\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}\right) = \Im_\flat\left(\tilde{z}_{o_i}, \tilde{w}_{o_j}, \tilde{\kappa}\right)$,
$(\mathcal{SM})^4$ *for every* $\tilde{w}_{o_i}, \tilde{z}_{o_j} \& \tilde{t}_{p_k} \tilde{\in} \tilde{\chi}_\varphi$, $\tilde{\kappa} > \tilde{0} \implies \Im_\flat\left(\tilde{w}_{o_i}, \tilde{t}_{p_k}, \tilde{\kappa} \oplus \tilde{l}\right) \geq \Im_\flat\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}\right) \tilde{\circledcirc} \Im_\flat\left(\tilde{z}_{o_j}, \tilde{t}_{p_k}, \tilde{l}\right)$,
$(\mathcal{SM})^5$ $\Im_\flat\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}, .\right) : (0,\infty)(\varphi) \longrightarrow [0,1](\varphi)$ *be a continuous map.*

Soft fuzzy metric $\Im_\flat$ *along with absolute soft set* $\tilde{\chi}_\varphi$ *called as soft fuzzy metric space* $(\mathcal{SFZMS})$ *and denoted as* $\left(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc}\right)$.

Example 2.1. *Let's assume* $\mathcal{SMS}$ $(\tilde{\chi}_\varphi, \mathfrak{L})$ *has*

$$\tilde{\zeta} \circledcirc \tilde{v} = \min\{\tilde{\zeta}, \tilde{v}\} \text{ or } \tilde{\zeta} \circledcirc \tilde{v} = \tilde{\zeta} \circ \tilde{v}$$

lets define the mapping $\Im_\flat : \mathcal{K}_\rho(\tilde{\chi}_\varphi) \times \mathcal{K}_\rho(\tilde{\chi}_\varphi) \times (0,\infty)(\varphi) \longrightarrow [0,1](\varphi)$ *as*

$$\frac{\tilde{\kappa}}{\tilde{\kappa} \oplus \mathfrak{L}\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}\right)} = \Im_\flat\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}\right) \text{ where } \tilde{w}_{o_i}, \tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_\varphi \text{ and } \tilde{\kappa} > \tilde{0}.$$

thus, $\left(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc}\right)$ *be a* $\mathcal{SFZMS}$, *Subsequently* $\Im_\flat$ *induced by soft metric* $\mathfrak{L}$ *is called as a standard soft fuzzy metric.*

Definition 2.12. [8] $\mathcal{S}_\flat = \{(\tilde{x}_{p_i}, \upsilon_{\mathcal{S}_\flat}(\tilde{x}_{p_i})) \mid \tilde{x}_{p_i} \in \tilde{\chi}_\varphi, p_i \in \varphi\}$, *be a soft fuzzy set of ordered pairs in* $\tilde{\chi}_\varphi$ *where* $\upsilon_{\mathcal{S}_\flat}$ *is a membership function having map* $\tilde{\chi}_\varphi \to [0,1](\varphi)$ *and* $\upsilon_{\mathcal{S}_\flat}(\tilde{x}_{p_i})$ *be the soft membership grade of soft point* $\tilde{x}_{p_i} \in \mathcal{S}_\flat$.

Example 2.2. $\tilde{\zeta} \circledcirc \tilde{v} = \max\{\tilde{0}, \tilde{\zeta} \oplus \tilde{v} \ominus \tilde{1}\}$, $\tilde{\zeta} \circledcirc \tilde{v} = \min\{\tilde{\zeta}, \tilde{v}\}$ *and* $\tilde{\zeta} \tilde{\circledcirc} \tilde{v} = \tilde{\zeta} \circ \tilde{v}$.

Definition 2.13. [19] *Let's suppose* $\mathcal{SFZMS}$ $\left(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc}\right)$ *having soft set*

$$\mathcal{M} = \{(\mathcal{V}, \varphi) : (\mathcal{V}, \varphi) \tilde{\subset} (\chi, \varphi)\}$$

we say that $\mathcal{M}$ as soft open cover for $\tilde{\chi}_\varphi$ if for every $(\mathcal{V},\varphi)\in\mathcal{M}$ is soft open and

$$\tilde{\chi}_\varphi\tilde{\subset}\tilde{U}_{(\mathcal{V},\varphi)\in\mathcal{M}}(\mathcal{V},\varphi)$$

We say $\mathcal{SFZMS}$ $\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ is a compact if every soft open cover $\tilde{\chi}_\varphi$ in $\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ has soft open sets

$$\{(\mathcal{V}_1,\varphi_1),(\mathcal{V}_2,\varphi_2),\ldots,(\mathcal{V}_n,\varphi_n)\}$$

whereas $(\mathcal{V}_i,\varphi_i)\in\mathcal{M}$ for all $i\in\{1,2,\ldots,n\}$ satisfying $\tilde{\chi}_\varphi\tilde{\subset}\tilde{U}_{i=1}^{n}(\mathcal{V}_i,\varphi_i)$.

Definition 2.14. [32] *Every Cauchy sequences in $\mathcal{SFZMS}$ is convergent whenever $\mathcal{SFZMS}$ is a complete.*

Definition 2.15. [32] *Every soft fuzzy sequences in $\tilde{\chi}_\varphi$ admit at least one convergent soft subsequence then $\mathcal{SFZMS}$ is a $\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ is compact.*

Definition 2.16. [32] *Let's consider $\mathcal{SFZMS}$ $\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ and $\left\{\tilde{w}_{o_i}^m\right\}$ be any soft sequence and*

$$\lim_{m\to\infty}\Im_\flat\left(\tilde{w}_{o_i}^m,\tilde{z}_{o_j},\tilde{\kappa}\right)=\bar{1},\quad \text{for every } \tilde{\kappa}>\bar{0}$$

then we say that soft sequence $\left\{\tilde{w}_{o_i}^m\right\}$ is convergent in $\mathcal{SFZMS}$ $\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$, which means, any $\bar{0}<\tilde{\lambda}<\bar{1}$ and $\tilde{\kappa}>\bar{0}$, which has $\mathbb{N}_0$ is positive integer such that

$$\tilde{w}_{o_i}^m\in SS\left(\mathcal{B}_{\Im_\flat}\left(\tilde{z}_{o_j},\tilde{\lambda},\tilde{\kappa}\right)\right),\quad \text{for every } m\geq\mathbb{N}_0$$

where as $\mathcal{B}_{\Im_\flat}\left(\tilde{z}_{o_j},\tilde{\lambda},\tilde{\kappa}\right)$ be a soft open ball which has a centred at $\tilde{z}_{o_j}$ and radius $\tilde{\lambda}$ with respect to $\tilde{\kappa}$. which gives,

$$\Im_\flat\left(\tilde{w}_{o_i}^m,\tilde{z}_{o_j},\tilde{\kappa}\right)>\bar{1}\odot\tilde{\lambda},\quad \forall\tilde{\kappa}>\bar{0}, m\geq\mathbb{N}_0$$

Definition 2.17. [32] *Let's consider $\mathcal{SFZMS}$ $\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ and $\left\{\tilde{w}_{o_i}^m\right\}$ be any soft Cauchy sequence inside $\mathcal{SFZMS}$ if,*

$$\lim_{m\to\infty}\Im_\flat\left(\tilde{w}_{o_i}^m,\tilde{w}_{o_i}^t,\tilde{\kappa}\right)=\bar{1},\quad \text{for every } \tilde{\kappa}>\bar{0}$$

which means, any $\bar{0}<\tilde{\lambda}<\bar{1}$ and $\tilde{\kappa}>\bar{0}$, which has $\mathbb{N}_0$ is positive integer such that

$$\Im_\flat\left(\tilde{w}_{o_i}^m,\tilde{w}_{o_i}^t,\tilde{\kappa}\right)>\bar{1}\odot\tilde{\lambda},\quad \text{for every } m,t\geq\mathbb{N}_0.$$

3. Main Results

Definition 3.1. *Let's assume $\mathcal{SFZMS}$ $\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ and $(\digamma,\vartheta):\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)\to\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ be a soft mapping which satisfying the following soft contraction,*

$$\Im_\flat\left(\tilde{w}_{o_i},\tilde{z}_{o_j},\frac{\tilde{\kappa}}{\bar{\eta}}\right)\leqslant\Im_\flat\left((\digamma,\vartheta)\tilde{w}_{o_i},(\digamma,\vartheta)\tilde{z}_{o_j},\tilde{\kappa}\right)\tag{1}$$

for every $\tilde{w}_{o_i},\tilde{\tilde{z}}_{o_j}\tilde{\in}\tilde{\chi}_\varphi$, $0\leqslant\bar{\eta}<1$ and $\tilde{\kappa}>\bar{0}$.

Definition 3.2. *Suppose $\mathcal{SFZMS}$ $\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$. The map $(\digamma,\vartheta):\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)\to\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ is called as ϖ-contraction mapping if there exist $0\leqslant\bar{\eta}<1$ having following condition:*

$$\Im_\flat\left(\tilde{w}_{o_i},\tilde{z}_{o_j},\varpi(\frac{\tilde{\kappa}}{\bar{\eta}})^{\tilde{m}}\right)\leqslant\Im_\flat\left((\digamma,\vartheta)\tilde{w}_{o_i},(\digamma,\vartheta)\tilde{z}_{o_j},\varpi(\tilde{\kappa})^{\tilde{m}}\right)\tag{2}$$

where, for every $\tilde{w}_{o_i},\tilde{z}_{o_j}\tilde{\in}\tilde{\chi}_\varphi$, $\tilde{\kappa},\tilde{m}>\bar{0}$ and ϖ is a Ψ-function.

Definition 3.3. (1) *ϖ is continuous at $\tilde{\kappa}=\bar{0}$,*

(2) *ϖ is left continuous at $\tilde{\kappa}>\bar{0}$,*

(3) *$\tilde{\kappa}=\bar{0}$ if and only if $\varpi(\tilde{\kappa})=\bar{0}$,*

(4) $\varpi(\tilde{\kappa}) \to \infty_\iota$ *as* $\tilde{\kappa} \to \infty$ *gives* ϖ *is increasing.*
then mapping $\varpi : \Re(\varphi) \longrightarrow [0,\infty)(\varphi)$ *called as* Ψ*-function.*

Example 3.1. *Consider collection of all soft real numbers as* $\Re(\varphi)$ *having soft topology and* $[0,\infty)(\varphi)$ *be a non-negative part of* $\Re(\varphi)$*. Lets define mapping* $\varpi : \Re(\varphi) \longrightarrow [0,\infty)(\varphi)$ *as :*

$$\varpi(\tilde{\kappa}) = \begin{cases} (\tilde{\kappa})^{\frac{1}{3}} & if \quad \tilde{\kappa} \neq \bar{0} \\ \\ \bar{0} & if \quad \tilde{\kappa} = \bar{0} \end{cases}$$

Here, ϖ *satisfies all four conditions for a* Ψ*-function.*

Theorem 3.1. *Assume* $\mathcal{SFZMS}$ $\left(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc}\right)$ *be a complete,*

$$\Im_\flat\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}, \frac{\tilde{\kappa}}{\bar{\eta}}\right) \leqslant \Im_\flat\left((\digamma,\vartheta)\tilde{w}_{o_i}, (\digamma,\vartheta)\tilde{z}_{o_j}, \tilde{\kappa}\right), \text{ for every } \tilde{w}_{o_i}, \tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_\varphi. \tag{3}$$

and subsequently when we apply limit as $\tilde{\kappa}$ *goes to positive infinity then we say* $\Im_\flat\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}\right)$ *gives the value* $\bar{1}$*. Then* $(\digamma,\vartheta)$ *on* $\tilde{\chi}_\varphi$ *hold a unique soft fixed point.*

Proof. Let's assume soft sequence $\left\{\tilde{w}^m_{o_i}\right\}$ and a soft point $\tilde{w}^0_{o_i} \tilde{\in} \tilde{\chi}_\varphi$ which has $\tilde{w}^m_{o_i} = (\digamma,\vartheta)^m \tilde{w}^0_{o_i}$. By applying induction,

$$\Im_\flat\left(\tilde{w}^0_{o_i}, \tilde{w}^1_{o_i}, \tilde{\kappa}/\bar{\eta}^m\right) \leqslant \Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+1}_{o_i}, \tilde{\kappa}\right). \tag{4}$$

from the definition of (2.11) $\mathcal{SFZMS}$-$(\mathcal{SM})^4$, (4) and for every positive integer β we write,

$$\Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+1}_{o_i}, \tilde{\kappa}/\bar{c}\right) \overbrace{\tilde{\circledcirc}\ldots\tilde{\circledcirc}}^{c-\text{ times}} \Im_\flat\left(\tilde{w}^{m+c-3}_{o_i}, \tilde{w}^{m+c-2}_{o_i}, \tilde{\kappa}/\bar{c}\right) \tilde{\circledcirc}$$
$$\tilde{\circledcirc}\Im_\flat\left(\tilde{w}^{m+c-2}_{o_i}, \tilde{w}^{m+c-1}_{o_i}, \tilde{\kappa}/\bar{c}\right) \tilde{\circledcirc}\Im_\flat\left(\tilde{w}^{m+c-1}_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}/\bar{c}\right) \leqslant \Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}\right),$$
$$\Im_\flat\left(\tilde{w}^0_{o_i}, \tilde{w}^1_{o_i}, \tilde{\kappa}/\bar{c}\bar{\eta}^m\right) \overbrace{\tilde{\circledcirc}\ldots\tilde{\circledcirc}}^{c-\text{ times}} \Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \tilde{\kappa}/\bar{c}\bar{\eta}^{m+c-3}\right) \tilde{\circledcirc}$$
$$\tilde{\circledcirc}\Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \tilde{\kappa}/\bar{c}\bar{\eta}^{m+c-2}\right) \tilde{\circledcirc}\Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \tilde{\kappa}/\bar{c}\bar{\eta}^{m+c-1}\right) \leqslant \Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}\right).$$

using statement (3) of our Main theorem and apply as $\lim_{\kappa\to\infty}$,

$$\lim_{\kappa\to+\infty} \Im_\flat\left(\tilde{w}^0_{o_i}, \tilde{w}^1_{o_i}, \tilde{\kappa}/\bar{c}\bar{\eta}^m\right) \overbrace{\tilde{\circledcirc}\ldots\tilde{\circledcirc}}^{c-\text{ times}} \lim_{\kappa\to+\infty} \Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \tilde{\kappa}/\bar{c}\bar{\eta}^{m+c-3}\right) \tilde{\circledcirc}$$
$$\tilde{\circledcirc} \lim_{\kappa\to+\infty} \Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \tilde{\kappa}/\bar{c}\bar{\eta}^{m+c-2}\right) \tilde{\circledcirc} \lim_{\kappa\to+\infty} \Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \tilde{\kappa}/\bar{c}\bar{\eta}^{m+c-1}\right) \leqslant \lim_{\kappa\to+\infty} \Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}\right).$$

$$\lim_{\tilde{\kappa}\to+\infty} \Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}\right) \geq \overbrace{\bar{1}\tilde{\circledcirc}\tilde{1}\tilde{\circledcirc}\tilde{1}\ldots\tilde{\circledcirc}\bar{1}}^{c-\text{ times}}$$

$$\lim_{\tilde{\kappa}\to+\infty} \Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}\right) \geq \bar{1}$$

then we say $\left\{\tilde{w}^m_{o_i}\right\}$ is a soft cauchy fuzzy sequence in $\left(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc}\right)$, which implies $\left\{\tilde{w}^m_{o_i}\right\}$ is convergent as $\left(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc}\right)$ is complete. Letting $\left\{\tilde{w}^m_{o_i}\right\} \to \tilde{z}_{o_j}$ and $\tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_\varphi$ that is

$$\lim_{m\to\infty} \Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{z}^t_{o_j}, \tilde{\kappa}\right) = \bar{1}, \tag{5}$$

then,

$$\Im_\flat\left((\digamma,\vartheta)\tilde{z}_{o_j}, (\digamma,\vartheta)\tilde{w}^m_{o_i}, \tilde{\kappa}/\bar{2}\right) \tilde{\circledcirc}\Im_\flat\left((\digamma,\vartheta)\tilde{w}^m_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}/\bar{2}\right) \leqslant \Im_\flat\left((\digamma,\vartheta)\tilde{z}_{o_j}, \tilde{z}_{o_j}, \tilde{\kappa}\right)$$
$$\lim_{m\to\infty} \Im_\flat\left(\tilde{z}_{o_j}, \tilde{w}^m_{o_i}, \tilde{\kappa}/\bar{2}\bar{\eta}\right) \tilde{\circledcirc} \lim_{m\to\infty} \Im_\flat\left(\tilde{w}^{m+1}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}/\bar{2}\right) \leqslant \lim_{m\to\infty} \Im_\flat\left((\digamma,\vartheta)\tilde{z}_{o_j}, \tilde{z}_{o_j}, \tilde{\kappa}\right)$$

using equation (5) we write,

$$\lim_{m\to\infty} \Im_\flat\left((\digamma,\vartheta)\tilde{z}_{o_j},\tilde{z}_{o_j},\tilde{\kappa}\right) \geqslant \bar{1}\tilde{\circledcirc}\bar{1},$$

$$\lim_{m\to\infty} \Im_\flat\left((\digamma,\vartheta)\tilde{z}_{o_j},\tilde{z}_{o_j},\tilde{\kappa}\right) \geqslant \bar{1}.$$

or we write,

$$\lim_{m\to\infty} \Im_\flat\left((\digamma,\vartheta)\tilde{z}_{o_j},\tilde{z}_{o_j},\tilde{\kappa}\right) = \bar{1}.$$

which implies

$$(\digamma,\vartheta)\tilde{z}_{o_j} = \tilde{z}_{o_j} \tag{6}$$

Hence, we write $\tilde{z}_{o_j}$ be the soft fixed point of $(\digamma,\vartheta)$ and the uniqueness we can verify easily. □

Our following example validate every conditions given in Theorem (3.1).

Example 3.2. *Soft t-norm defined as* $\tilde{c}\tilde{\circledcirc}\tilde{d} = \min\{\tilde{c},\tilde{d}\}$, *a set* $\chi = \{\tilde{w},\tilde{z},t\}$ *and a parameter set* $\varphi = \{a,b\}$ *then we write*

$$\mathcal{K}_\rho\left(\tilde{\chi}_\varphi\right) = \left\{\tilde{w}_a,\tilde{w}_b,\tilde{z}_a,\tilde{z}_b,\tilde{t}_a,\tilde{t}_b\right\}.$$

Let's define mapping $\Im_\flat : \mathcal{K}_\rho\left(\tilde{\chi}_\varphi\right) \times \mathcal{K}_\rho\left(\tilde{\chi}_\varphi\right) \times (0,\infty)(\varphi) \longrightarrow [0,1](\varphi)$ *for every* $o_i, o_j \in \varphi$, *we write*

$$\Im_\flat\left(\tilde{z}_{o_j},\tilde{w}_{o_i},\tilde{\kappa}\right) = \Im_\flat\left(\tilde{w}_{o_i},\tilde{z}_{o_j},\tilde{\kappa}\right) = \begin{cases} \bar{0} & \textit{whenever} \quad \tilde{\kappa} = \bar{0} \\ \bar{0.8} & \textit{whenever} \quad \bar{0} < \tilde{\kappa} \leq \bar{2} \\ \bar{1} & \textit{whenever} \quad \tilde{\kappa} > \bar{2} \end{cases}$$

$$\Im_\flat\left(\tilde{t}_{p_i},\tilde{z}_{o_j},\tilde{\kappa}\right) = \Im_\flat\left(\tilde{z}_{o_j},\tilde{t}_{p_i},\tilde{\kappa}\right) = \Im_\flat\left(\tilde{w}_{o_i},\tilde{t}_{p_j},\tilde{\kappa}\right) = \Im_\flat\left(\tilde{t}_{p_j},\tilde{w}_{o_i},\tilde{\kappa}\right) =$$
$$= \begin{cases} \bar{0} & \textit{whenever} \quad \tilde{\kappa} = \bar{0} \\ \bar{0.5} & \textit{whenever} \quad \bar{0} < \tilde{\kappa} \leq \bar{4} \\ \bar{1} & \textit{whenever} \quad \tilde{\kappa} > \bar{4}, \end{cases}$$

$\Im_\flat\left(\tilde{w}_{o_i},\tilde{z}_{o_j},\tilde{\kappa}\right) = \bar{1}$ *if and only if* $\tilde{w}_{o_i} = \tilde{z}_{o_j}$, *for every* $\tilde{w}_{o_i},\tilde{\tilde{z}}_{o_j}\tilde{\in}\tilde{\chi}_\varphi$ *and* $\tilde{\kappa} > \bar{0}$. *Hence we say,* $\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ *is a complete* $\mathcal{SFZMS}$. *Now, let's suppose a soft mapping* $(\digamma,\vartheta)$ *on* $\tilde{\chi}_\varphi$,

$$(\digamma,\vartheta)\left(\tilde{w}_a\right) = \tilde{z}_a, (\digamma,\vartheta)\left(\tilde{w}_b\right) = \tilde{z}_b, (\digamma,\vartheta)\left(\tilde{z}_a\right) = \tilde{z}_b,$$
$$(\digamma,\vartheta)\left(\tilde{z}_b\right) = \tilde{z}_b, (\digamma,\vartheta)\left(\tilde{t}_a\right) = \tilde{w}_b, (\digamma,\vartheta)\left(\tilde{t}_b\right) = \tilde{w}_a.$$

So, $(\digamma,\vartheta)$ *be a soft contraction maping on* $\mathcal{SFZMS}$ $\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ *which follows every conditions given in Theorem (3.1). then it has only fixed point* $\tilde{z}_b$.

In our next theorem we take under consideration a continuous soft t-norm and then we prove our contraction condition admits a unique common fixed point for $\mathcal{SFZMS}$.

Theorem 3.2. *Assume* $\mathcal{SFZMS}$ $\left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ *be a complete and having property of continuous soft t-norm* $\tilde{\circledcirc}$ *along with we consider mapping* $(\digamma,\vartheta) : \left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right) \to \left(\tilde{\chi}_\varphi,\Im_\flat,\tilde{\circledcirc}\right)$ *be a* ϖ*-contraction if there exist* $0 \leqslant \bar{\eta} < 1$ *having following condition:*

$$\Im_\flat\left(\tilde{w}_{o_i},\tilde{z}_{o_j},\varpi\left(\frac{\tilde{\kappa}}{\bar{\eta}}\right)^{\tilde{m}}\right) \leqslant \Im_\flat\left((\digamma,\vartheta)\tilde{w}_{o_i}(\digamma,\vartheta)\tilde{z}_{o_j},\varpi(\tilde{\kappa})^{\tilde{m}}\right) \tag{7}$$

where, for every $\tilde{w}_{o_i},\tilde{z}_{o_j}\tilde{\in}\tilde{\chi}_\varphi$, $\tilde{\kappa},\tilde{m} > \bar{0}$ *and* ϖ *is a* Ψ*-function and additionally when we apply limit as* $\tilde{\kappa}$ *goes to positive infinity then we say* $\Im_\flat\left(\tilde{w}_{o_i},\tilde{z}_{o_j},\tilde{\kappa}\right)$ *gives the value* $\bar{1}$. *Then* $(\digamma,\vartheta)$ *admits unique soft fixed point.*

Proof. Let's suppose soft sequence $\{\tilde{w}^m_{o_i}\}$ and a soft point $\tilde{w}^0_{o_i}\tilde{\in}\tilde{\chi}_\varphi$ which has $\tilde{w}^m_{o_i} = (F, \vartheta)^m \tilde{w}^0_{o_i}$. By applying condition 1) and 3) from given definition (3.3) for any $\tilde{\kappa} > \bar{0}\ \exists\ \tilde{l} > \bar{0}$ such that $\tilde{\kappa} > \varpi(\tilde{l})$. By using induction process,

$$\Im_\flat\left(\tilde{w}^0_{o_i}, \tilde{w}^1_{o_i}, \tilde{\kappa}/\bar{\eta}^m\right) \leqslant \Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+1}_{o_i}, \tilde{\kappa}\right). \tag{8}$$

from the definition of (2.11) $\mathcal{SFZMS}$-$(\mathcal{SM})^4$, (8) and for every positive integer β we write,

$$\Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \varpi(\tilde{l})^{\tilde{m}}\right) \leqslant \Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}\right)$$

$$\Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+1}_{o_i}, \varpi(\tilde{l}/\bar{c})^{\tilde{m}}\right) \overbrace{\tilde{\circledcirc}\ldots\tilde{\circledcirc}}^{c-\text{ times}} \Im_\flat\left(\tilde{w}^{m+c-3}_{o_i}, \tilde{w}^{m+c-2}_{o_i}, \varpi(\tilde{l}/\bar{c})^{\tilde{m}}\right)\tilde{\circledcirc}$$

$$\Im_\flat\left(\tilde{w}^{m+c-2}_{o_i}, \tilde{w}^{m+c-1}_{o_i}, \varpi(\tilde{l}/\bar{c})^{\tilde{m}}\right)\tilde{\circledcirc}\Im_\flat\left(\tilde{w}^{m+c-1}_{o_i}, \tilde{w}^{m+c}_{o_i}, \varpi(\tilde{l}/\bar{c})^{\tilde{m}}\right) \leqslant \Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}\right),$$

$$\Im_\flat\left(\tilde{w}^0_{o_i}, \tilde{w}^1_{o_i}, \varpi(\tilde{l}/\bar{c}\tilde{\eta}^m)^{\tilde{m}}\right) \overbrace{\tilde{\circledcirc}\ldots\tilde{\circledcirc}}^{c-\text{ times}} \Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \varpi(\tilde{l}/\bar{c}\tilde{\eta}^{m+c-3})^{\tilde{m}}\right)\tilde{\circledcirc}$$

$$\tilde{\circledcirc}\Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \varpi(\tilde{l}/\bar{c}\tilde{\eta}^{m+c-2})^{\tilde{m}}\right)\tilde{\circledcirc}\Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \varpi(\tilde{l}/\bar{c}\tilde{\eta}^{m+c-1})^{\tilde{m}}\right) \leqslant \Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}\right).$$

using statement of our theorem(8) and apply $\lim_{\tilde{\kappa}\to+\infty_\iota}$, we get

$$\lim_{\tilde{\kappa}\to+\infty_\iota}\Im_\flat\left(\tilde{w}^0_{o_i}, \tilde{w}^1_{o_i}, \varpi(\tilde{l}/\bar{c}\tilde{\eta}^m)^{\tilde{m}}\right) \overbrace{\tilde{\circledcirc}\ldots\tilde{\circledcirc}}^{c-\text{ times}} \lim_{\tilde{\kappa}\to+\infty_\iota}\Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \varpi(\tilde{l}/\bar{c}\tilde{\eta}^{m+c-3})^{\tilde{m}}\right)\tilde{\circledcirc}$$

$$\tilde{\circledcirc}\lim_{\tilde{\kappa}\to+\infty_\iota}\Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \varpi(\tilde{l}/\bar{c}\tilde{\eta}^{m+c-2})^{\tilde{m}}\right)\tilde{\circledcirc}\lim_{\tilde{\kappa}\to+\infty_\iota}\Im_\flat\left(\tilde{w}^0_{o_i}\tilde{w}^1_{o_i}, \varpi(\tilde{l}/\bar{c}\tilde{\eta}^{m+c-1})^{\tilde{m}}\right) \leqslant$$

$$\leqslant \lim_{\tilde{\kappa}\to+\infty_\iota}\Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}\right).$$

$$\lim_{\tilde{\kappa}\to+\infty_\iota}\Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}\right) \geq \overbrace{\tilde{1}\tilde{\circledcirc}\tilde{1}\tilde{\circledcirc}\ldots\tilde{\circledcirc}\tilde{1}\tilde{\circledcirc}\tilde{1}}^{c-\text{ times}},$$

$$\lim_{\tilde{\kappa}\to+\infty_\iota}\Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{w}^{m+c}_{o_i}, \tilde{\kappa}\right) \geq \tilde{1}.$$

then inside $\left(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc}\right)$ soft fuzzy sequence $\{\tilde{w}^m_{o_i}\}$ is a Cauchy sequence which implies it's convergent as we have $\left(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc}\right)$ is complete. letting $\{\tilde{w}^m_{o_i}\} \to \tilde{z}_{o_j}$ *and* $\tilde{z}_{o_j}\tilde{\in}\tilde{\chi}_\varphi$ that is

$$\lim_{m\to\infty}\Im_\flat\left(\tilde{w}^m_{o_i}, \tilde{z}^t_{o_j}, \tilde{\kappa}\right) = \bar{1} \tag{9}$$

subsequently,

$$\Im_\flat\left((F,\vartheta)\tilde{z}_{o_j}, (F,\vartheta)\tilde{w}^m_{o_i}, \tilde{\kappa}/\bar{2}\right)\tilde{\circledcirc}\Im_\flat\left((F,\vartheta)\tilde{w}^m_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}/\bar{2}\right) \leqslant \Im_\flat\left((F,\vartheta)\tilde{z}_{o_j}, \tilde{z}_{o_j}, \tilde{\kappa}\right)$$

$$\Im_\flat\left(\tilde{z}_{o_j}, \tilde{w}^m_{o_i}, \varpi(\tilde{l}/\bar{2}\eta)^{\tilde{m}}\right)\tilde{\circledcirc}\Im_\flat\left(\tilde{w}^{m+1}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}/\bar{2}\right) \leqslant \Im_\flat\left((F,\vartheta)\tilde{z}_{o_j}, \tilde{z}_{o_j}, \tilde{\kappa}\right)$$

using $\tilde{\circledcirc}$ is a continuous soft t-norm and from (9), we write

$$\lim_{m\to\infty}\Im_\flat\left(\tilde{z}_{o_j}, \tilde{w}^m_{o_i}, \varpi(\tilde{l}/\bar{2}\bar{\eta})^{\tilde{m}}\right)\tilde{\circledcirc}\lim_{m\to\infty}\Im_\flat\left(\tilde{w}^{m+1}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}/\bar{2}\right) \leqslant \lim_{m\to\infty}\Im_\flat\left((F,\vartheta)\tilde{z}_{o_j}, \tilde{z}_{o_j}, \tilde{\kappa}\right),$$

$$\Im_\flat\left((F,\vartheta)\tilde{z}_{o_j}, \tilde{z}_{o_j}, \tilde{\kappa}\right) \longrightarrow \bar{1} \text{ whenever } m\to\infty,$$

which gives (F,ϑ) having $\tilde{z}_{o_j}$ is a soft fixed point. It's easy to verify uniqueness of a soft fixed point for ϖ-contraction function (F,ϑ) on $\left(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc}\right)$. □

Theorem 3.3. *Suppose Complete* $\mathcal{SFZMS}$ $\left(\tilde{\chi}_{\varphi}, \Im_{\flat}, \tilde{\circledcirc}\right)$ *with continuous soft t-norm* $\tilde{\circledcirc}$ *and a* ϖ*-contraction mapping* $(F, \vartheta) : \left(\tilde{\chi}_{\varphi}, \Im_{\flat}, \tilde{\circledcirc}\right) \to \left(\tilde{\chi}_{\varphi}, \Im_{\flat}, \tilde{\circledcirc}\right)$. *We consider* $\tilde{w}^{0}_{o_i} \tilde{\in} \tilde{\chi}_{\varphi}$ *be a soft point and soft sequence* $\left\{\tilde{w}^{m}_{o_i}\right\}$ *formed by* $\tilde{w}^{m}_{o_i} = (F, \vartheta)\tilde{w}^{m-1}_{o_i}$ *where* $m = 1, 2, 3, \ldots$ *is convergent. then we say, soft fixed point of* (F, ϑ) *exists and unique in* $\left(\tilde{\chi}_{\varphi}, \Im_{\flat}, \tilde{\circledcirc}\right)$ *and converges* $\left\{\tilde{w}^{m}_{o_i}\right\}$.

Proof. Let's Suppose $\left(\tilde{\chi}_{\varphi}, \Im_{\flat}, \tilde{\circledcirc}\right)$ with ϖ-contraction mapping (F, ϑ) which has a soft real number $\bar{0} \leqslant \eta < \bar{1}$ holding following condition,

$$\Im_{\flat}\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}, \varpi\left(\frac{\tilde{\kappa}}{\tilde{\eta}}\right)^{\tilde{m}}\right) \leqslant \Im_{\flat}\left((F, \vartheta)\tilde{w}_{o_i}(F, \vartheta)\tilde{z}_{o_j}, \varpi(\tilde{\kappa})^{\tilde{m}}\right) \tag{10}$$

for every $\tilde{w}_{o_i}, \tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_{\varphi}$, $\tilde{\kappa}, \tilde{m} > \bar{0}$ and ϖ is a Ψ-function. By the condition (1) and (3) given in Definition (3.3), any $\tilde{\kappa}, \tilde{m} > \bar{0}$, $\exists\, \tilde{l} > \bar{0}$ such that $\tilde{\kappa} > \varpi(\tilde{l})^{\tilde{m}}$.
Then we write,

$$\Im_{\flat}\left(\tilde{w}^{0}_{o_i}, \tilde{w}^{1}_{o'}, \varpi\left(\tilde{l}/\bar{\eta}^{m}\right)^{\tilde{m}}\right) \leqslant \Im_{\flat}\left(\tilde{w}^{m}_{o_i}, \tilde{w}^{m+1}_{o_i}, \tilde{\kappa}\right). \tag{11}$$

we apply $\lim_{m\to\infty}$ to condtion (11), which gives $\Im_{\flat}\left(\tilde{w}^{m}_{o_i}, \tilde{w}^{m+1}_{o_i}, \tilde{\kappa}\right) \longrightarrow \bar{1}$, as we have $\left\{\tilde{w}^{m}_{o_i}\right\}$ is convergent then there exist a soft point $\tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_{\varphi}$ such that $\left\{\tilde{w}^{m}_{o_i}\right\} \to \tilde{z}_{o_j}$ that is

$$\Im_{\flat}\left(\tilde{w}^{m}_{o_i}\tilde{z}^{t}_{o_j}, \tilde{\kappa}\right) \longrightarrow \bar{1} \text{ as } m \to \infty \tag{12}$$

which implies

$$\Im_{\flat}\left((F, \vartheta)\tilde{z}_{o_j}, (F, \vartheta)\tilde{w}^{m}_{o_i}, \tilde{\kappa}/\bar{2}\right) \tilde{\circledcirc} \Im_{\flat}\left((F, \vartheta)\tilde{w}^{m}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}/\bar{2}\right) \leqslant \Im_{\flat}\left((F, \vartheta)\tilde{z}_{o_j}, \tilde{z}_{o_j}, \tilde{\kappa}\right),$$

$$\Im_{\flat}\left(\tilde{z}_{o_j}, \tilde{p}^{m}_{p_i}, \varpi(\tilde{l}/\bar{2}\bar{\eta})^{\tilde{m}}\right) \tilde{\circledcirc} \Im_{\flat}\left(\tilde{w}^{m+1}_{o_i}, \tilde{\tilde{z}}_{o_j}, \tilde{\kappa}/\bar{2}\right) \leqslant \Im_{\flat}\left((F, \vartheta)\tilde{z}_{o_j}, \tilde{z}_{o_j}, \tilde{\kappa}\right).$$

By using (12) and $\tilde{\circledcirc}$ is a continuous soft t-norm, we write

$$\lim_{m\to\infty} \Im_{\flat}\left(\tilde{z}_{o_j}, \tilde{p}^{m}_{p_i}, \varpi(\tilde{l}/\bar{2}\bar{\eta})^{\tilde{m}}\right) \tilde{\circledcirc} \lim_{m\to\infty} \Im_{\flat}\left(\tilde{w}^{m+1}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa}/\bar{2}\right) \leqslant \lim_{m\to\infty} \Im_{\flat}\left((F, \vartheta)\tilde{z}_{o_j}, \tilde{z}_{o_j}, \tilde{\kappa}\right),$$

$$\Im_{\flat}\left((F, \vartheta)\tilde{z}_{o_j}, \tilde{\tilde{z}}_{o_j}, \tilde{\kappa}\right) \longrightarrow \bar{1} \text{ as } m \to \infty.$$

Hence, $\tilde{z}_{o_j}$ be a fixed point of (F, ϑ) and finally we can easily verify uniqueness of a soft fixed point of the ϖ-contraction map (F, ϑ) on $\left(\tilde{\chi}_{\varphi}, \Im_{\flat}, \tilde{\circledcirc}\right)$. □

Theorem 3.4. *Let's suppose complete* $\mathcal{SFZMS}$ $\left(\tilde{\chi}_{\varphi}, \Im_{\flat}, \tilde{\circledcirc}\right)$ *with a continuous soft t-norm* $\tilde{\circledcirc}$ *written as* $\tilde{\zeta}\tilde{\circledcirc}\tilde{\upsilon} = \min\{\tilde{\zeta}, \tilde{\upsilon}\}$. *Alongwith we assume* $\varpi-$ *contraction mapping*

$$(F, \vartheta) : \left(\tilde{\chi}_{\varphi}, \Im_{\flat}, \tilde{\circledcirc}\right) \to \left(\tilde{\chi}_{\varphi}, \Im_{\flat}, \tilde{\circledcirc}\right)$$

Then, (F, ϑ) *admits a unique common soft fixed point.*

Proof. Consider soft sequence $\left\{\tilde{w}^{m}_{o_i}\right\}$ which has a soft point $\tilde{w}^{0}_{o_i} \tilde{\in} \tilde{\chi}_{\varphi}$ can be written as,

$$\tilde{w}^{m}_{o_i} = (F, \vartheta)^{m}\tilde{w}^{0}_{o_i}. \tag{13}$$

Using Theorem (3.3) we can prove this theorem easily, but reaffirming that $\left\{\tilde{w}^{m}_{o_i}\right\}$ is a Cauchy soft sequence. Suppose $\left\{\tilde{w}^{m}_{o_i}\right\}$ is not a Cauchy soft sequence, which gives there exist a soft real numbers $\tilde{\kappa} > \bar{0}$ and $\tilde{\lambda} > \bar{0}$ satisfying for any $\mathbb{N}_0$ is a poitive integer $\exists\, m(\mathbb{N}_0)$ and $t(\mathbb{N}_0) \geqslant \mathbb{N}_0$ such that

$$\Im_{\flat}\left(\tilde{w}^{m(\mathbb{N}_0)}_{o_i}, \tilde{w}^{t(\mathbb{N}_0)}_{o_i}, \tilde{\kappa}\right) > \bar{1} \ominus \tilde{\lambda}, \tag{14}$$

using $m(\mathbb{N}_0) < t(\mathbb{N}_0)$, so that $t(\mathbb{N}_0)$ is the lowest positive integer w. r. t. the $m(\mathbb{N}_0)$ which hold condition (14), then $\exists\ \tilde{\kappa} > \bar{0}$ *and* $\tilde{\lambda} > \bar{0}$ which has $\{m(\mathbb{N}_0)\}$ and $\{t(\mathbb{N}_0)\}$ be two increasing sequence such that $m(\mathbb{N}_0) < t(\mathbb{N}_0)$ satisfies the following,

$$\Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{w}_{o_i}^{t(\mathbb{N}_0)-1}, \tilde{\kappa}\right) \tilde{\leq} \bar{1} \ominus \tilde{\lambda} \tag{15}$$

and

$$\Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{w}_{o_i}^{t(\mathbb{N}_0)}, \tilde{\kappa}\right) > \bar{1} \ominus \tilde{\lambda}. \tag{16}$$

to form such sequence we need to find $\tilde{w}_{o_i}^{t(\mathbb{N}_0)}$ s.t.

$$\tilde{w}_{o_i}^{t(\mathbb{N}_0)} \tilde{\notin} \left\{\tilde{z}_{o_j} : \Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{z}_{o_j}, \tilde{\kappa}\right) \tilde{\leq} \bar{1} \ominus \tilde{\lambda}\right\} \tag{17}$$

and

$$\tilde{w}_{o_i}^{t(\mathbb{N}_0)-1} \tilde{\in} \left\{\tilde{z}_{o_j} : \Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{z}_{o_j}, \tilde{\kappa}\right) \tilde{\leq} \bar{1} \ominus \tilde{\lambda}\right\} \tag{18}$$

as we have considered $\left\{\tilde{w}_{o_i}^m\right\}$ is not a Cauchy soft sequence and $\tilde{z}_{o_j} \tilde{\in} \tilde{\chi}_\varphi, \tilde{\lambda} > \bar{0}$ and $\bar{0} < \tilde{\kappa_1} < \tilde{\kappa_2}$,

$$\left\{\tilde{z}_{o_j} : \Im_\flat\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa_1}\right) \tilde{\leq} \bar{1} \ominus \tilde{\lambda}\right\} \tilde{\subset} \left\{\tilde{z}_{o_j} : \Im_\flat\left(\tilde{w}_{o_i}, \tilde{z}_{o_j}, \tilde{\kappa_2}\right) \tilde{\leq} \bar{1} \ominus \tilde{\lambda}\right\},$$

whcih follow the sequence formation is attainable for all $\tilde{\kappa} > \bar{0}, \tilde{\lambda} > \bar{0}$, then the sequence $\left\{\tilde{w}_{o_i}^{m(\mathbb{N}_0)}\right\}$ and $\left\{\tilde{w}_{o_i}^{t(\mathbb{N}_0)}\right\}$ hold Conditions (15) and (16) for any $\tilde{l} > \bar{0}, \tilde{\lambda} > \bar{0}$ where $\tilde{l} < \tilde{\kappa}$. as we have ϖ is Ψ-function and for every $\tilde{\kappa}, \tilde{m} > \bar{0}$, which has $\tilde{l} > \bar{0}$ such that $\tilde{\kappa} > \varpi(\tilde{l})^{\tilde{m}}$. Then, we use $\tilde{\kappa}$ in to (15) and (16) as $\tilde{\kappa} = \varpi\left(\tilde{\kappa_1}\right)^{\tilde{m}}$ for some $\tilde{\kappa_1} \bar{>} 0$ such that $\varpi\left(\tilde{\kappa_1}/\bar{\eta}\right)^{\tilde{m}} > \varpi\left(\tilde{\kappa_1}\right)^{\tilde{m}}$ then the choice is possible through condition 1) and 3) given in Definition (3.3), By applying (15) and (16), we get

$$\Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{w}_{o_i}^{t(\mathbb{N}_0)-1}, \varpi\left(\tilde{\kappa_1}\right)^{\tilde{m}}\right) \tilde{\leq} \bar{1} \ominus \tilde{\lambda}. \tag{19}$$

and

$$\Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{w}_{o_i}^{t(\mathbb{N}_0)}, \varpi\left(\tilde{\kappa_1}\right)^{\tilde{m}}\right) > \bar{1} \ominus \tilde{\lambda}, \tag{20}$$

then

$$\Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{w}_{o_i}^{t(\mathbb{N}_0)}, \varpi\left(\tilde{\kappa_1}\right)^{\tilde{m}}\right) > \bar{1} \ominus \tilde{\lambda},$$

$$\Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)-1}, \tilde{w}_{o_i}^{t(\mathbb{N}_0)-1}, \varpi\left(\tilde{\kappa_1}/\bar{\eta}\right)^{\tilde{m}}\right) \geqslant \bar{1} \ominus \tilde{\lambda}.$$

or

$$\Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)-1}, \tilde{w}_{o_i}^{t(\mathbb{N}_0)-1}, \varpi\left(\tilde{\kappa_1}/\bar{\eta}\right)^{\tilde{m}}\right) > \bar{1} \ominus \tilde{\lambda}.$$

as we have $\varpi\left(\tilde{\kappa_1}\right)^{\tilde{m}} < \varpi\left(\tilde{\kappa_1}/\bar{\eta}\right)^{\tilde{m}}$, now use $\tilde{\kappa}$ as $\tilde{\kappa} < \left\{\varpi\left(\tilde{\kappa_1}/\bar{\eta}\right)^{\tilde{m}} \ominus \varpi\left(\tilde{\kappa_1}\right)^{\tilde{m}}\right\}$ which means

$$\varpi\left(\tilde{\kappa_1}/\bar{\eta}\right)^{\tilde{m}} \ominus \tilde{\kappa} > \varpi\left(\tilde{\kappa_1}\right)^{\tilde{m}}$$

now using (11) and Theorem (3.3) we select $\mathbb{N}_0$ large enough such that

$$\Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{w}_{o_i}^{m(\mathbb{N}_0)-1}, \tilde{\kappa}\right) < \bar{1} \ominus \tilde{\lambda}_1, \text{ for every } \bar{0} < \tilde{\lambda}_1 < \tilde{\lambda}. \tag{21}$$

using (19) and (21), the choice of $\mathbb{N}_0$ and $\tilde{\kappa}$ we get

$$\Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{w}_{o_i}^{t(\mathbb{N}_0)-1}, \varpi\left(\tilde{\kappa_1}/\bar{\eta}\right)^{\tilde{m}}\right) > \bar{1} \ominus \tilde{\lambda},$$

$$\Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{w}_{o_i}^{t(\mathbb{N}_0)-1}, \left(\varpi\left(\tilde{\kappa_1}/\bar{\eta}\right)^{\tilde{m}} \ominus \tilde{\kappa}\right)\right) \tilde{\circledcirc} \Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)-1}, \tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{\kappa}\right) \geqslant \bar{1} \ominus \tilde{\lambda},$$

$$\Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{w}_{o_i}^{t(\mathbb{N}_0)-1}, \varpi\left(\tilde{\kappa_1}\right)^{\tilde{m}}\right) \tilde{\circledcirc} \Im_\flat\left(\tilde{w}_{o_i}^{m(\mathbb{N}_0)-1}, \tilde{w}_{o_i}^{m(\mathbb{N}_0)}, \tilde{\kappa}\right) \geqslant \bar{1} \ominus \tilde{\lambda},$$

$$(\bar{1} \ominus \tilde{\lambda}) \tilde{\circledcirc} \left(\bar{1} \ominus \tilde{\lambda}_1\right) \geqslant \bar{1} \ominus \tilde{\lambda}.$$

by applying the fact $\tilde{\lambda}_1 < \tilde{\lambda}$, we get $(\bar{1} \ominus \tilde{\lambda}_1) > (\bar{1} \ominus \tilde{\lambda})$ which shows contradiction to our main assumption so $\{\tilde{w}^m_{o_i}\}$ is Cauchy sequence. □

In the following section we use some numerical examples to validate our established main result from section (3), where the soft fuzzy contraction principle described in theorem (3.1) and confirmed by examples number (4.1) subsequently example (4.2) validate theorem (3.4) along with this we already proved Example (3.2) holding theorem (3.1).

4. Illustrations

Example 4.1. *Suppose* $\chi = \mathcal{U} \cup \mathcal{V}$ *which has* $\mathcal{U} = \left\{\frac{1}{2}, \frac{1}{3}\right\}, \mathcal{V} = [4,5]$, *a parameter set* $\mathcal{P} = \{1,2\}$ *and the mapping* $\mathfrak{L} : \mathcal{K}_\rho(\tilde{\chi}_\mathcal{P}) \times \mathcal{K}_\rho(\tilde{\chi}_\mathcal{P}) \to \Re(\mathcal{P})^*$ *having*

$$\mathfrak{L}(\tilde{w}_o, \tilde{z}_q) = |\bar{w} - \bar{z}| + |\bar{p} - \bar{q}| \text{ for every } \tilde{w}_o, \tilde{z}_q \in \mathcal{K}_\rho(\tilde{\chi}_\mathcal{P}).$$

Inside $(\tilde{\chi}_\mathcal{P}, \mathfrak{L})$, *we define following operation* $\tilde{w}\tilde{\circledcirc}\tilde{z} = \tilde{w} \circ \tilde{z}$ *or* $\tilde{w}\tilde{\circledcirc}\tilde{z} = \min\{\tilde{w}, \tilde{z}\}$ *with mapping* $\Im_\flat : \mathcal{K}_\rho(\tilde{\chi}_\varphi) \times \mathcal{K}_\rho(\tilde{\chi}_\varphi) \times (0,\infty)(\varphi) \longrightarrow [0,1](\varphi)$ *we define as,*

$$\frac{\tilde{\kappa}}{\tilde{\kappa} \oplus \mathfrak{L}(\tilde{w}_o, \tilde{z}_q)} = \Im_\flat(\tilde{w}_o, \tilde{z}_q, \tilde{\kappa}), \text{ for every } \tilde{w}_o, \tilde{z}_q \tilde{\in} \tilde{\chi}_\varphi \text{ and } \tilde{\kappa} > \bar{0}.$$

which implies $(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc})$ *be a complete* $\mathcal{SFZMS}$.
Let's define mapping $(F, \vartheta) : \tilde{\chi}_\varphi \to \tilde{\chi}_\varphi$ *as*

$$(F,\vartheta)(\tilde{w}_o) = \begin{cases} \left(\frac{\bar{1}}{2}\right)_1 & \text{whenever } \tilde{w}_o \in \mathcal{K}_\rho(\mathcal{V}_\mathcal{P}) \\ \left(\frac{\bar{1}}{3}\right)_2 & \text{otherwise.} \end{cases}$$

So, all the condition given in the Theorem 1 are satisfied hence (F, ϑ) *is a soft contraction map on* $\mathcal{SFZMS}$ $(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc})$ *and it gives* $\left(\frac{\bar{1}}{3}\right)_2$ *as a soft fixed point.*

Example 4.2. *Let's suppose the set* $\chi = \left\{\frac{5}{8}, \frac{3}{4}, \frac{8}{9}\right\}$ *having parameter set* $\varphi = \{1,2\}$ *along with a soft t-norm, defined by* $\hbar\tilde{\circledcirc}\ell = \min\{\hbar, \ell\}$ *for* $\hbar, \ell \in [0,1](\varphi)$, *so*

$$\mathcal{K}_\rho(\tilde{\chi}_\varphi) = \{\frac{5}{81}, \frac{5}{82}, \frac{3}{41}, \frac{3}{42}, \frac{8}{91}, \frac{8}{92}\}$$

and the mapping for every $o, q \in \varphi$, $\Im_\flat : \mathcal{K}_\rho(\tilde{\chi}_\varphi) \times \mathcal{K}_\rho(\tilde{\chi}_\varphi) \times (0,\infty)(\varphi) \longrightarrow [0,1](\varphi)$ *as*

$$\Im_\flat\left(\frac{\bar{5}}{8}_o, \frac{\bar{3}}{4}_q, \tilde{\kappa}\right) = \Im_\flat\left(\frac{\bar{3}}{4}_q, \frac{\bar{5}}{8}_o, \tilde{\kappa}\right) = \begin{cases} \bar{1} & \text{whenever } \tilde{\kappa} > \bar{3} \\ \bar{0} & \text{whenever } \tilde{\kappa} = \bar{0} \\ 0.\bar{9} & \text{whenever } \bar{0} < \tilde{\kappa} \leq \bar{3} \end{cases}$$

$$\Im_\flat\left(\frac{\bar{8}}{9}_o, \frac{\bar{3}}{4}_q, \tilde{\kappa}\right) = \Im_\flat\left(\frac{\bar{8}}{9}_q, \frac{\bar{5}}{8}_o, \tilde{\kappa}\right) = \Im_\flat\left(\frac{\bar{3}}{4}_q, \frac{\bar{8}}{9}_o, \tilde{\kappa}\right) = \Im_\flat\left(\frac{\bar{5}}{8}_o, \frac{\bar{8}}{9}_q, \tilde{\kappa}\right) =$$

$$= \begin{cases} \bar{1} & \text{whenever } \tilde{\kappa} > \bar{8} \\ \bar{0} & \text{whenever } \tilde{\kappa} = \bar{0} \\ 0.\bar{6} & \text{whenever } \bar{0} < \tilde{\kappa} \leq \bar{8} \end{cases}$$

$\Im_\flat(\tilde{w}_o, \tilde{z}_q, \tilde{\kappa}) = \bar{1}$ *if and only if* $\tilde{w}_o = \tilde{z}_q$, *for every* $\tilde{w}_o, \tilde{\tilde{z}}_q \tilde{\in} \tilde{\chi}_\varphi$ *and* $\tilde{\kappa} > \bar{0}$. *Hence we say,* $(\tilde{\chi}_\varphi, \Im_\flat, \tilde{\circledcirc})$ *is a complete* $\mathcal{SFZMS}$. *Now, let's suppose a soft mapping* (F, ϑ) *on* $\tilde{\chi}_\varphi$,

$$(F,\vartheta)(\frac{\bar{5}}{8}_1) = \frac{\bar{3}}{4}_1, (F,\vartheta)(\frac{\bar{5}}{8}_2) = \frac{\bar{3}}{4}_1, (F,\vartheta)(\frac{\bar{3}}{4}_1) = \frac{\bar{3}}{4}_1,$$
$$(F,\vartheta)(\frac{\bar{3}}{4}_2) = \frac{\bar{5}}{8}_2, (F,\vartheta)(\frac{\bar{8}}{9}_1) = \frac{\bar{8}}{9}_2, (F,\vartheta)(\frac{\bar{8}}{9}_2) = \frac{\bar{8}}{9}_1.$$

if we use $\varphi(\kappa) = (\kappa)^{\frac{1}{3}}$ *which implies* $\kappa \in \varpi$. $(\digamma, \vartheta)$ *be a soft contraction maping on* $\mathcal{SFZMS}$ $\left(\tilde{\chi}_{\varphi}, \Im_{\flat}, \tilde{\circledcirc}\right)$ *which follows every conditions given in Theorem (3.4). then it has only fixed point* $\overline{\frac{3}{4}}_1$.

5. Conclusion

In this two fold's of manuscript Firstly, we introduce the uniqueness and existance of fixed point for ϖ-contraction mapping principle using Continuity of soft-t-norm under $\mathcal{SFZMS}$ along with we taken some restriction on soft fuzzy metric space between a soft point of the absolute soft set taken under consideration. We expanded concept of altering distance functions and proved the uniqueness and existance of fixed point for ϖ-contraction mapping in the context of $\mathcal{SFZMS}$.
Secondly, we applied some appropriate results and illustrations to support of the newly devloped fixed point Theorems (3.1), (3.2) and (3.4). Additionally, The presented work in our paper is extended version of some well known results from litrature like soft fuzzy b-metric spaces, soft fuzzy partial spaces, neutrosophic fuzzy soft metric spaces and so on. This result can be further extended to a proximity fixed point.

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References

[1] A. Kharal and B. Ahmad., *Mappings on soft classes*, New Mathematics and Natural Computation, **7(3)** (2011), 471-481,
[2] A. Azzam, Z. A. Ameen, T. M. Al-shami and M. E. El-Shafei, *Generating soft topologies via soft set operators*, Symmetry, **14** (2022), 914-954, https://doi.org/10.3390/sym14050914,
[3] A. R. Alar, Yigit Dakmaz, E. Sola Erduran and F.Gezici, *On soft fuzzy metric spaces and topological structure*, J. Adv. Stud. Topol., **9** (2018), 61–70,
[4] A. Z. Ameen and S. Al Ghour, *Minimal soft topologies*, New Math. Nat. Comput., (2022), 1–13, https://doi.org/10.1142/S1793005722500466,
[5] A. F. Sayed and A. Alahmari, *Fuzzy soft* α *-* ψ *- contractive type mappings and some fixed point theorems in fuzzy soft metric spaces*, Annals Fuzzy Math. Inform., **15(1)** (2018), 73-87.
[6] B. Mohammadi, A. Hussain, V. Parvaneh, N. Saleem and R. J. Shahkoohi, *Fixed point results for generalized fuzzy contractive mappings in fuzzy metric spaces with application in integral equations,* J. Math. (2021), Article ID 9931066, https://doi.org/10.1155/2021/9931066,
[7] D. Molodtsov, *Soft set theory—First results*, Comput. Math. Appl., **37** (1999), 19-31,
[8] F.S. Erduran, E. Yigit, R. Alar and A. Gezici, *Soft fuzzy metric spaces*, Gen. Lett. Math., **3** (2017), 91–101,
[9] G. A. Cigdem, B. Sadi and C. Vefa, *Fixed point theorems on soft S-metric space*, Communications in Mathematics and Applications, **9(4)** (2018), 725-733,
[10] H. Hosseinzadeh, *Fixed point theorem on soft metric spaces*, Journal of Fixed Point Theory and Applications, **18(4)** (2017), 1-22,
[11] I. Kramosil and J. Michalek, *Fuzzy metric and statistical metric spaces*, Kybernetica, **11** (1975), 326–334,
[12] J. C. R. Alcantud, *Softarisons: Theory and practice*, Neural Comput. Appl., **33** (2021), 6759–16771, https://doi.org/10.1007/s00521-021-06272-4,
[13] J. C. R. Alcantud, T. M. Al-shami and A. A. Azzam, *Caliber and chain conditions in soft topologies*, Mathematics, **9** (2021), 2349-2370, https://doi.org/10.3390/math9192349,

[14] M. J. Borah and B. Hazarika, *Soft ideal topological space and mixed fuzzy soft ideal topological space*, Bol. Soc. Paran. Mat, **37(1)** (2019), 141-151,

[15] M. I, Yazar, C. Gunduz and S. Bayramov, *Fixed point theorems of soft contractive mappings*, Filomat, **30** (2016), 269–279,

[16] M. Riaz and Z. Fatima, *Certain properties of soft metric spaces*, Journal of Fuzzy Mathematics, **25(3)** (2017), 543–560,

[17] P. K. Maji, R. Biswas and A. R. Roy, *Soft set theory*, Comput. Math. Appl., **45** (2003), 555–562,

[18] R. Bhardwaj, S. Singh, S. Pandey, B. R. Wadkar, *Fixed point results in soft metric space*, International Journal of Modern Agriculture, **10(2)** (2021), 1792-1803,

[19] R. I. Sabri, M. N. Mohammedali, M. Rasheed and S. Shihab, *Compactness of soft fuzzy metric space*, J. Southwest Jiaotong Univ., **55** (2020), 1–10,

[20] R. Bhardwaj, *Fixed Point results in Compact Rough Metric spaces*, Int. J. Emerg. Technol. Adv. Eng., **12** (2022), 107–110,

[21] S. Sonam, C. S. Chauhan, R. Bhardwaj, and S. Narayan, *Fixed point results in soft rectangular b-metric spaces*, Non linear Funct. Anal. Appl., **2023**,

[22] S. Al Ghour and Z. A. Ameen, *Maximal soft compact and maximal soft connected topologies*, Appl.Comput. Intell. Soft Comput., (2022), 1–7, https://doi.org/10.1155/2022/9860015,

[23] S. Sonam, R. Bhardwaj, F. Beg and S. Narayan, *Fixed-point results through altering distance*, J. Basic Sci., **22** (2022), 162–167,

[24] S. A. Khandait, C. Singh, R. Bhardwaj and A. K. Mishra, *Theorems on Soft Fuzzy Metric Spaces by Using Control Function*, In Fuzzy Intelligent Systems: Methodologies, Techniques, and Applications, Scrivener Publishing LLC: USA, (2021), pp. 413–430,

[25] S. Rathee, R. Girdhar and K. Dhingra, *Fixed point theorems in soft multiplicative metric spaces*, Communications in Mathematics and Applications., **11** (2020), 425-442,

[26] S. Das and S. Samanta, *Soft real sets, soft real numbers and their properties*, J. Fuzzy Math., **20** (2012), 551–576,

[27] S. Das and S. Samanta, *On soft metric space*, J. Fuzzy Math., **21** (2013), 707–735,

[28] S. Das and S. Samanta, *Soft metric,* Ann. Fuzzy Math. Inform., **6** (2013), 77–94,

[29] T. Beaulaa and R. Raja, *Completeness in Fuzzy Soft Metric Space*, Malaya J. Mat., **5(2)** (2015), 438-442,

[30] T. M. Al-shami, *Soft somewhat open sets: Soft separation axioms and medical application to nutrition*, Comput. Appl. Math.,(2022),

[31] V. Gupta and A. Gondhi, *Soft Tripled Coincidence Fixed Point Theorems in Soft Fuzzy Metric Space*, J. Sib. Fed. Univ. Math. Phys., **16** (2023), 397–407,

[32] Yigit Dakmaz, R. Alar, Sola Erduran and A. Gezici, *On soft fuzzy metric spaces and topological structure*, J. Adv. Stud. Topol., **9** (2018), 61–70.