Inventory model for Imperfect Production System with and without Disruption and Rework

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Abstract

Production system may disrupted due to labor strick, machine break down, power breakdown etc. Along with this problem, machine may produce imperfect items. In this paper, we have suggested a production policy for an imperfect production system with disruption. The model is compared with disruption and without disruption. The rework started just after the regular production. The profit function has been derived and obtained the regular production time, rework time and disrupted production time. The proposed model is analyzed analytically, graphically and numerically.

1 Introduction

Every manufacturer have to confirm production level, and finding the most economical quantity. Now days, this is a general problem and admit the general solution, however much it may be advisable to exercise judgment in that particular case and such solution assisted by a knowledge of the general solution to decide the policy should include all the features involve in that case.

Production planning is another aspect that manufacturers (practitioners) and researchers are attracted to recovered overages and shortages of the items. There are many reason to disrupt the production system, like machine breakdown, labor strike unexpected events etc. With these uncertainties production system may produce some imperfect items, so the problem become more complex with disruption and imperfect production. The classical EOQ model does not includes chances of disruption in supply. The classical EMQ model considered the all produced/manufactured quantity are of perfect quantity. However in real life production system due to destruction and other failure generate the defective items, there are two type of defective items, those imperfect items which may convert into perfect items through rework process, called as reworkable items and those imperfect items can not be convert into perfect items, called as scrap items.

Harris (1913) was the first mathematician who used the word EMQ in inventory management. Kul *et al.* (1995) presented a model in which they have considered rework on imperfect items at the end of regular production process. They have developed a very simple procedure to compute the optimal threshold policy. Hayek *et al.* (2001) developed a finite production model in which they studied the effect of imperfect quality to minimize the total inventory cost. Chiu *et al.* (2007) developed an optimal replenishment policy for imperfect EMQ model with the help of differential calculus approach. They also solved the same problem by the algebraic approach and suggested a differential calculus approach which is much better than the algebraic approach. Chiu *et al.* (2008) considered a finite production model with a random defective items production rate and applied the rework process on defective items. They assumed that a portion of defective items produced in scrap and must be discarded before rework on repairable defective items. They also suggested an optimal policy for production lot-size and back order level.

Haji *et al.* (2008) considered good and defective item both in the production system and rework rate is assumed to be a function of the random variable. Talaizadeh *et al.* (2013) developed an EPQ model with random defective items production rate by considering with rewokable and non reworkable items allowing with shortage. They determined the optimal period length of back order quantities and minimized the total expected cost. Chiu *et al.* (2014) introduced a mathematical model in which they minimized the total production cost, delivery cost for the EPQ and also incorporated the rework process and multi delivery cost. Kumar *et al.* (2016) considered a probabilistic inventory model for deteriorating items with constant deterioration rate and ramp type demand under stock dependent consumption. Sang *et al.* (2016) designed the imperfect manufacturing system using various cases of two-echelon supply chain consisting of a single manufacturer and single retailer. They determined the optimal retail price, replenishment cycle and a number of shipment for exponential deteriorating items.

Kumar *et al.* (2017) considered a multiple production setup and single rework setup of EOQ model for imperfect items. They also determine the number of production setup and optimize production time by minimizing the total inventory cost. He *et al.* (2010) proposed a production inventory model for a deteriorating item which is constant and the production disruption under a different situation. This method helps the manufacturer to reduce the losses caused by production disruption. Khedlekar *et al.* (2014) formulated a production inventory model for deteriorating item with production disruption and analyzed the system under a different situation. Chiu*et al.* (2011) derived some special cases in EMQ model with rework and multiple shipments. They optimized the total quantity in terms of production rate and regular time.
In this paper, we have following objectives.

- To optimize production items.

- To optimize total defective items.
- To optimize that solution has no defective items.

based on above literature and consideration we motivated to develop a model for two cases first one is EMQ model which depends on regular production time considering constant demand. In this case, we optimized regular production time and total production cost. The second one is the EMQ model considering with disruption which depends on production time assuming constant demand. In this case, we have optimized disrupted production time and total production cost.

2 Assumptions & Notations

We have considered an imperfect quality EMQ model, in which production rate is constant and it is larger than the demand rate. The production process may randomly generate defective items at the rate ν_1 $(0 \leq \nu_1 \leq 1)$, the defective items are θ_1. We also assumed that the total defective items are found in two category, first one is reworkable and second one is non reworkable which is called scrap items. The rework process starts just after the end of regular production. Let ν_2 $(0 \leq \nu_2 \leq 1)$ denotes the quantity of defective items, which can not be rewokable during the rework process, and became scrap. After the rework process, only good items are delivered to the customer.
The used notations are given below.

P_1 : Production rate,

ϕ : Demand rate for the time horizon T,

ν_1 : Portion of defective items,

θ_1 : Production of defective items,

C : Production cost,

C_R : Rework cost,

P_2 : Rework production rate,

K_1 : Delivery cost per shipment,

θ_2 : Production rate of scrap items during the rework process

ν_2 : Production rate of scrap item,

h_1 : Holding cost per quantity per unit time,

C_C : Disposal cost per scrap items,

Figure 1: On hand inventory of perfect items

h_2 : Holding cost per reworkable items,

t_1 : Regular production time,

t_2 : Time required for reworking of defective items,

C_T : Delivery cost per items,

t_3 : Time to send finished Items,

n : Number of installments,

K : Setup cost per order,

T : Total horizon time time,

Q : Total produced quantity,

$I(t)$: On hand inventory of perfect quality items at time t,

$I_d(t)$: On hand inventory of defective items at time t,

$TC(t_1)$: Total production inventory delivery cost per cycle,

$TC_1(t_1)$: Total production inventory delivery cost per cycle for special case1,

δP : Change of production rate due to disruption,

t_d : Regular production time on disrupted production system,

t_1^P : Disrupted production time,

t_2^P : Rework time in disrupted production system,

$TC^*(t_d^P)$: Total production inventory delivery cost for disrupted case,

$TC_1^*(t_d^P)$: Total production inventory delivery cost per cycle for disruption case (special case).

3 Model for Imperfect Production with Reworkable and few Scrapable Items

In this model we consider a manufacturing process starts with a constant production rate at P_1, which is larger than the demand rate. Because of imperfect production of defective items produced at rate ν_1, then the total defective items are $\theta_1 = P_1\nu_1$. Defective items have reworked a rate of P_2 which is started after the end of regular production. The rework process randomly generate scrap items at a rate ν_2, then the total scrapable items are $\theta_2 = P_2\nu_2$. The finished products of good items delivered to costumer in equal n parts of interval time t_3. Let regular production time is t_1, rework production time of a defective item is t_2, delivery time of the finished product is t_3, on-hand inventory of regular production H and on-hand inventory of rework production H^*, then the production cycle length T can be written as

$$T = t_1 + t_2 + t_3 \tag{3.1}$$

let Q, be the total quantity including perfect items, imperfect items and scrap items.
Then

$$Q = P_1 t_1 \tag{3.2}$$

$$H = (P_1 - \theta_1)t_1 \tag{3.3}$$

$$H^* = (1 - \nu_1\nu_2)P_1 t_1 \tag{3.4}$$

$$t_2 = \left(\frac{\nu_1 P_1}{P_2}\right) t_1 \tag{3.5}$$

$$t_3 = T - t_1 - t_2 = \left(\frac{P(1-\nu_1\nu_2)}{\theta} - \frac{\nu_1 P_1}{P_2} - 1\right) t_1 \tag{3.6}$$

Total number of defective items at the time t_1 is $\theta_1 t_1$, then

$$\theta_1 t_1 = P_1\nu_1 t_1, \qquad where \;\; \nu_1 = \frac{\theta_1}{P_1} \tag{3.7}$$

Total number of scrap items at the length cycle T is $\nu_2\theta_1 t_1$, then

$$\nu_2\theta_1 t_1 = P_1\nu_1\nu_1 t_1, \qquad where \;\; \nu_2 = \frac{\theta_2}{P_2} \tag{3.8}$$

Thus, we have to find total production cost and delivery cost at the time t_1 (Using Appendix 1)

$$\begin{aligned} TC(t_1) =& CP_1 t_1 + K + C_R(\nu_1 P_1 t_1) + C_S(\nu_1\nu_2 P_1 t_1) + nK_1 \\ &+ C_T((1-\nu_1\nu_2)P_1 t_1) + h_2.\frac{P_2 t_2}{2} t_2 \\ &+ h_1\left(\frac{H+\theta_1 t_1}{2} t_1 + \frac{H+H^*}{2} t_2\right) + h_1\left(\frac{n-1}{2n}\right) h t_3. \end{aligned} \tag{3.9}$$

Total production inventory delivery cost at time t_1 is

$$\begin{aligned}TC(t_1) &= (K + nK_1) + (C + C_R\nu_1 + C_S\nu_1\nu_2 + C_T(1-\nu_1\nu_2))\, P_1 t_1 \\ &\quad + h_2 \frac{(\nu_1 P_1)^2}{2P_2} {t_1}^2 + h_1 \left(\frac{P_1}{2} + \frac{\nu_1 P1^2}{2P_2}(1-\nu_1\nu 2)\right) {t_1}^2 + h_1 \left(\frac{n-1}{2n}\right) \\ &\quad \left(\frac{{P_1}^2(1-\nu_1\nu_2)}{\phi} - (1-\nu_1\nu_2)P_1 - \frac{\nu_1(1-\nu_1\nu_2){P_1}^2}{P_2}\right) {t_1}^2\end{aligned}$$

The optimal inventory replenishment lot size can be obtained by minimizing the cost function at time t_1. So average production inventory delivery cost at time t_1 is

$$\begin{aligned}TC(t_1) = \frac{\phi}{P_1(1-\nu_1\nu_2)} &\Bigg(\frac{(K+nK_1)}{t_1} + \Big(C + C_R\nu_1 \\ &+ C_S\nu_1\nu_2 + C_T(1-\nu_1\nu_2)\Big) P_1 + h_2 \frac{(\nu_1 P_1)^2}{2P_2} t_1 \\ &+ h_1 \left(\frac{P_1}{2} + \frac{\nu_1 {P_1}^2}{2P_2}(1-\nu_1\nu_2)\right) t_1 \\ &+ h_1 \left(\frac{n-1}{2n}\right) \left(\frac{{P_1}^2(1-\nu_1\nu_2)}{\phi} - (1-\nu_1\nu_2)P_1 - \frac{\nu_1(1-\nu_1\nu_2){P_1}^2}{P_2}\right) t_1 \Bigg)\end{aligned} \tag{3.10}$$

We differentiate from eq. (13) with respect to t_1

$$\begin{aligned}\frac{TC(t_1)}{dt_1} = \frac{\phi}{P_1(1-\nu_1\nu_2)} &\Bigg(\frac{-(K+nK_1)}{{t_1}^2} + h_2 \frac{(\nu_1 P_1)^2}{2P_2} \\ &+ h_1 \left(\frac{P_1}{2} + \frac{\nu_1 {P_1}^2}{2P_2}(1-\nu_1\nu_2)\right) \\ &+ h_1 \left(\frac{n-1}{2n}\right) \left(\frac{{P_1}^2(1-\nu_1\nu_2)}{\phi} - (1-\nu_1\nu_2)P_1 - \frac{\nu_1(1-\nu_1\nu_2){P_1}^2}{P_2}\right) \Bigg)\end{aligned} \tag{3.11}$$

and

$$\frac{d^2TC(t_1)}{d{t_1}^2} = \frac{2\phi(K+nK_1)}{P(1-\nu_1\nu_2){t_1}^3} \geq 0. \quad where\ (1-\nu_1\nu_2) \geq 0. \tag{3.12}$$

The optimal time ${t_1}^*$ is given by

$$\begin{aligned}\frac{(K+nK_1)}{{t_1}^2} &= h_2 \frac{(\nu_1 P_1)^2}{2P_2} + h_1 \left(\frac{P_1}{2} + \frac{\nu_1 {P_1}^2}{2P_2}(1-\nu_1\nu_2)\right) \\ &+ h_1 \left(\frac{n-1}{2n}\right) \left(\frac{{P_1}^2(1-\nu_1\nu_2)}{\phi} - (1-\nu_1\nu_2)P - \frac{\nu_1(1-\nu_1\nu_2){P_1}^2}{P_2}\right)\end{aligned}$$

So, the optimal time ${t_1}^*$ is

$${t_1}^* = \sqrt{\frac{(K+nK_1)}{h_2 \frac{(\nu_1 P_1)^2}{2P_2} + h_1 \left(\frac{P_1}{2} + \frac{\nu_1 {P_1}^2}{2P_2}(1-\nu_1\nu_2)\right) + h_1 \left(\frac{n-1}{2n}\right) \left(\frac{{P_1}^2(1-\nu_1\nu_2)}{\phi} - (1-\nu_1\nu_2)P - \frac{\nu_1(1-\nu_1\nu_2){P_1}^2}{P_2}\right)}}$$

If $\nu_2 = 0$, therefore all imperfect items are reworkable.

Total production inventory and delivery cost per cycle at the time t_1 is

$$\begin{aligned} TC_1(t_1) =& CP_1t_1 + K + C_R(\nu_1 P_1 t_1) + nK_1 + C_T P_1 t_1 + \\ & h_2 \frac{P_2}{2}.{t_2}^2 + h_1 \left(\frac{H + \theta_1 t_1}{2} t_1 + \frac{H + H^*}{2} t_2 \right) + h_1 \left(\frac{n-1}{2n} \right) ht_3 \end{aligned} \tag{3.13}$$

$$\begin{aligned} TC_1(t_1) =& (K + nK_1) + (C + C_R\nu_1 + C_T)\, P_1 t_1 + h_2 \frac{(\nu_1 P_1)^2}{2P_2} {t_1}^2 \\ & + h_1 \left(\frac{P_1}{2} + \frac{\nu_1 {P_1}^2}{2P_2} \right) {t_1}^2 + h_1 \left(\frac{n-1}{2n} \right) \left(\frac{P^2}{\phi} - P_1 - \frac{\nu_1 {P_1}^2}{P_2} \right) {t_1}^2 \end{aligned} \tag{3.14}$$

The optimal inventory replenishment lot size can be obtained by minimizing the cost function at time t_1. So average production inventory and delivery cost at time t_1 is

$$\begin{aligned} TC_1(t_1) = \frac{\phi}{P_1} \Bigg[& \frac{(K + nK_1)}{t_1} + \Big(C + C_R\nu_1 + C_T \Big) P_1 \\ & + h_2 \frac{(\nu_1 P_1)^2}{2P_2} t_1 + h_1 \left[\frac{P_1}{2} + \frac{\nu_1 {P_1}^2}{2P_2} \right] t_1 \\ & + h_1 \left(\frac{n-1}{2n} \right) \left[\frac{{P_1}^2}{\phi} - P_1 - \frac{\nu_1 {P_1}^2}{P_2} \right] t_1 \Bigg] \end{aligned} \tag{3.15}$$

If $\nu_1 \geq 0$, then the total production inventory delivery cost $TC_1(t_1)$ is convex function in t_1, and optimal regular production regular production time t_1^* will be

$$ {t_1}^* = \sqrt{\frac{(K + nK_1)}{h_2 \frac{(\nu_1 P_1)^2}{2P_2} + h_1 \left(\frac{P_1}{2} + \frac{\nu_1 {P_1}^2}{2P_2} \right) + h_1 \left(\frac{n-1}{2n} \right) \left(\frac{{P_1}^2}{\phi} - P_1 - \frac{\nu_1 {P_1}^2}{P_2} \right)}} $$

Proof.

$$\begin{aligned} \frac{dTC_1(t_1)}{dt_1} =& \frac{\phi}{P_1} \Bigg[\frac{-(K + nK_1)}{{t_1}^2} + h_2 \frac{(\nu_1 P_1)^2}{2P_2} + h_1 \left(\frac{P_1}{2} + \frac{\nu_1 {P_1}^2}{2P_2} \right) \\ & + h_1 \left(\frac{n-1}{2n} \right) \left(\frac{{P_1}^2}{\phi} - P_1 - \frac{\nu_1 {P_1}^2}{P_2} \right) \Bigg] \end{aligned} \tag{3.16}$$

$$\frac{d^2 TC_1(t_1)}{d{t_1}^2} = \frac{2\phi(K + nK_1)}{P_1 {t_1}^3} \tag{3.17}$$

From eq. (14), the optimal time ${t_1}^*$ is given by

$$\frac{(K + nK_1)}{{t_1}^2} = h_2 \frac{(\nu_1 P_1)^2}{2P_2} + h_1 \left(\frac{P_1}{2} + \frac{\nu_1 {P_1}^2}{2P_2} \right) + h_1 \left(\frac{n-1}{2n} \right) \left(\frac{{P_1}^2}{\phi} - P_1 - \frac{\nu_1 {P_1}^2}{P_2} \right)$$

So, the optimal regular production time

$$ {t_1}^* = \sqrt{\frac{(K + nK_1)}{h_2 \frac{(\nu_1 P_1)^2}{2P_2} + h_1 \left(\frac{P_1}{2} + \frac{\nu_1 {P_1}^2}{2P_2} \right) + h_1 \left(\frac{n-1}{2n} \right) \left(\frac{{P_1}^2}{\phi} - P_1 - \frac{\nu_1 {P_1}^2}{P_2} \right)}} \tag{3.18}$$

Figure 2: Compression of inventory of finished items with and without disruption

4 Model for Imperfect Production with Rework and Disruption

In the real-life, most of the production system may be disrupted, due to uncertainty. We consider a production process starts with a constant production rate of (P_1), which is larger than the demand rate of ($P_1 > \phi$). We also considered rework and disruption time for imperfect production. Let t_d be the regular production time and after time disrupted system at time $t_1^{P_1}$ the production reduced by δP_1. After time $t_1^{P_2}$ the rework process starts with rate $(P_2$, during the rework process. We deliver the finished product of good items to customer in equal n parts in t_3 time interval. Let regular production time be t_1, disrupted production time be $t_1^{P_1}$, rework production time of defective items be $t_2^{P_1}$, and delivery time of finished product be t_3, on hand inventory levels H_1, H_2 and H_3 respectably, then the production cycle length T can be written as.

$$T = t_d + t_1^{P_1} + t_2^{P_1} + t_3 \tag{4.1}$$

let Q, be the total quantity including perfect items, imperfect items and scrap items.
Then

$$Q = Pt_d + (P_1 + \delta P_1)t_1^{P_1} \tag{4.2}$$

$$H_1 = (P_1 - \theta_1)t_d \tag{4.3}$$

$$H_2 = Q - \theta(t_d + t_1^{P_1}) \tag{4.4}$$

$$H_3 = \left(1 + (P_2 - \theta_1)\frac{\nu_1}{P_2}\right) Q - \theta_1(t_d + t_1^{P_1}) \tag{4.5}$$

$$t_2^P = \frac{\nu_1}{P_2}Q \tag{4.6}$$

$$t_3 = T - t_d - t_1^p - \frac{\nu_1}{P_2}Q \tag{4.7}$$

Suppose H^* is the total production inventory level without disruption and let H_3 be a total inventory level while system get disrupted. Then the production time t_1^P is

$$t_1^P = \frac{(1 - \nu_1\nu_2)(t_1 - t_d)P_1}{(1 + (1 - \nu_2)\nu_1)(P_1 + \delta P_1) - \nu_1 P_1} \tag{4.8}$$

Proof. Let the total production inventory level is same for both cases, then

$$H_3 = H^*$$

from eq. (26) and eq. (4)

$$(1 + (P_2 - \theta_1)\frac{\nu_1}{p_2})Q - \theta_1(t_d + {t_1}^P) = (1 - \nu_1\nu_2)P_1 t_1$$

$$(1+(1-\nu_2)\nu_1)\,P_1t_d+(1+(1-\nu_2)\nu_1)\,(P+\delta P)t_1^P-\nu_1P_1t_d-\nu_1P_1t_1^P=(1-\nu_1\nu_2)P_1t_1$$
$$((1+(1-\nu_2)\nu_1)\,(P_1+\delta P_1)-\nu_1P_1)\,t_1^P=(1-\nu_1\nu_2)P_1t_1-((1+(1-\nu_2)\nu_1)-\nu_1)P_1t_d$$

So, the production time after disruption

$$t_1^P=\frac{(1-\nu_1\nu_2)(t_1-t_d)P_1}{((1+(1-\nu_2)\nu_1)\,(P_1+\delta P_1)-\nu_1P_1)} \tag{4.9}$$

from eq. (27) the rework production time after disruption t_2^P is

$$t_2^P=\frac{\nu_1}{P_2}P_1t_1^P$$

$$t_2^P=\frac{(1-\nu_1\nu_2)(t_1-t_d)\nu_1{P_1}^2}{P_2\,((1+(1-\nu_2)\nu_1)\,(P_1+\delta P_1)-\nu_1P_1)}$$

The total production inventory and delivery cost at the time t_d^P is

$$TC^*(t_1^P)=\frac{\phi}{(1-\nu_1\nu_2)}\left(\frac{A}{(t_d+t_1^P)}+(B+D+E)P_1(t_d+t_1^P)\right) \tag{4.10}$$

where

$$A=\frac{K+nK_1}{P_1}$$
$$B=h_2\frac{{\nu_1}^2P_1}{2P_2}+h_1\left(\frac{1}{2}+\nu_1(1-\nu_1\nu_2)\frac{P_1}{2P_2}\right)$$
$$D=h_1\frac{n-1}{n}\left(\frac{P_1(1-\nu_1\nu_2)^2}{\phi}-(1-\nu_1\nu_2)-\frac{(1-\nu_1\nu_2)\nu_1P_1}{P_2}\right)$$
$$E=(C+C_R\nu_1+C_S\nu_1\nu_2+C_T(1-\nu_1\nu_2))$$

If the scrap items rate is $\nu_2=0$, i.e. all imperfect items are reworkable. Suppose H^* is the total production inventory level and let H_3 is a total inventory level often system gets disrupted. Then the production time t_1^P is

$$t_1^P=\frac{(t_1-t_d)P_1}{(1+\nu_1)(P_1+\delta P_1)-\nu_1P_1} \tag{4.11}$$

Proof. Let the total production inventory level is same for both cases, then

$$H_3=H^*$$

from eq. (26) and eq. (4)

$$(1+\nu_1)P_1t_d+(1+\nu_1)(P_1+\delta P_1)t_1^P-\nu_1P_1t_d+\nu_1P_1t_1^P=P_1t_1$$

$$(1+\nu_1)P_1t_d+(1+\nu_1)(P_1+\delta P_1)t_1^P-\nu_1P_1t_d+\nu_1P_1t_1^P=P_1t_1$$
$$P_1t_d+((1+\nu_1)(P_1+\delta P_1)-\nu_1P_1)t_1^P=P_1t_1$$

So the production time after disruption

$$t_1^P = \frac{(t_1 - t_d)P_1}{(1+\nu_1)(P_1 + \delta P_1) - \nu_1 P_1} \tag{4.12}$$

the rework production time after disruption t_2^P is

$$t_2^P = \frac{\nu_1 P_1}{P_2} t_1^P$$

$$t_2^P = \frac{(t_1 - t_d){P_1}^2}{((1+\nu_1)(P_1 + delta P_1) - \nu_1 P_1)P_2} \tag{4.13}$$

The total production inventory delivery cost at the time t_1^P is

$$TC^*(t_1^P) = \phi\left(\frac{A}{(t_d + t_1^P)} + (B + D + E)P_1(t_d + t_1^P)\right) \tag{4.14}$$

where

$$A = \frac{K + nK_1}{P_1}$$

$$B = h_2\frac{{\nu_1}^2 P_1}{2P_2} + h_1\left(\frac{1}{2} + \frac{\nu_1 P_1}{2P_2}\right)$$

$$D = h_1\frac{n-1}{n}\left(\frac{P_1}{\phi} - 1 - \frac{\nu_1 P_1}{P_2}\right)$$

$$E = (C + C_R\nu_1 + C_T)$$

5 Numerical Examples

5.1 Numerical Example for Case I (without disruption)

Assume that a manufacture produces items at the rate of 100 units per year and has a flat demand rate of 50 units per year. A portion $\nu_1 = 0.1$ is rate of defective items during the production time. A portion $\nu_2 = 0.1$ is the rate of scrap items in work process. Let production cost $C = 4$ unit per items, setup cost $K = 20000$ per production run, the fixed delivery cost $K_1 = 10000$ per shipment, delivery cost from shipment to customers $C_T = 10$ per items, unit rework cost $C_R = 4$, for each rework items, disposal cost $C_S = 2$, for each scrap items, holding cost $h_1 = 0.10$ per items per year, rework holding cost h_2 =0.02 per items reworks per unit time.

The optimal regular production time $t_1 = 15.1092$, and the rework production time t_2= 0.3021 are obtained. The production inventory delivery cost $TC(t_1) = 270761.9$ is calculated by using eq. (3.11) as well.

5.2 Numerical Example for Case II (with disruption)

Let the model be disrupted at the time t_d= 6, and let δP = -10 unit per year then by using eq. (4.18), eq. (4.19) and eq. (4.20) the optimal disrupted regular production time on ${t_1}^P$ = 10.1328, and the disrupted rework production time ${t_2}^P$= 0. are obtained. The production inventory delivery cost $TC^*({t_d}^P)$ = Rs. 75434.39. and the production inventory delivery cost for special case is $TC_1^* t_d^P$ = Rs. 75031.33 shown in Fig.(5).

If the defective rate increases then the regular production time without disruption (t_1) and with production (t_d^P) are decreases accordingly and rework production time without disruption (t_2) and with disruption (t_2^P) are increases sharply. Moreover high defective rate leads more production time for both with and without disruption. Due to this, the total cost increases accordingly as shown in table 1.

Table 1: Effect of defective rate (θ_1) on optimal policy with and without disruption for ($\theta_2 > 0$.)

	t_1	t_2	$TC(t_1)$	t_d^P	t_2^P	$TC^*(t_d^P)$
0.10	13.6712	2.7342	65562.36	11.1487	2.2297	75434.39
0.11	13.0402	2.8688	68386.87	10.37073	2.2815	78603.60
0.12	12.4881	2.9971	71101.11	9.69050	2.3257	81639.06
0.13	11.9999	3.1199	73718.90	9.08940	2.3632	84557.53

If the scrap rate (θ_2) increases, then the regular production time without disruption (t_1) and with disruption (t_d^P) increases as well as the rework production time without disruption (t_2) and with disruption (t_2^P) are increasing sharply. Moreover, high scrap rate leads to more production time for both with and without disruption. Due to this, the total cost increases accordingly shown in table 2.

Table 2: Effect of scrap item rate (θ_2) on optimal policy with and without disruption for ($\theta_1 > 0$.)

	t_1	t_2	$TC(t_1)$	t_d^P	t_2^P	$TC^*(t_d^P)$
0.10	13.6712	2.7342	65562.11	11.1487	2.2297	75434.39
0.11	13.6781	2.7356	65595.14	11.1576	2.2315	75475.05
0.12	13.6850	2.7370	65627.97	11.1664	2.3232	75515.78
0.13	13.6919	2.7383	65660.85	11.1664	2.2350	75556.58

Figure 3: Total cost with respect to regular time

If the defective rate increases then the regular production time without disruption (t_1) and with disruption (t_d^P) are increases accordingly and rework production time without disruption

Figure 4: Total cost with respect to disrupted time

(t_2) and with disruption (t_2^P) are increasing sharply. Moreover, high defective rate leads to more production time for both with and without disruption. Due to this, the total cost increases accordingly as shown in table 3.

Table 3: Effect of defective rate (θ_1) on optimal policy with and without disruption for ($\theta_2 = 0$.)

	t_1	t_2	$TC(t_1)$	t_d^P	t_2^P	$TC^*(t_d^P)$
0.10	13.6030	2.7206	65237.47	11.0610	2.2122	75031.33
0.11	12.9688	2.8531	68012.53	10.2786	2.2613	78137.62
0.12	12.4138	2.9793	70920.35	9.59430	2.3026	81106.71
0.13	11.9226	3.0999	73238.35	8.98910	2.3371	83955.43

6 Conclusion and Suggestions

This paper presents a economic production problem policy by considering imperfect production and rework. Furthermore, the model is developed in two situations (i) production model with disruption (ii) production model without disruption. We first derived the production inventory delivery cost function for two sub cases, first one is assuming defective and scrap items exists in the system, and second one is only defective items exist in the system. In the first case we optimized the regular production time, rework production time and total production cost.

In the second case, we derived the production inventory delivery cost function for EMQ model with disruption for two sub cases. In the first sub case we assumed defective and scrap items are existing in the system, and in the second sub cases only defective items are existing the system. We have optimized the disrupted production time, rework production time and total production cost. The sensitive analysis reveals that without disruption cost is higher than the production cost with disruption. It is suggestion for inventory manager that, to reduce the disruptions in the production system for earning more profit.

The model can be extended with variable production rate and price sensitive demand. Also we can incorporate multiple shipment and variable demand.

Appendix

Theorem .1 *If n, is the number of installment of fixed quantity of finished batch, which are delivered to costumer in a fixed time interval. Then the holding cost during the time interval t_3 is given by*

$$h_1\left(\frac{n-1}{2n}\right)H^*t_3 \tag{.1}$$

Proof. The inventory level of finished production system is shown fig.(1).
The part of this theorem is given by induction method.
For $n=1$, total holding costs in delivery time t_3 is zero,
For $n=2$, total holding costs in delivery time t_3 become

$$h_1\left(\frac{H^*}{2}\times\frac{t_3}{2}\right)=h_1\left(\frac{1}{2^2}\right)H^*t_3$$

For $n=3$, total holding costs in delivery time t_3 is

$$h_1\left(\frac{2H^*}{3}\times\frac{t_3}{3}+\frac{H^*}{3}\times\frac{t_3}{3}\right)=h_1\left(\frac{2+1}{3^2}\right)H^*t_3$$

For $n=4$, total holding costs in delivery time t_3 is

$$h_1\left(\frac{3H^*}{4}\times\frac{t_3}{4}+\frac{2H^*}{4}\times\frac{t_3}{4}+\frac{H^*}{4}\times\frac{t_3}{4}\right)=h_1\left(\frac{3+2+1}{4^2}\right)H^*t_3$$

Therefore, the following general term for total holding costs during delivery time t_3 can be obtained by

$$h_1\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n} i\right)H^*t_3=h_1\left(\frac{1}{n^2}\right)\left(\frac{n(n-1)}{2}\right)H^*t_3=h_1\left(\frac{n-1}{2n}\right)H^*t_3$$

Delivery cost can be formulated as the total delivery cost for n shipments in a cycle is

$$n\left(K_1+C_T\left(\frac{H^*}{n}\right)\right)=nK_1+C_T(1-\nu_1\nu_2)P_1t_1 \tag{.2}$$

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