AL - STRUCTURE ON F- RELATION L - FUZZY TOPOLOGICAL SYSTEMS

M.ANNALAKSHMI, N.SUGANTHAMEENA

V.H.N.Senthikumara Nadar College (Autonomous)
Virudhunagar - 626001.
Tamilnadu, India.
annalakshmi@vhnsnc.edu.in
suganthameena.n@vhnsnc.edu.in

Abstract. We introduced, Algebric structure on L - fuzzy topological system which is described in fuzzy relation . Moreover, the paper Algebric structure on F - relation L - fuzzy topological system provide with cloudless examples.

AMS Classification: 03F55, 03E72, 08A72

Key words : Fuzzy Topology, Fuzzy Relations, TM-Algebra

1. Introduction

Fuzzy concept introduced by L.A.Zadeh [25] at 1965 and developed the Fuzzy relation at 1971. Chang [18], Wong [24] , Lowen [22] and others developed the fuzzy topological spaces. In 2010, Tamilarasi and Manimegalai proposed a new type of algebras known as TM-algebras [23].
we introduced the concept in [1], Fuzzy Topological subsystem on a TM-algebra. We studied in [2] $L-$ Fuzzy Topological TM-system. We developed the concept in [3] $L-$ Fuzzy Topological TM-subsystem. we studied in [4], [5] Fuzzy Supratopological TM-system, Fuzzy $\alpha-$ supracontinuous functions. In this paper, discuss the notion of an AL - Structure on F-Relation L - Fuzzy Topological Systems and investigate some simple properties

2. Preliminaries

Definition 2.1. *Let X be a non-empty set. A mapping $\mu : X \to [0,1]$ is called a fuzzy set of X.*

Definition 2.2. *μ_1 and μ_2 are the fuzzy sets of a set X.*
Then $\mu_1 \subset \mu_2$ is defined by $\mu_1(x) \leq \mu_2(x)$ $forall$ $x \in X$

Definition 2.3. *μ is a fuzzy set of X. The complement of μ is denoted by μ' and defined by $\mu'(x) = 1 - \mu(x)$ $forall$ $x \in X$*

Definition 2.4. *μ_1 and μ_2 are the fuzzy sets of a set X.*
The union of fuzzy sets μ_1, μ_2 is :

$$(\mu_1 \cup \mu_2)(x) = MAX\{\mu_1(x), \mu_2(x)\} \quad forall\ x \in X.$$

The intersection of fuzzy sets μ_1, μ_2 is :

$$(\mu_1 \cap \mu_2)(x) = MIN\{\mu_1(x), \mu_2(x)\} \quad forall\ x \in X.$$

Definition 2.5. *A family T of fuzzy sets in X, the fuzzy topology which is satisfies the conditions as*

i) $\phi, X \in T$ ii) If $\mu, \sigma \in T$ then $\mu \cap \sigma \in T$ iii) If $\mu_i \in T$ for each $i \in I$ then $\cup_I \mu_i \in T$ where I is an indexing set.

Definition 2.6. *A non-empty set X with a constant 0 and a binary operation $*$ of TM-Algebra ($X, *, 0$) is satisfying the following conditions : i) $x*0 = x$ ii) $(x*y)*(x*z) = z*y$ $for\ all$ $x, y, z \in X$.*

Definition 2.7.

A fuzzy Subalgebra μ of a TM-Algebra ($X, *, 0$) if , for all $x, y \in X$,
$\mu(x * y) \geq min\{\mu(x)\ , \mu(y)\}$

Definition 2.8. *μ and σ are two fuzzy sets in a fuzzy topological space (X, T). σ is said to be an interior of μ if μ is a neighbourhood of σ and $\mu \supset \sigma$.*

Definition 2.9. *Fuzzy Relation*
Consider the cartesian product $A \times B = \{(x, y) : x \in A, y \in B\}$ where A and B in universal sets U and V correspondingly. A fuzzy relation on $A \times B$ denoted by R or R(x, y) is defined as the set $R = \{(x, y), \mu_R(x, y) : (x, y) \in A \times B, \mu_R(x, y) \in [0, 1]\}$

Definition 2.10. *The union of fuzzy relations R_1 and R_2 is denoted by $R_1 \cup R_2$ is defined by $\mu_{R_1 \cup R_2}(x, y) = min\{\mu_{R_1}(x, y), \mu_{R_2}(x, y)\}$, $(x, y) \in A \times B$*
The intersection of fuzzy relations R_1 and R_2 is denoted by $R_1 \cap R_2$ is defined by $\mu_{R_1 \cap R_2}(x, y) = max\{\mu_{R_1}(x, y), \mu_{R_2}(x, y)\}$, $(x, y) \in A \times B$

3. AL - STRUCTURE ON F- RELATION L - FUZZY TOPOLOGICAL SYSTEMS

Definition 3.1.
AL - Structure on F - Relation L - Fuzzy Topological Systems:
X, Y are TM-Algebras. $R_1(x, y), R_2(x, y)$ are the fuzzy relations of X, Y. AL - structrue on F - relation L - Fuzzy Topological System is a family T of L - fuzzy subalgebras in (X, Y, T) which is satisfies the conditions :

i) $\phi, X \in T$ *ii) If* $\mu(x,y), \sigma(x,y) \in T$ *then* $\mu(x,y) \cap \sigma(x,y) \in T$ *iii) If* $\mu_i(x,y) \in T$ *for each* $i \in I$ *then* $\cup_I \mu_i(x,y) \in T$ *where I is an indexing L -subalgebra.*

Example 3.2.
The set $X = \{0,1,2\}$, $Y = \{0,1,2\}$ with the cayley table

$*$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

The fuzzy relations $R_1(x,y)$ and $R_2(x,y)$ are on the sets X , Y is ,

X,Y	0	1	2
0	$(0,0)$	$(0,1)$	$(0,2)$
1	$(1,0)$	$(1,1)$	$(1,2)$
2	$(2,0)$	$(2,1)$	$(2,2)$

The F - relations L - subalgebras $\mu_i : X \to [0,1], i = 1,2,3$ $\nu_i : Y \to [0,1], i = 1,2,3$ are

$$\mu_1(x,y) = \begin{cases} t_5 & if\ (0,0) \\ t_3 & if\ (0,1) \\ t_1 & if\ (0,2) \end{cases} \quad \mu_2(x,y) = \begin{cases} t_8 & if\ (1,0) \\ t_6 & if\ (1,1) \\ t_7 & if\ (1,2) \end{cases}$$

$$\mu_3(x,y) = \begin{cases} t_6 & if\ (2,0) \\ t_3 & if\ (2,1) \\ t_1 & if\ (2,2) \end{cases} \quad \nu_1(x,y) = \begin{cases} t_6 & if\ (0,0) \\ t_4 & if\ (0,1) \\ t_2 & if\ (0,2) \end{cases}$$

$$\nu_2(x,y) = \begin{cases} t_4 & if\ (1,0) \\ t_2 & if\ (1,1) \\ t_1 & if\ (1,2) \end{cases} \quad \nu_3(x,y) = \begin{cases} t_5 & if\ (2,0) \\ t_4 & if\ (2,2) \\ t_2 & if\ (2,3) \end{cases}$$

A family $T = \{\phi, X, \mu_1, \mu_2, \mu_3, \nu_1, \nu_2, \nu_3\}$ which is satisfying the AL - structure on F - relation L - fuzzy topological system (X, Y, T)

Definition 3.3.
X, Y are TM-Algebras. $R_1(x,y), R_2(x,y)$ are the fuzzy relations of X, Y. AL - structrue on F - relation L - Fuzzy Topological System (X, Y, T). F - relation L - fuzzy subalgebra $\mathcal{N}$ in L - fuzzy topological TM-system, is an F - relation L - fuzzy neighbourhood of an F - relation L - fuzzy subalgebra $\mathcal{M}$ if there exist an T-open F- relation L - fuzzy subalgebra $\mathcal{D}$ such that $\mathcal{M} \subset \mathcal{D} \subset \mathcal{N}$
ie $\mathcal{M} \leq \mathcal{D} \leq \mathcal{N}$ for all $x \in X$, $y \in Y$

Example 3.4.
$\mu_i(x,y)$, $i = 1,2,3$, $\nu_i(x,y)$, $i = 1,2,3$ are F - relation subalgebras of the L - fuzzy topological TM-system given in example 3.4
A family $T = \{\phi, X, \mu_1, \mu_2, \mu_3, \nu_1, \nu_2, \nu_3\}$ is an AL - structrue on F - relation L - Fuzzy

Topological System (X, Y, T).
$\mu_2(x, y)$ is F - relation L - fuzzy neighbourhood of an F - relation L - fuzzy subalgebra $\mu_1(x, y)$ for $\mu_1(x, y) \leq \nu_2(x, y) \leq \mu_2(x, y)$

Definition 3.5.
X, Y are TM-Algebras. $R_1(x, y), R_2(x, y)$ are the fuzzy relations of X, Y. AL - structrue on F - relation L - Fuzzy Topological System (X, Y, T). $\mu(x, y)$ is F - relation L - fuzzy subalgebra in (X, Y, T). The collection of F - relation L - fuzzy neighbourhood of $\mu(x, y)$ is the set $\mathfrak{U}(x, y)$ is said to be a F - relation L - fuzzy neighbourhood system of $\mu(x, y)$.

Definition 3.6.
X, Y are TM-Algebras. $R_1(x, y), R_2(x, y)$ are the fuzzy relations of X, Y. AL - structrue on F - relation L - Fuzzy Topological System (X, Y, T). The F- relation L - fuzzy interior of $\mu(x, y)$ is the union of all F - relation open L - fuzzy subalgebras contained in $\mu(x, y)$ and it is denoted by $(\mu)^{\circ}(x, y)$.
That is $(\mu)^{\circ}(x, y) = \cup\{\mu(x, y) : \mu(x, y) \subseteq \mu(x, y), \mu(x, y) \in (X, Y, T)\}$

Example 3.7.
$\mu_i(x, y)$, $i = 1, 2, 3$ $\nu_i(x, y)$, $i = 1, 2, 3$ are F - relation L - fuzzy subalgebras of the TM-system given in example 3.4
A family $T = \{\phi, X, \mu_1, \mu_2, \mu_3, \nu_1, \nu_2, \nu_3\}$ is an AL - structrue on F - relation L - Fuzzy Topological System (X, Y, T).
The F - relation L - fuzzy interior of $(\mu_3)^{\circ}(x, y) = \cup\{\mu_1, \nu_2, \nu_4\} = \mu_1(x, y)$

Theorem 3.8.
X, Y are TM-Algebras. $R_1(x, y), R_2(x, y)$ are the fuzzy relations of X, Y. AL - structrue on F - relation L - Fuzzy Topological System (X, Y, T). F - relation L - fuzzy subalgebra $\mathcal{U}(x, y)$ is open in (X, Y, T) if and only if for each F - relation L - fuzzy subalgebra $\mathcal{V}(x, y)$ contained in $\mathcal{U}(x, y)$, $\mathcal{U}(x, y)$ is F- relation L - fuzzy neighbourhood of $\mathcal{V}(x, y)$.
Proof:
AL - structrue on F - relation L - Fuzzy Topological System (X, Y, T). The F - relation L - fuzzy subalgebra $\mathcal{U}(x, y)$ is open in (X, Y, T) .
$\mathcal{V}(x, y)$ is any F - relation L - fuzzy subalgebra contained in $\mathcal{U}(x, y)$. Since $\mathcal{U}(x, y)$ is open, and $\mathcal{V}(x, y) \subset \mathcal{U}(x, y)$, $\mathcal{V}(x, y) \subset \mathcal{U}(x, y) \subset \mathcal{U}(x, y)$
$\therefore$ $\mathcal{U}(x, y)$ is F - relation L - fuzzy neighbourhood of $\mathcal{V}(x, y)$.
Conversely , for each F - relation L - fuzzy subalgebra $\mathcal{V}(x, y)$ contained in $\mathcal{U}(x, y)$, $\mathcal{U}(x, y)$ is L - fuzzy neighbourhood of $\mathcal{V}(x, y)$.
for $\mathcal{U}(x, y) \subset \mathcal{U}(x, y)$, by our assumption, $\mathcal{U}(x, y)$ is F - relation L - fuzzy neighbourhood of $\mathcal{U}(x, y)$.
Hence there exits an open F - relation L - fuzzy subalgebra $\mathcal{O}(x, y)$ such that $\mathcal{U}(x, y) \subset \mathcal{O}(x, y) \subset \mathcal{U}(x, y)$
Hence $\mathcal{U}(x, y) = \mathcal{O}(x, y)$ and $\mathcal{U}(x, y)$ is open in (X, Y, T) .

Theorem 3.9.

X, Y are TM-Algebras. $R_1(x,y), R_2(x,y)$ are the fuzzy relations of X, Y. AL - structrue on F - relation L - Fuzzy Topological System (X,Y,T). $\mu(x,y)$ is F - relation L - fuzzy subalgebra in (X,Y,T). $\mathfrak{U}(x,y)$ is F - relation L - fuzzy neighbourhood system of F - relation L - fuzzy subalgebra $\mu(x,y)$. then

(1) *The finite intersections of F - relation L - fuzzy subalgebras of $\mathfrak{U}(x,y)$ belong to $\mathfrak{U}(x,y)$*
(2) *F - relation L - fuzzy subalgebra of (X,Y,T) which contain a F - relation L - fuzzy subalgebra of $\mathfrak{U}(x,y)$ belong to $\mathfrak{U}(x,y)$*

Proof:

(1) *AL - structrue on F - relation L - Fuzzy Topological System (X,Y,T).*
$\mu(x,y)$ is F - relation L - fuzzy subalgebra in (X,Y,T). $\mathfrak{U}(x,y)$ is F - relation L - fuzzy neighbourhood system of $\mu(x,y)$.
The F- relation L - fuzzy subalgebras $g(x,y), h(x,y) \in \mathfrak{U}(x,y)$. Hence $g(x,y)$ and $h(x,y)$ are F - relation L -fuzzy neighbourhood of $\mu(x,y)$.
Thus there exits open F - relation L - fuzzy subalgebras $g_0(x,y)$ and $h_0(x,y)$ Such that $\mu(x,y) \subset g_0(x,y) \subset g(x,y)$ and $\mu(x,y) \subset h_0(x,y) \subset h(x,y)$ respectively.
Hence $\mu(x,y) \subset g_0(x,y) \cap h_0(x,y) \subset g(x,y) \cap h(x,y)$
$\Rightarrow$ $g(x,y) \cap h(x,y)$ is F - relation L -fuzzy neighbourhood of $\mu(x,y)$.
Hence the intersection of two F - relation L - fuzzy subalgebras of $\mathfrak{U}(x,y)$ is again a F- relation L - fuzzy subalgebra of $\mathfrak{U}(x,y)$
Hence the intersection of any finite number of F- relation L - fuzzy subalgebras of $\mathfrak{U}(x,y)$ is again a F - relation L - fuzzy subalgebra of $\mathfrak{U}(x,y)$

(2) *$g(x,y)$ is F - relation L - fuzzy subalgebra that contains a F- relation L - fuzzy subalgebra of $\mathfrak{U}(x,y)$ say $u(x,y)$.*
Hence $g(x,y)$ contains a F- relation L - fuzzy neighbourhood $u(x,y)$ of $\mu(x,y)$.
That is $u(x,y) \subset g(x,y)$, $u(x,y) \in \mathfrak{U}(x,y)$
since $u(x,y)$ is a F - relation L - fuzzy neighbourhood of $\mu(x,y)$ then by definition there exists a open F- relation L - fuzzy subalgebra $o(x,y)$.
$\Rightarrow \mu(x,y) \subset o(x,y) \subset u(x,y) \subset g(x,y)$.
Therefore $\mu(x,y) \subset o(x,y) \subset g(x,y)$
$\Rightarrow$ $g(x,y)$ is F - relation L- fuzzy neighbourhood of $\mu(x,y)$.
$\therefore$ $g(x,y) \in \mathfrak{U}(x,y)$

Theorem 3.10.

X, Y are TM-Algebras. $R_1(x,y), R_2(x,y)$ are the fuzzy relations of X, Y. AL - structrue on F - relation L - Fuzzy Topological System (X,Y,T). $\mu(x,y)$ is a F - relation L - fuzzy subalgebra in (X,Y,T).

(1) *$\nu^\circ(x,y)$ is open in (X,Y,T) and is the largest open F - relation L - fuzzy subalgebra contained in $\mu(x,y)$.*

(2) *F - relation L - fuzzy subalgebra* $\mu(x,y)$ *is open in* (X,Y,T) *if and only if* $\mu(x,y)=\nu^{\circ}(x,y)$

Proof:

(1) *AL - structrue on F - relation L - Fuzzy Topological System* (X,Y,T). $\mu(x,y)$ *is F - relation L - fuzzy subalgebra in* (X,Y,T).
Then by definition of F - relation L - fuzzy interior , $\sigma^{\circ}(x,y)$ *is again F - relation interior L - subalgebra of* $\mu(x,y)$.
Hence there exist an T- open F - relation L - fuzzy subalgebra $o(x,y)$ *such that* $\nu^{\circ}(x,y)\subset o(x,y)\subset \mu(x,y)$.
But $o(x,y)$ *is F - relation L - fuzzy interior L - fuzzy subalgebra of* $\mu(x,y), o(x,y)\subset \nu^{\circ}(x,y)$ *Hence* $\nu^{\circ}(x,y)=o(x,y)$.
Thus $\nu^{\circ}(x,y)$ *is open in* (X,Y,T) *and is the largest open F - relation L - fuzzy subalgebra contained in* $\mu(x,y)$.

(2) *Suppose F - relation L - fuzzy subalgebra* $\mu(x,y)$ *is open.*
If $\mu(x,y)$ *is open , then* $\mu(x,y)\subset\nu^{\circ}(x,y)$ *and* $\nu^{\circ}(x,y)$ *is F - relation L - fuzzy interior of* $\mu(x,y)$.
Hence $\mu(x,y)=\nu^{\circ}(x,y)$
Conversely , Suppose $\mu(x,y)=\nu^{\circ}(x,y)$
Hence by definition of F - relation L- fuzzy interior, the union of all F - relation L- fuzzy interior of $\mu(x,y)$ *is called the interior of* $\mu(x,y)$ *and is denoted by* $\nu^{\circ}(x,y)$.
$\therefore$ $\mu(x,y)$ *is a F - relation L - fuzzy neighbourhood of* $\nu^{\circ}(x,y)$.
Therefore F - relation L - fuzzy subalgebra $\mu(x,y)$ *is open in* (X,Y,T) .

References

[1] Annalakshmi.M,Chandramouleeswaran.M, Fuzzy Topological subsystem on a TM-algebra, Intetnational Journal of Pure and Applied Mathematics,Vol.94,No.3 2014, 439-449

[2] Annalakshmi.M,Chandramouleeswaran.M, On $L-$ Fuzzy Topological TM-system, Intetnational Journal of Mathematical Science and Engineering Applications, Vol.8,No.IV (July 2014) PP. 135 -145.

[3] Annalakshmi.M ,Chandramouleeswaran.M, On $L-$ Fuzzy Topological TM- subsystem, Italian Journal of Pure and Applied Mathematics - N. 33 - 2014 (359-368)

[4] Annalakshmi.M, Chandramouleeswaran.M, Fuzzy supratopological TM-system , Intetnational Journal of Pure and Applied Mathematics,Vol.98,No.5 2015, 55-61

[5] Annalakshmi.M, Chandramouleeswaran.M, Fuzzy and $L-$ Fuzzy compactness in a Fuzzy Topological TM-system , Global Journal of Pure and Applied Mathematics,Vol11, No.1 2015, PP491-498

[6] Annalakshmi.M, Chandramouleeswaran.M, Fuzzy $\alpha-$ supracontiuous functions, Intetnational Journal of Mathematical Archive -6(5),2015,116-119.

[7] Annalakshmi.M, Chandramouleeswaran.M, Interior and Closure of Fuzzy open sets in a Fuzzy Topological TM-system, Glopal Journal of Pure and Applied Mathematics,Vol.11,No.5 (2015), PP 3157-3164.

[8] Annalakshmi.M, Chandramouleeswaran.M, Fuzzy Connected on Fuzzy Topological TM-system, International Journal of Pure and Applied Mathematics- Vol-112, No.5, pp.93-102

[9] Annalakshmi.M, Chandramouleeswaran.M Fuzzy Supratopological TM-Subsystem, Mathematical Sciences International Research Journal, Vol 6, Spl Issue Dec 2017 , PP 243-247

[10] Annalakshmi.M, Chandramouleeswaran.M An L- Fuzzy $\alpha-$ Supratopological TM-Subsystem, International Journal of Computer Science, ISSN:2348-6600

[11] Annalakshmi.M, Chandramouleeswaran.M,An L - Fuzzy Connected on L - Fuzzy Topological TM-system, International Journal of Mathematical Archive- Vol-5, No.5, May 2018 pp.1-6

[12] Annalakshmi.M, An L - Fuzzy $\alpha-$ supracontiuous in $\alpha-$ supratopological TM-System, International Journal of Mathematical Archieve-10(11), 2019 ISSN 2229-5046 PP 1- 6

[13] Annalakshmi.M, An L - Fuzzy supratopological TM-System, International Journal of Computer Science, Jan 2018 ISSN 2348-6600 PP 410- 416

[14] Annalakshmi.M, L-Fuzzy connected on L-Fuzzy Supratopological TM-Systems, Recent advances in Arts,Science and social sciences, ISBN:978-81-951746-3-8

[15] Annalakshmi.M, An Intuitionistic L-Fuzzy topological TM-systems, Journal of Information and Computational Science - ISSN 1548-7741, vol.12,Issue.1-2022,pg 26-33.

[16] Annalakshmi.M, An Intuitionistic Fuzzy Topological TM-Subsystems, International Journal of Research and Analytical Reviews, Vol 9, Issue 1,660-666

[17] Annalakshmi.M, An Intuitionistic L-Fuzzy Topological TM-Subsystems, International Journal of All Research Education and Scientific Methods, Vol 10,Issue2

[18] Chang C.L., Fuzzy Topological Spaces, Journal of Mathematical Analysis and Applications, Vol.24, (1968), pp. 182-190.

[19] Gyanendra kumar panda, On the theory of fuzzy relations, Technical research organisation ,India, ISSN:2395-7786,vol 2,Issue4.

[20] Imai.Y and Iseki.K., On axiom systems of Propositional Calculi , XVI Proc. Japan Acad. Ser A. Math. Sci., 42 (1966), 19-22.

[21] Iseki. K. On BCI-algebras, Math. Seminar Notes.,11(1980), pp.313-320.

[22] Lowen.R, Fuzzy topological spaces and fuzzy compactness,J.Math.Anal.Appl.56(1976),621-633.

[23] Tamilarasi.A and Megalai.K , $TM-$ algebra an introduction, CASCT .,(2010).

[24] Wong.C.K ,Fuzzy Topology:Product and quotient theorems,J.Math. Anal.Appl.45(1974),512-521.

[25] Zadeh.L.A , Fuzzy sets, Inform.control 8 (1965),338-353

M.Annalakshmi, N.Sugantha Meena, V.H.N.Senthikumara Nadar College (Autonomous) , Virudhunagar - 626001. India.