Semi Analytic-Numerical Solution of Fingero - Imbibition Phenomenon in Heterogenous Porous Medium with Magnetic Field Effect

**Amit K Parikh**

*Professor & Principal,*

*Mehsana Urban Institute of Sciences, Ganpat University,Kherva-384012, Mehsana,Gujarat, India*

**Aruna Sharma**

*Research Scholar,*

*Department of Mathematics, Mehsana Urban Institute of Sciences, Ganpat University, Kherva-384012, Mehsana, Gujarat, India.*

ABSTRACT

The present paper studies the magnetic field effect on Fingero- Imbibition phenomenon in a double phase flow in a heterogenous porous media. Fingero- Imbibition Phenomenon arises due to the simultaneous occurrence of two important phenomena namely Imbibition and Fingering. Imbibition is the displacement of a non- wetting fluid by a wetting fluid in a porous media, where oil is the non-wetting fluid and water is considered as the wetting fluid. Fingering arises when a fluid with higher viscosity is displaced by a fluid with lower viscosity due to which instead of regular displacement of the whole front, protuberances occur which shoot through the porous medium at a relatively very high speed, resulting in fingers being developed. Fingero-Imbibition phenomenon has tremendous importance in secondary oil recovery process. The mathematical formulation of this phenomenon leads to a highly non-linear partial differential equation which is solved using the Multistep Hybrid Differential Transform Finite Difference Method. In the Semi Analytic-Numerical Multistep Hybrid Differential Transform Finite Difference Method, the solution is in the form of an infinite series. The solution to this equation enables the measurement of the saturation of the injected water in a double phase flow at different distances and time and the effect of magnetic field on the saturation is also analyzed.

The Multistep Hybrid Differential Transform Finite Difference Method is a combination of the Multistep Differential Transform Method and Finite Difference Method. The flexibility of the Differential Transform Method is integrated with the efficiency of the Finite Difference Method. A straightforward iterative approach is employed to acquire the numerical solution, which speeds up computation compared to the conventional Differential Transform Method. This approach has been discovered to be reliable and effective. By making all calculations simpler, it lessens computing work. In this method the Multistep Differential Transform Method (MDTM) and Finite Difference Method (FDM) have been used to achieve the solution for large values of time. Using MATLAB, the numerical solution and graphical representation were obtained. The results obtained were compared with the existing results and found to be in close agreement.

Keywords— Fingering, Imbibition, Hybrid Differential transform Method, Double phase flow, Heterogenous porous media,magenetic field

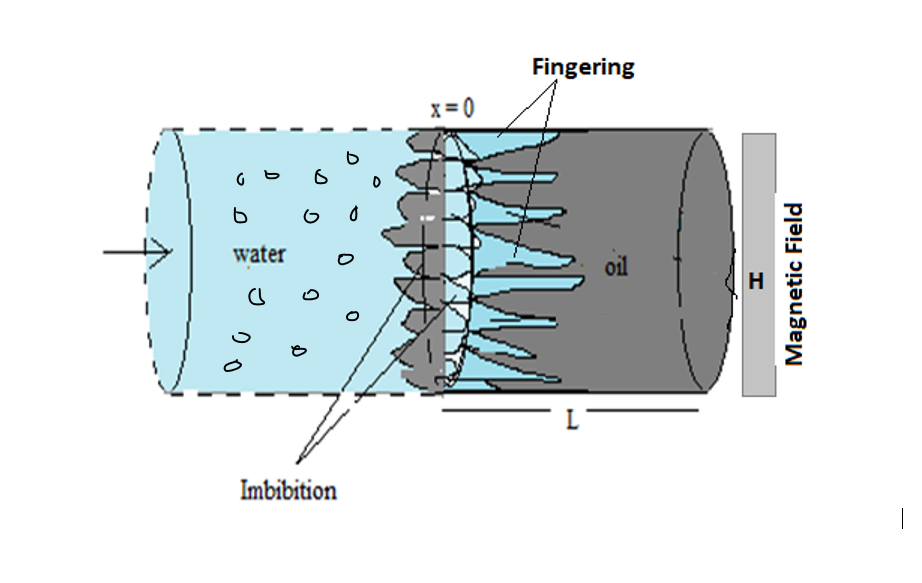
1. **INTRODUCTION**

The primary aim here is to study the effect of magnetic field on Fingero- Imbibition phenomenon in heterogenous porous media. When a porous media filled with a nonwetting fluid (oil) comes into contact with a wetting fluid (water), there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the nonwetting fluid out of the medium. This is referred to as the imbibition phenomena that arises due to differences in fluid wetting abilities. Again, instead of a normal displacement of the entire front when a porous media filled with one fluid is displaced by another fluid of lower viscosity, protuberances occur which shoot through the porous medium at a considerably high speed and give rise to fingers. Fingero-Imbibition Phenomenon is the result of the simultaneous occurrence of fingering and imbibition. This phenomenon is observed in many engineering disciplines, including petroleum technology, soil mechanics, agricultural engineering, hydrology, etc. Many researchers are also investigating this phenomenon.

Shah and Verma [5] studied this phenomenon in a heterogenous porous medium with magnetic field effect. Desai [21] has used the similarity transformation. Composite expansion of finger imbibition phenomenon in heterogenous porous medium was analyzed by Mehta and Verma [3]. Fingero-imbibition in double phase flow through heterogenous porous media was studied by Patel, K. R., Mehta, M. N. and Patel, T. R., [4]. Parikh, A. K., Mehta, M. N. and Pradhan, V. H., [10], discussed the fingero-imbibition phenomenon in homogeneous porous medium with magnetic field effect in vertical downward direction.

The main purpose of present paper is to obtain the saturation of injected water at a distance ‘x’ and time ‘t’and analyse the effect of magnetic field on the saturation for Fingero-Imbibition phenomenon in heterogenous porous media. The mathematical formulation gives rise to a non-linear partial differential equation which was solved using the Multistep Hybrid Differential Transform Finite Difference Method with appropriate initial and boundary conditions.

1. **Statement of Problem**



Consider that there is uniform water with magnetic particles injected in an heterogenous porous medium which is fully saturated with native fluid oil(o). Assume that the injected water is conductive and oil is nonconductive. In secondary oil recovery process, the simultaneous occurrence of imbibition and fingering phenomenon for preferentially wetting phase and a less viscous phase leads to this Fingero- imbibition Phenomenon. Rock properties such as porosity and permeability vary from place to place in a heterogeneous porous media. In a heterogeneous porous medium, porosity is considered to be a function of space ''. The size and shape of the fingers vary, but we assume they are rectangles and consider just the average cross- sectional area filled by the fingers, ignoring the size and shape of the individual fingers. The effect of variable magnetic field is to increase the velocity of injected water by a gradient ,where is the permeability of magnetic field

The saturation (of injected water is defined as the average cross-sectional area occupied by the injected water at a distance and time’’. The size and shape may vary but the average cross-sectional area filled by fingers is considered.

1. **Mathematical Formulation**

Assuming Darcy’s Law [14,15] to be valid, the velocity of injected water and velocity of oil can be expressed as

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

The equation of continuity of injected water is

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

where,

|  |  |  |
| --- | --- | --- |
| ∅= | | represents the porosity of the heterogenous medium |
|  | Variable permeability in | | |
|  | relative permeability of water | | |
|  | relative permeability of oil | | |
|  | pressure of water (injected fluid) | | |
|  | pressure of oil (native fluid) | | |
|  | seepage velocity of water (m /s ) | | |
|  | seepage velocity of oil (m /s ) | | |
|  | constant kinematic viscosity of water | | |
|  | constant kinematic viscosity of oil | | |
|  | porosity of a medium | | |

The sum of the velocities of the injected water and native oil is zero in counter current imbibition Phenomenon [25].Therefore,

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

From (1) and(2) and (4) in (5) we get,

|  |  |  |
| --- | --- | --- |
|  |  |  |

Substitute for ,we get,

|  |  |  |
| --- | --- | --- |
|  |  |  |

Simplifying we get,

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

Simplifying ,

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

According to Scheidegger [14] it is assumed ,

(7)

Substitute equation (6) and (7) in equation (1) ,we get,

Therefore,

(8)

|  |  |  |
| --- | --- | --- |
|  |  |  |

Substituting in equation (3) we get,

|  |  |  |
| --- | --- | --- |
|  | Assuming A The capillary pressure is considered as a linear function [19]  where,is a constant. | (9)  (10) |

Due to Scheidegger and Johnson [1], the standard relationship between phase saturation and relative permeability is considered as,

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

where,is a constant.

For uniform heterogenous porous medium we assume the porosity and permeability are functions of only [2]

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | (12) |

where .

Since , cannot exceed unity we assume ,

For simplicity we consider [12]

|  |  |  |
| --- | --- | --- |
|  |  | (13) |

where is a constant.

Considering the magnetic field only in the ‘’ direction, we write H as [4,22]

H=

Where is a constant parameter and ‘n’ is an integer. Using the value of H for n=1 in equation (9) with equations (10),(11), and (13), we get,

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

|  |  |  |
| --- | --- | --- |
|  |  | (15) |

Using dimensionless variables,

|  |  |  |
| --- | --- | --- |
|  |  | (16) |

Since,

(neglecting higher order terms in )

Therefore,

Substituting equation (16) simplifies to,

|  |  |  |
| --- | --- | --- |
|  | where A=, B= and | (17) |

(17) is the non-linear partial differential equation for the finger-imbibition phenomenon in the heterogenous porous medium and its solution gives the saturation of injected water at time ‘’ and distance’’.

Assuming initial and boundary conditions as follows[9]:

Initial conditions : , 0

Boundary conditions : , T

= , T

**IV. Problem Solution**

Consider Fingero-Imbibition phenomenon equation

|  |  |  |
| --- | --- | --- |
|  |  |  |

The governing equation is a nonlinear partial differential equation.

Therefore, the initial condition is ,

|  |  |  |
| --- | --- | --- |
|  | , , | (18) |

Taking appropriate boundary conditions as

|  |  |  |
| --- | --- | --- |
|  | , T  = , T | (19) |
|  |  |  |

(17) is solved using the Hybrid Differential Transform and the Finite Difference Method (HDTFDM) for small values of “T” and Multistep Hybrid Differential Transform Method for large values of ‘T’.

**SOLUTION BY HYBRID DIFFERENTIAL TRANSFORM FINITE DIFFERENCE METHOD (HDTFDM)**

**A. Methodology**

The Hybrid Differential Transform and Finite Difference Method (HDTFDM) is used to solve the non-linear partial differential equation [17]. The Differential Transform Method (DTM) and the Finite Difference Method (FDM) are combined in this method.

The spatial variables are approximated using the Finite Difference Method, while the time variable is approximated using the Differential Transform Method.

For the solution of linear and nonlinear differential equations in electrical circuit analysis, Zhou [25] proposed the DTM. Yu, L.T., Chen, C.K [20] used the hybrid method.

For the 'T' variable, the differential transform is used, and on the 'X variable, the finite difference method is used. The hybrid method is efficient in solving linear and nonlinear partial differential equations as it is found to converge rapidly with a few iterations.

**B. Preliminaries**

The Differential Transform of the kth derivative of applied to the ‘t’ variable is given as

|  |  |  |
| --- | --- | --- |
|  |  | (20) |

The inverse Transformation of is given as

|  |  |  |
| --- | --- | --- |
|  |  | (21) |

where in lower case letters represents the original function and in upper case letters represents transformed function.

|  |  |  |
| --- | --- | --- |
|  |  | (22) |

where i  ­i

The finite difference step interval is denoted by h.

The theorems stated below follow from [25] and [26]

**Theorem 1**.

**Theorem 2.**

**Theorem 3**.=

**Theorem 4**.=

**Theorem 5.** =

**Theorem 6.**

**Theorem 7**.

**C. Numerical Solution**

The Hybrid Differential Transform and Finite Difference Method (HDTFDM) and theorems given above are applied to Equation (17)

|  |  |  |
| --- | --- | --- |
|  | subject to  Initial Conditions: , ,  Boundary Conditions: , T  = , T |  |

Taking

Applying ,Differential Transformation to the ‘T’ variable and finite difference to the ‘X’ variable and using theorems given above, we have,

|  |  |  |  |
| --- | --- | --- | --- |
|  | = | | (29) |
|  | | |
| where is the original function and,­i  i­i transformed function   |  |  |  | | --- | --- | --- | |  | Initial Conditions: ,  Boundary Conditions: |  | | | |
|  | | |
|  | | |
| where N is the no. of spatial segments.  Substituting in (17) we get, according to the hybrid method, the following recurrence  relation,  (23)  where for differential transform coefficients are  obtained. The approximate solutions for various values ‘X’ and ‘T’ are found using the inverse  transformation,   |  |  |  | | --- | --- | --- | |  |  | (24) |   i=0,   |  |  |  | | --- | --- | --- | |  |  | (25) |   where I = mesh points for  **V**. **Results and Discussion**  The numerical values of the saturation obtained from equation (24) for various distances ‘’ at fixed time =0.001,0.002,0.003,0.004,0.005,0.006,0.007,0.008,0.009,.01 are obtained by using MATLAB and presented in **Table 1** below.  **Table 1: Saturation (,) for different values of distance’ ’ and time ‘’ by Hybrid Differential Transform and Finite Difference method** | | |
| |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | **X\T** | **0.001** | **0.002** | **0.003** | **0.004** | **0.005** | **0.006** | **0.007** | **0.008** | **0.009** | **0.01** | | **0** | **0.0002** | **0.0004** | **0.0006** | **0.0008** | **0.001** | **0.0012** | **0.0014** | **0.0016** | **0.0018** | **0.002** | | **0.1** | **0.0024** | **0.0028** | **0.0032** | **0.0035** | **0.0039** | **0.0042** | **0.0046** | **0.0049** | **0.0052** | **0.0056** | | **0.2** | **0.0084** | **0.0089** | **0.0093** | **0.0097** | **0.0101** | **0.0105** | **0.0109** | **0.0113** | **0.0117** | **0.0121** | | **0.3** | **0.0184** | **0.0189** | **0.0193** | **0.0197** | **0.0202** | **0.0206** | **0.021** | **0.0214** | **0.0219** | **0.0223** | | **0.4** | **0.0324** | **0.0329** | **0.0333** | **0.0337** | **0.0342** | **0.0346** | **0.035** | **0.0355** | **0.0359** | **0.0363** | | **0.5** | **0.0504** | **0.0509** | **0.0513** | **0.0517** | **0.0521** | **0.0526** | **0.053** | **0.0534** | **0.0538** | **0.0542** | | **0.6** | **0.0724** | **0.0728** | **0.0732** | **0.0737** | **0.0741** | **0.0745** | **0.0749** | **0.0753** | **0.0757** | **0.0761** | | **0.7** | **0.0984** | **0.0988** | **0.0992** | **0.0996** | **0.1** | **0.1004** | **0.1008** | **0.1011** | **0.1015** | **0.1019** | | **0.8** | **0.1284** | **0.1287** | **0.1291** | **0.1295** | **0.1299** | **0.1302** | **0.1306** | **0.131** | **0.1314** | **0.1318** | | **0.9** | **0.1623** | **0.1627** | **0.1631** | **0.1635** | **0.1639** | **0.1643** | **0.1648** | **0.1652** | **0.1656** | **0.1661** | | **1** | **0.2006** | **0.2012** | **0.2018** | **0.2024** | **0.203** | **0.2036** | **0.2042** | **0.2048** | **0.2054** | **0.206** |  1. **Graphical representation**:   **Fig 3** : **Saturation () for different values of distance’ ’ and time ‘’ by Hybrid Differential Transform and Finite Difference method** | | |

From the graph in Fig.3 we observe that the saturation (X,T) of injected water is increasing when distance ‘ X’ is increasing for fixed time T’ and saturation is also increasing when time ‘T’ is increasing for fixed distance ‘X’.

1. **Limitation of Hybrid Differential transform Method** :

The HDFDTM is used to approximate solutions to a wide variety of nonlinear problems with convergent series with easily computed components; however, it has some drawbacks: The series solution always converges quickly in a small region and slowly in a larger region. In a very small region, this series solution provides a good approximation to the true solution. The Multi-step DTM (MDTM) and finite Difference Method (FDM) are improved methods that accelerate series solution convergence over a large region while also improving DTM accuracy.

**VI. Multistep Hybrid Differential Transform Finite Difference Method**  (MHDTFDM):

In many modelling situations, the computation interval [0, T] is not always small, and in order to accelerate the rate of convergence and improve calculation accuracy, the entire domain T is divided into ‘N’ subdomains.

The main benefit of splitting the domain is that it only requires a few Taylor series terms to construct the solution in a small-time interval. It is worth noting that the time interval can be chosen arbitrarily small if necessary. As a result, the differential equations in each subdomain can be solved.

1. **METHODOLGY**

DTM solutions have a small interval of convergence, whereas Multi-step DTM solutions have a large interval of convergence. This demonstrates that the MDTM increases the interval of convergence for the series solution. Of course, the accuracy can be improved by decreasing the step size h and increasing the number of terms in each subinterval.

Definition 1:

The Differential Transform is defined as follows

|  |  |  |
| --- | --- | --- |
|  |  | (26) |

where is the original function and ) is the transformed function.

The inverse transformation is defined as,

|  |  |  |
| --- | --- | --- |
|  |  | (27) |

and the function is considered as a series with finite terms

|  |  |  |
| --- | --- | --- |
|  |  | (28) |

where m represents no. of Taylor series components. Increasing the no. of terms can improve the accuracy of the solution.

**B. Solution of Partial Differential equation in in domain**

We divide the domain into N sections where H= T/N is the length of each subdomain.

So, a separate function is considered for each sub domain.

|  |  |  |
| --- | --- | --- |
|  | where | (29) |

|  |  |  |
| --- | --- | --- |
|  | where | (30) |

|  |  |  |
| --- | --- | --- |
|  | where | (31) |

The solution of equation (21) is of the form

,for

Applying Central finite difference to the spatial variable and MDTM to the variable

From (17), the following recurrence relation can be obtained

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | is the differential transform of  Here, separate functions are considered in each sub domain i.e   |  |  |  | | --- | --- | --- | |  | =  **(x,t)** = where | (33)3) | | (32) |

where is the original function and,­i ­i transformed function

Applying Multistep Differential Transform Method (MDTM) to the ‘T’ variable and Finite Difference Method ( FDM ) on the initial conditions, we have

|  |  |  |
| --- | --- | --- |
|  | Initial Conditions: ,  Boundary Conditions: |  |

where N is the no. of spatial segments.

Here and we use the continuity condition in each time subdomain.

Firstly, the DTM is applied to the given PDE over the interval [0,.005]. For the next time step, the value at T = .005 is used as an initial condition in the interval[.005,.01] i.e in general,for solving the problem in whole of the time subdomains, we must use the continuity condition in each time subdomain. These conditions can be expressed as

For 1,1

MDTM is implemented by dividing the solution interval [0,1] into 200 subintervals of equal step size given by H = 0.005.

**VII. RESULTS AND DISCUSSION**

The numerical values of the saturation obtained from equation (32) and (33) for various distances ‘’ at fixed time =0.01,0.02,0.03,0.04,0.05,0.06,0.07,0.08,0.09 and .1 are obtained by using MATLAB and presented in **Table 2** below.

T**able 2**: **Saturation (X,T) for different values of distance 'X' and time 'T’ by Hybrid Multistep Differential Transform and Finite Difference method for**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **X\T** | **0.01** | **0.02** | **0.03** | **0.04** | **0.05** | **0.06** | **0.07** | **0.08** | **0.09** | **0.1** |
| **0** | **0.002** | **0.004** | **0.006** | **0.008** | **0.01** | **0.012** | **0.0140** | **0.016** | **0.018** | **0.02** |
| **0.1** | **0.0056** | **0.0086** | **0.0114** | **0.0141** | **0.0167** | **0.0193** | **0.0217** | **0.0242** | **0.0266** | **0.029** |
| **0.2** | **0.0121** | **0.0158** | **0.0193** | **0.0226** | **0.0257** | **0.0287** | **0.0317** | **0.0346** | **0.0374** | **0.0402** |
| **0.3** | **0.0223** | **0.0264** | **0.0302** | **0.0339** | **0.0374** | **0.0408** | **0.0442** | **0.0474** | **0.0507** | **0.0539** |
| **0.4** | **0.0363** | **0.0405** | **0.0446** | **0.0485** | **0.0523** | **0.056** | **0.0597** | **0.0633** | **0.0669** | **0.0704** |
| **0.5** | **0.0543** | **0.0585** | **0.0626** | **0.0666** | **0.0706** | **0.0746** | **0.0785** | **0.0825** | **0.0864** | **0.0902** |
| **0.6** | **0.0761** | **0.0803** | **0.0844** | **0.0886** | **0.0928** | **0.097** | **0.1012** | **0.1054** | **0.1096** | **0.1138** |
| **0.7** | **0.102** | **0.106** | **0.1102** | **0.1146** | **0.119** | **0.1235** | **0.128** | **0.1326** | **0.1371** | **0.1417** |
| **0.8** | **0.1318** | **0.1361** | **0.1405** | **0.1452** | **0.15** | **0.1548** | **0.1597** | **0.1647** | **0.1696** | **0.1746** |
| **0.9** | **0.1661** | **0.1709** | **0.176** | **0.1812** | **0.1865** | **0.1919** | **0.1973** | **0.2027** | **0.2081** | **0.2135** |
| **1** | **0.206** | **0.212** | **0.218** | **0.224** | **0.23** | **0.236** | **0.242** | **0.248** | **0.254** | **0.26** |

From Table2, we observe that the values of saturation converge in this interval by using MDTM.

We continue finding for the remaining intervals [.1,.2], [.2,.3] and so on and saturation at T=.1,.2,.3…… is shown in TABLE 3.

T**able 3**: **Saturation (X,T) for different values of distance 'X' and time 'T’ by Multistep Hybrid Differential Transform and Finite Difference method for**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **X/T** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **1** |
| **0** | **0.02** | **0.04** | **0.06** | **0.08** | **0.1** | **0.1200** | **0.1400** | **0.16** | **0.18** | **0.2000** |
| **0.1** | **0.029** | **0.0528** | **0.0759** | **0.0988** | **0.1214** | **0.1436** | **0.1656** | **0.1871** | **0.2083** | **0.2292** |
| **0.2** | **0.0402** | **0.0675** | **0.0938** | **0.1197** | **0.1449** | **0.1694** | **0.1934** | **0.2166** | **0.2391** | **0.2609** |
| **0.3** | **0.0539** | **0.0847** | **0.1142** | **0.143** | **0.1709** | **0.1978** | **0.2239** | **0.2489** | **0.2729** | **0.2957** |
| **0.4** | **0.0704** | **0.1046** | **0.1374** | **0.1692** | **0.1999** | **0.2294** | **0.2577** | **0.2847** | **0.3103** | **0.3342** |
| **0.5** | **0.0902** | **0.1278** | **0.164** | **0.1988** | **0.2326** | **0.2649** | **0.2955** | **0.3247** | **0.352** | **0.3772** |
| **0.6** | **0.1138** | **0.1548** | **0.1945** | **0.2328** | **0.2697** | **0.305** | **0.3385** | **0.3701** | **0.3994** | **0.4262** |
| **0.7** | **0.1417** | **0.1864** | **0.2299** | **0.272** | **0.3126** | **0.3514** | **0.3883** | **0.4228** | **0.4545** | **0.4832** |
| **0.8** | **0.1746** | **0.2235** | **0.2713** | **0.3177** | **0.3628** | **0.4061** | **0.4473** | **0.4858** | **0.5211** | **0.5524** |
| **0.9** | **0.2135** | **0.2673** | **0.3204** | **0.3725** | **0.4234** | **0.4729** | **0.5205** | **0.5654** | **0.6069** | **0.6435** |
| **1** | **0.26** | **0.32** | **0.38** | **0.44** | **0.5** | **0.56** | **0.62** | **0.68** | **0.74** | **0.8** |

As we can see, the main advantage of the MDTM is that the obtained series solution converges for wide time region, which is not possible by the traditional DTM

From the table. we observe that the saturation () of injected water is increasing when distance ‘ ’ is increasing for fixed time ‘ ’ and saturation is also increasing when time ‘’ is increasing for fixed distance ‘’.

**Graphical Representation:**

Figure 4 : **Saturation () for different values of distance' ' and time '' by Multistep Hybrid Differential Transform Finite Difference Method**

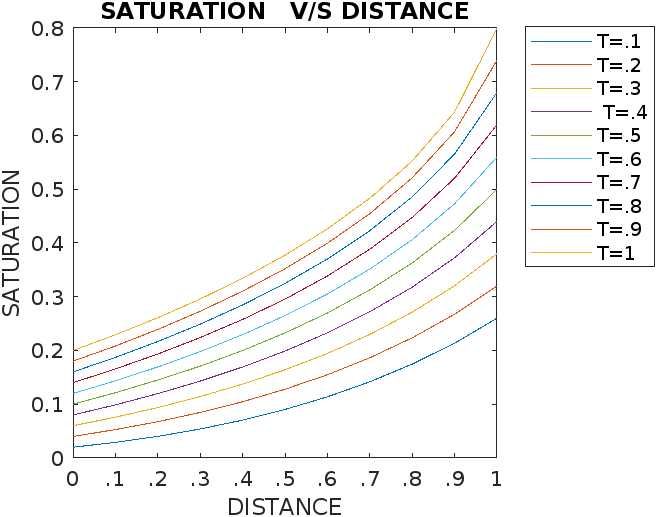


Figure 5 : **3 D plot of** **Saturation () for different values of distance' ' and time '' by Multistep Hybrid Differential Transform Finite Difference Method**

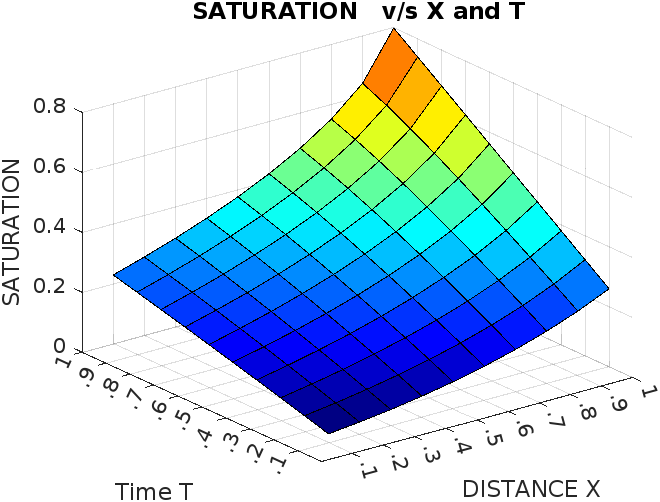


Fig.5 is a 3D plot of Saturation V/s Distance and Time. Here we can clearly observe that the saturation () of injected water is increasing when distance ‘ ’ is increasing for fixed time ‘ ’ and also increasing with time ‘’ for fixed distance ’’.

The above solution was compared with Homotopy Analysis Method [9] and the results were found to be in close agreement.

Table 4**. COMPARISON OF MULTISTEP HYBRID DIFFERENTIAL TRANSFORM METHOD AND HOMOTOPY ANALYSIS METHOD [9]**



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **X/T** | **0.1** | |  | **0.3** | |  | **0.5** | |  | **0.7** | |  | **0.9** | |
|  | **HDTFD** | **HAM** |  | **HDTFD** | **HAM** |  | **HDTFD** | **HAM** |  | **HDTFD** | **HAM** |  | **HDTFD** | **HAM** |
| **0** | **0.02** | **0.0200** |  | **0.06** | **0.06** |  | **0.1** | **0.1** |  | **0.1400** | **0.14** |  | **0.18** | **0.18** |
| **0.1** | **0.029** | **0.0295** |  | **0.0759** | **0.0759** |  | **0.1214** | **0.1213** |  | **0.1656** | **0.1654** |  | **0.2083** | **0.2081** |
| **0.2** | **0.0402** | **0.0410** |  | **0.0938** | **0.0939** |  | **0.1449** | **0.1447** |  | **0.1934** | **0.1931** |  | **0.2391** | **0.2387** |
| **0.3** | **0.0539** | **0.0548** |  | **0.1142** | **0.1142** |  | **0.1709** | **0.1706** |  | **0.2239** | **0.2235** |  | **0.2729** | **0.2723** |
| **0.4** | **0.0704** | **0.0714** |  | **0.1374** | **0.1374** |  | **0.1999** | **0.1995** |  | **0.2577** | **0.2572** |  | **0.3103** | **0.3094** |
| **0.5** | **0.0902** | **0.0912** |  | **0.164** | **0.1639** |  | **0.2326** | **0.2322** |  | **0.2955** | **0.295** |  | **0.352** | **0.351** |
| **0.6** | **0.1138** | **0.1146** |  | **0.1945** | **0.1944** |  | **0.2697** | **0.2693** |  | **0.3385** | **0.3379** |  | **0.3994** | **0.3982** |
| **0.7** | **0.1417** | **0.1422** |  | **0.2299** | **0.2297** |  | **0.3126** | **0.3121** |  | **0.3883** | **0.3875** |  | **0.4545** | **0.453** |
| **0.8** | **0.1746** | **0.1749** |  | **0.2713** | **0.2711** |  | **0.3628** | **0.3624** |  | **0.4473** | **0.4464** |  | **0.5211** | **0.5191** |
| **0.9** | **0.2135** | **0.2136** |  | **0.3204** | **0.3202** |  | **0.4234** | **0.4231** |  | **0.5205** | **0.5197** |  | **0.6069** | **0.6045** |
| **1** | **0.26** | **0.2600** |  | **0.38** | **0.38** |  | **0.5** | **0.5** |  | **0.62** | **0.62** |  | **0.74** | **0.74** |

Table 4 . shows the comparison of Multistep Hybrid Differential Transform Finite Difference Method with Homotopy Analysis Method and the results are found to be in close agreement.

Table 5**. COMPARISON OF SATURATION OF INJECTED WATER WITH MAGNETIC EFFECT AND WITHOUT MAGNETIC EFFECT**



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **With Magnetic**  **Field** | **Without**  **Magnetic field** |  | **With Magnetic**  **Field** | **Without**  **Magnetic field** |  | **With Magnetic**  **Field** | **Without**  **Magnetic field** |  | **With Magnetic**  **Field** | **Without**  **Magnetic field** |  | **With Magnetic**  **Field** | **Without**  **Magnetic field** |
| **x/t** | **0.1** | **0.1** |  | **0.3** | **0.3** |  | **0.5** | **0.5** |  | **0.7** | **0.7** |  | **0.9** | **0.9** |
| **0.1** | **0.029** | **0.0289** |  | **0.0759** | **0.0758** |  | **0.1214** | **0.1212** |  | **0.1656** | **0.1654** |  | **0.2083** | **0.2082** |
| **0.2** | **0.0402** | **0.04** |  | **0.0938** | **0.0937** |  | **0.1449** | **0.1446** |  | **0.1934** | **0.1931** |  | **0.2391** | **0.2388** |
| **0.3** | **0.0539** | **0.0537** |  | **0.1142** | **0.114** |  | **0.1709** | **0.1706** |  | **0.2239** | **0.2235** |  | **0.2729** | **0.2726** |
| **0.4** | **0.0704** | **0.0702** |  | **0.1374** | **0.1371** |  | **0.1999** | **0.1995** |  | **0.2577** | **0.2572** |  | **0.3103** | **0.3098** |
| **0.5** | **0.0902** | **0.09** |  | **0.164** | **0.1637** |  | **0.2326** | **0.2322** |  | **0.2955** | **0.295** |  | **0.352** | **0.3515** |
| **0.6** | **0.1138** | **0.1136** |  | **0.1945** | **0.1942** |  | **0.2697** | **0.2693** |  | **0.3385** | **0.3381** |  | **0.3994** | **0.3989** |
| **0.7** | **0.1417** | **0.1415** |  | **0.2299** | **0.2296** |  | **0.3126** | **0.3122** |  | **0.3883** | **0.3879** |  | **0.4545** | **0.454** |
| **0.8** | **0.1746** | **0.1744** |  | **0.2713** | **0.2711** |  | **0.3628** | **0.3625** |  | **0.4473** | **0.4469** |  | **0.5211** | **0.5206** |
| **0.9** | **0.2135** | **0.2134** |  | **0.3204** | **0.3202** |  | **0.4234** | **0.4232** |  | **0.5205** | **0.5202** |  | **0.6069** | **0.6066** |
| **1** | **0.26** | **0.26** |  | **0.38** | **0.38** |  | **0.5** | **0.5** |  | **0.62** | **0.62** |  | **0.74** | **0.74** |

From table (5) we can observe that the Magnetic Field Effect increases the Saturation of injected water as compared to saturation of injected water without magnetic field effect

**Convergence Criteria**

From equation (21) we get the series solution for the nonlinear PDE as

then the convergence of the power series in ‘t’ can be found as per the following theorem [27]

**Theorem**

If ,then the series solution ,stated in equation above, follows the following criteria.

It is convergent if ,such that

It is divergent if ,such that

**VIII. CONCLUSIONS**

Numerical solution and graphs for obtaining the saturation of injected water in heterogenous porous media during the Fingero- imbibition phenomenon considering magnetic field effect, with regard to distance and time are obtained using MATLAB.

The graph given by Figure. 2 shows that saturation of injected water increases as distance X increases for given time T. The saturation of water increases with distance as well as with time, which is consistent with physical phenomena. Due to additional magnetic effect the saturation of injected water is more than the saturation of injected water without magnetic field effect as shown in Table 5.

Therefore,it can be concluded that the magnetic field helps in increasing the saturation of injected water in the fingero-imbibition phenomenon..

Multi-step DTM, a reliable modification of the DTM, was used in this work for the variable 'T,' which improves the convergence of the series solution. The method gives approximate numerical solutions to both linear and nonlinear differential equations. Table 1. shows that using DTM we get solutions which converge over a small interval of convergence, whereas Multi-step DTM solutions have a wide interval of convergence, as shown in Table 3. This demonstrates that the MDTM increases the interval of convergence for the series solution.

Comparative study of obtaining the result by the two methods, namely, Hybrid Differential transform and Finite difference Method and Homptopy analysis method shows that results closely agree with each other.

Complex symbolic computation is not necessary because the Hybrid Differential Transform Finite Difference Method calculates numerical solutions through an iterative process. It has been shown that the suggested approach can produce very precise numerical approximations and that Multistep DTM improves the obtained solution. The last and most important advantage is that we do not use linearization.

As a result, these methods can be used to solve a wide range of difficult partial differential equations, both linear and nonlinear, without the need for linearization or perturbation.

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