

Local Isolate Domination In Graphs

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Abstract

In this Communication a new parameter called Local Isolate Domination in graphs is defined and Studied. A Dominating Set S is a Local Dominating set iff for each u in S , $\langle N(u) \rangle$ has an isolate.

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1 Introduction

Throughout this paper , Simple Finite Graphs without loops and multiple edges are considered. For terminologies and notations refer Chartrand And Lesniak[3].

Domination and related topics are dealt in [1 , 4, 5, 6].

A subset S of the vertex set $V(G)$ is a Dominating set if each vertex in the set $V \setminus S$ is adjacent to a vertex in S . Minimum cardinality of a minimal dominating set is the Domination number of a Graph denoted by $\gamma(G)$.

It is an isolate dominating set if the induced graph $\langle S \rangle$ contains an isolate and is introduced and studied in [7].

A Dominating set S is called a Doubly Isolate Dominating set if both the induced graphs $\langle S \rangle$ and $\langle V \setminus S \rangle$ have isolates. Doubly Isolate Dominating set is introduced and studied in [2].

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Also when the concept of Isolate Dominating set is localized to the Neighbour set we arrive at a new variant called Local Isolate Domination in Graphs. This motivated us to define a new parameter that is introduced and studied in this communication.

2 Preliminary Results

Theorem 2.1 “For a Graph G with order atleast 3, $\Delta(G) = n-1$ and minimum degree atleast 2, G has no Local Isolate Dominating Set.”

Proof: From the hypothesis we observe that G is a graph without isolates and with Domination number as one and hence this dominating set is not a Local Isolate Dominating set of G . Suppose S is any dominating set of G and $S \setminus \{v\} \neq \phi$ where $\{v\}$ is a full degree vertex of G . Now for each u in $\{S \setminus \{v\}\}$ the induced graph $\langle N(u) \rangle$ has no isolated vertex. Hence S is not a Local Isolate Dominating Set of G .

Corollary 2.2 “The Local Isolate Dominating Set Does Not Exist For The Following Graphs:

- (i) Complete Graph K_n .
- (ii) Wheel Graph W_n .
- (iii) Fan Graph F_n .”

Observation 2.3 “The Local Isolate Dominating Set Does Not Exist For Complete r -Partite K_{n_1, n_2, \dots, n_r} , $r \geq 3$ Graph.”

3. Main results

Proposition 3.1

“(i) For the Paths P_n and the Cycles C_n we have $\gamma_{lo}(P_n) = \gamma_{lo}(C_n) = \left\lceil \frac{n}{3} \right\rceil$, $\Gamma_{lo}(P_n) = \left\lceil \frac{n}{2} \right\rceil$

and $\Gamma_{lo}(C_n) = \left\lceil \frac{n}{2} \right\rceil$

(ii) If G is a Graph of Order n , Then $\gamma(G^+) = \Gamma_{lo}(G^+) = n$, Where G^+ is the Graph that was produced from G by attaching at each of G 's vertex's e Edges.”

Proof.(i) “Obviously $\gamma_{lo}(P_4) = 2$ and when $n \neq 4$, any γ -set of P_n is a local isolate dominating set as well, so that $\gamma_{lo}(P_n) \leq \gamma(P_n)$.” Every Local Isolate D dominating set is

a dominating set so $\gamma(P_n) \leq \gamma_{lo}(P_n)$ thus $\gamma_{lo}(P_n) = \gamma(P_n)$ and so $\gamma_{lo}(P_n) = \left\lceil \frac{n}{3} \right\rceil$ as

$\gamma(P_n) \left\lceil \frac{n}{3} \right\rceil$. Now if $P_n = \{v_1, v_2, v_3, \dots, v_n\}$ then the set $s = \left\{ v_{2i-1} \mid 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \right\}$ is a minimal

isolate dominating set so that $\Gamma_{lo}(P_n) \geq \left\lceil \frac{n}{2} \right\rceil$. Additionally, since any set that contains

more than vertices of P_n is no longer able to be a minimal isolation dominating, we

have $\Gamma_{lo}(P_n) = \left\lceil \frac{n}{2} \right\rceil$. Similar to this, one may prove $\gamma_{lo}(C_n) = \left\lceil \frac{n}{3} \right\rceil$ and $\Gamma_{lo}(C_n) = \left\lceil \frac{n}{2} \right\rceil$

(ii) Each pendant vertex is required to be present in any minimal isolate-dominating set S of G^+ or one of its neighbours, in order to have at least n vertices. “Further, if $|S| > n$, S must consequently include a pendant vertex along with its support and so $S - \{v\}$, where v is the support, is an isolate dominating set of G^+ , a contradiction to the minimality of S .” Hence $|S| = n$.

Theorem 3.2 “For a Graph G of order at least 2, $\gamma_{lo}(G) = 1$ iff there exists a pair $u, v \in V(G)$, $\deg_G(u) = 1$ and $\deg_G(v) = n - 1$.”

Proof: Let G be a graph with $n \geq 2$. Suppose $\gamma_{lo}(G) = 1$. Let $S = \{v\}$ be a Local Isolate Dominating set of G . “Since S is a dominating set and $|V(G) \setminus S| = n - 1$, $\deg_G(v) = n - 1$.” Also since S is a γ_{lo} -set of G , $\langle n(v) \rangle$ has an isolate vertex, say u . Therefore u is a pendent vertex of G . Hence $\deg_G(u) = 1$. Conversely, $\{V\}$ is a dominant set of G since there is a vertex v with $\deg_G(v) = n$. Since $\deg_G(u) = 1$, u is a isolate vertex in $\langle n(v) \rangle$, thus $\gamma_{lo}(G) = 1$.

Corollary 3.3 “For a Star Graph S_n with $n \geq 2$, $\gamma_{lo}(S_n) = 1$.”

Theorem 3.4 “If G is a Tree with $n \geq 2$ then G has a Local Isolate Dominating Set.”

Proof : Let G be a Tree of Order $n \geq 2$ and S be any Dominating Set of G . “Suppose G has no Local Isolate Dominating Set, there exist a vertex $v \in S$, and $\langle n(v) \rangle$ has no isolate vertex.” Thus $\langle n(v) \rangle$ is a connected Graph. This implies $\langle n[v] \rangle$ contains a cycle, which contradicts that G is a Tree. Therefore G has a Local Isolate Dominating Set.

Corollary 3.5 “For any Tree T , $\gamma(T) = \gamma_o(T) = \gamma_{lo}(T)$ ”.

Theorem 3.6 “Let S be any Local Isolate Dominating Set of a Graph G and $U \in S$. Then there exist a vertex $v \in V(G)$ such that $uv \in E(G)$ and $N(U) \cap N(V) = \emptyset$.”

Theorem 3.7 “For a Complete bipartite Graph $K_{m,n}$, $\gamma_{lo}(K_{m,n}) = 2$, $m \geq 2$, $n \geq 2$.”

Theorem 3.8 “A Local isolate dominating set S of a graph G is minimal iff it is 1-minimal.”

proof : “Let S be a 1-minimal Local Isolate Dominating Set of a graph G . Suppose there exists a $S' \subset S$ that is also a Local Isolated Dominating Set of G , then for all v in S' , $\langle n(v) \rangle$ has an isolate vertex”. Since S' is a Dominating Set, for all vertex in u in $S \setminus S'$ is adjacent to at least one vertex in S' and either u is an isolate vertex in $\langle n(v) \rangle$, $v \in S'$ or $\langle n(v) \rangle$ has an isolated vertex in $V \setminus S$.

case(i): u is an isolate vertex in $\langle n(v) \rangle$, $v \in S'$ then $S' \setminus \{v\}$ is Local Isolate Dominating Set of G which contradicts the 1-minimality of S .

case(ii): $\langle n(v) \rangle$ has an isolated vertex in $V \setminus S$. Let $w \in \langle n(v) \rangle$ be isolate vertex in $V \setminus S$ then $S \setminus \{u\}$ is Local Isolate Dominating Set of G which contradicts the 1-minimality of S . Hence S is minimal.

Converse is obvious.

Theorem 3.9 “A Local Isolate Dominating Set S of a Graph G is Minimal iff every vertex in S has a Private Neighbor with respect to S .”

Proof: “Let S be a minimal Local Isolate Dominating set and u be a vertex of S .” u is a private neighbour of itself if it is an “isolate” in $\langle S \rangle$. “Suppose u is not an isolate of $\langle S \rangle$.” “The set $S \setminus \{u\}$ will be a Local Isolate Dominating set if u has no private neighbours with regard to S .” “This contradicts the minimality of S ”, hence “ u must have a private neighbour with regard to S ”. Conversely, “suppose S is a Local Isolate Dominating set of G and every vertex of S has a private neighbor with respect to S ”. If S is not minimal, then by theorem 3.8, as a result of S 's inability to be a “1-minimal Dominating Set” of G , S has a vertex u that makes $S \setminus \{u\}$ a Local Isolate Dominating Set of G . “Each vertex in $V \setminus (S \setminus u)$ must thus have at least one neighbour in $S \setminus u$, and as a result, the vertex u cannot have any private neighbour with regard to S .” This is a contradiction to our assumption, therefore S is minimal.

Corollary 3.10 “A minimal Local Isolate Dominating Set S of a Graph G is also a minimal Dominating Set of a Graph G .”

4. Join of Graphs

Observation 4.1 “Let G and H be any two Graphs of order $m, n \geq 3$ with isolate vertex and S be a Local Isolate Dominating Set of $G+H$. Then $S \cap V(G) \neq \emptyset$ and $S \cap V(H) \neq \emptyset$.”

Theorem 4.2 “Let G and H be any two Graphs. Then S a subset of

$V(G+H)$ is a Local Isolate Dominating Set of $G+H$ iff G and H have isolated vertices.”

Proof “Let G and H be any two Graphs and $S \subseteq V(G+H)$ be a Local Isolate Dominating Set of $G+H$.” Suppose G and H have no isolated vertex then for each $u \in S$, $\langle n(u) \rangle$ is connected, which is a contradiction. Therefore there are isolated vertex in both G and H .

Conversely, U and V be isolated vertices of G and H respectively, Then $S = \{u, v\}$ is a Dominating Set of $G + H$ and also $n(u) \geq V(H)$ and $n(v) \geq V(G)$. Thus $\langle n(u) \rangle$ and $\langle n(v) \rangle$ have isolated vertex. Therefore “ S is a Local Isolate Dominating Set of $G+H$.”

Corollary 4.3 “Let G and H be any Graphs with isolated vertex, Then $\gamma_{10}(G+H) \leq 2$.”

Proof. “Let G and H be Graphs with isolated vertex.” Suppose either $G = K_1$ or $H = K_1$ or $G = H = K_1$ Then Clearly, $\gamma_{10}(G + H) = 1$. Suppose $G \neq K_1$ and $H \neq K_1$, by the theorem 4.2, $\gamma_{10}(G + H) = 2$. Thus $\gamma_{10}(G + H) \leq 2$.

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