

# Performance of DF Relay Assisted Dual-Hop System over Generalized-K along with $\kappa$ - $\mu$ Fading Environments

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## ABSTRACT

The outage probability (OP), average bit-error probability (ABEP) as well as ergodic capacity of decode-and-forward (DF) relay-based dual-hop transmission are evaluated in this analysis. The mixed fading channel environments are considered to study the system performance. The channel connecting the source to relay experiences Generalized-K ( $K_G$ ) distribution, whereas the link joining the relay with the destination is assumed as  $\kappa$ - $\mu$  distribution. To analyse the system's performance, the probability density function (PDF) based approach is used. The BPSK as well as QPSK modulation schemes are applied to analyse the ABEP of the system. The results are validated by computer simulations.

**Keywords:** Dual-hop relaying, Generalized-K, Decode-and-forward,  $k - \mu$  fading, Outage probability, Ergodic capacity, ABEP.

## 1. INTRODUCTION

In dual-hop cooperative communication, a source node transmits to the destination node via an additional node named as relay node [1]. Whenever the direct transmission between the base station and the end user or the destination node is in a deep fade, the relay is suitable to improve the SNR. The relay is used to cooperate the communication process by retransmitting the signal and thereby improving the SNR at the destination. Communication can take place in dual phases. Dual-hop networks improve the data rate in wireless networks. There are several relay transmission protocols. The standard protocols are amplify and forward (AF) as well as decode and forward (DF) protocols. In case of AF protocol, the signal is amplified by a relay and later retransmit. It blindly amplifies the input signal and forwards it. Therefore the external input noise gets amplified in AF protocol. In DF communication, relay decodes the symbols transmitted by the source node and subsequently sends the symbols to end user [2]. The advantage of using DF communication is that it does not amplify the noise of the environment. [3].

In the available literature, cooperative transmission systems operating over fading channels are analysed in several works. In [4], the analysis of the dual-hop AF system subject to  $k - \mu$  shadowed distribution was

carried out. Asymptotic analysis at high SNR for ABEP was performed to depict the effect of channel parameters on the communication system behaviour. Dual-hop transmission models with the DF technique under Nakagami- $m$  fading channels were analysed in [5] [6] [7]. Dual-hop transmissions under Gamma fading channels were investigated in [8]. The performance of a dual-hop AF variable gain relaying system under Gamma distribution was investigated in [9]. The performance of DF based cooperative free-space optical (FSO) network with multiple relays was investigated in [10]. The asymptotic ABEP and asymptotic OP analysis were derived for the source to relay links. In [11], the performance of dual-hop AF transmission operating in a non-identical Rician fading environment was presented. A generic moment-based approach for the evaluation of dual-hop wireless communication employing AF systems under generalized fading channels was presented in [12]. In [13], the performance of multi-hop communication links was investigated under  $K_G$  fading environment. An expression for the MGF of the SNR of dual-hop with non-regenerative wireless communication systems over i.i.d  $\alpha$ - $\mu$  fading channels was derived in [14]. The expressions for outage probability and ABEP of the system were derived from the SNR expression. These works are based on symmetric fading condition, which is not suitable for fundamental wireless communication environment characteristics.

Nevertheless, several articles have also mentioned the performance of dual-hop relaying techniques over asymmetric fading conditions. The asymmetric channel models mean wireless channel conditions of every hop are different because of the possibility that receiving signals may be different for the two relay links [15]. These mixed fading propagation channels are more realistic for precisely modelling numerous practical dual-hop communications scenarios.

The performance of a two-hop AF system was studied in [16], considering links undergo Rayleigh and Rician fading, respectively. The performance of a two-hop AF cooperative system was presented in [17] for source-to-relay experience  $\eta - \mu$  fading, whereas the relay-to-destination undergoes  $k - \mu$  fading environment. The expressions were derived for both fixed-gain and CSI-

assisted transmission. The expressions for ASEP in addition to OP of two-hop AF relay system under Nakagami- $m$  as well as Rician faded wireless system environments were presented in [18]. The AF transmission scheme, where the links undergo Rayleigh, as well as Rician fading distribution, was analysed in [19]. The outage probability of dual-hop communication with AF system under consideration that relay node is corrupted by co-channel interferences was studied in [20]. The outage probability was examined for instantaneous signal-to-interference and noise ratio (SINR) of a dual-hop scheme with channel-state-information (CSI) operated relay under asymmetric fading conditions. The system performance of a dual-hop system was also carried over Nakagami- $m$  as well as Rician fading scenarios in [21] for the AF scheme. In [22], expressions for ABEP and outage probability of an AF relaying-based transmission model was investigated in mixed Rayleigh with Hoyt fading.

The analysis of an asymmetric dual-hop the DF relaying phenomena composed of radio-frequency (RF) and FSO links was carried out in [23]. The transmitted RF signal from the source was decoded by the relay and then generated an optical signal applying the sub-carrier intensity-modulation (SIM) scheme. The optical signal was transmitted under the FSO link. For mixed RF/FSO system, the end-to-end SNR expression was derived using Meijer's G function. In [24], the OP of the dual-hop relay assisted DF transmission model was investigated under Rayleigh as well as generalized Gamma fading distributions. In [25], the outage probability under asymmetric  $k - \mu$  as well as  $\eta - \mu$  fading channels was acquired for DF relaying, employing the CDF-based expression of  $\eta - \mu$  distribution. The analysis of two-hop DF relaying protocol experiencing mixed faded distributions was also explained in [15]. It was considered that the channel from the source to the relay node was subject to Rayleigh fading; however, the Weibull distributed fading model was between the relay and destination node. The expressions were derived for ABEP, the OP and ergodic capacity of the specified system model. The ABEP performance was studied for  $M$ -ary PSK signalling considering different constellation sizes.

This work uses  $K_G$  fading along with  $k - \mu$  fading distribution for the two hops, with DF intermediate relay assisted communication system which is not available in literature

## 2. SYSTEM AND CHANNEL DESCRIPTION

A dual-hop communication system is considered where the transmission from a source node S to the destination node D takes place through the cooperation of a relay node R. Transmission from S to D occurs in two different time slots. The source node S sends a signal to the relay node R in the first time slot whereas, in the second time slot, R decodes the received signal and sends the resulting decoded signal to D. The S is situated in an intensely shadowed environment; therefore it is considered that there is no direct sight between S and D node.

The SNR of  $S - R$  and  $R - D$  links are  $\gamma_1 = \frac{P_1 |h_{SR}|^2}{N_0}$  and  $\gamma_2 = \frac{P_2 |h_{RD}|^2}{N_0}$  respectively [16].

Where,  $P_1$  as well as  $P_2$  are the transmit power of S and R respectively.  $|h_{SR}|$  as well as  $|h_{RD}|$  are the fading amplitudes of the channels in the  $S - R$  and  $R - D$  links respectively.  $N_0$  is the power of the AWGN. The  $S - R$  link undergoes  $K_G$  fading distribution, whereas the  $R - D$  link is modeled by  $k - \mu$  fading. The PDF of SNR  $\gamma_1$  for  $K_G$  fading is given as [26]

$$f_{\gamma_1}(\gamma_1) = \frac{2\Xi^{\frac{\beta+1}{2}} \gamma_1^{\frac{\beta-1}{2}}}{\Gamma(m)\Gamma(k_1)} K_\alpha \left[ 2(\Xi\gamma_1)^{\frac{1}{2}} \right], \gamma_1 \geq 0 \quad (1)$$

Where,  $m$  and  $k_1$  are the distribution shaping parameters,  $\Gamma(\cdot)$  is the Gamma function,

$$\beta = k_1 + m - 1, \quad \alpha = k_1 - m, \quad \Xi = \frac{mk_1}{\gamma_1}, \quad \text{and } K_\alpha(\cdot)$$

is the modified Bessel function of order  $\alpha$ . Furthermore,  $\bar{\gamma}_1$  is the average SNR. The expression (1) can narrate various fading as well as shadowing models by utilizing different values for  $k_1$  and/or  $m$ . The  $K_G$  is an adaptable distribution that includes many of the familiar models for multipath and shadow fading. This fading model can approximate numerous fading distributions, such as, Rayleigh, Nakagami- $m$ , Nakagami- $n$ , and Rayleigh-Lognormal (R-L) [26].

If the link experiences  $k - \mu$  fading distribution, the PDF of SNR  $\gamma_2$  is given as [27]

$$f_{\gamma_2}(\gamma_2) = \frac{\mu(1+k_2)^{\frac{\mu+1}{2}} \gamma_2^{\frac{\mu-1}{2}} e^{-\frac{\mu(1+k_2)\gamma_2}{\gamma_2}}}{k_2^{\frac{\mu-1}{2}} (\bar{\gamma}_2)^{\frac{\mu+1}{2}} e^{\mu k_2}} \times I_{\mu-1} \left( 2\mu \sqrt{\frac{k_2(1+k_2)\gamma_2}{\bar{\gamma}_2}} \right). \quad (2)$$

The parameter  $k_2$  is the ratio between total power of the dominant component and total power of the scattered waves. The symbol  $\mu$  represents the number of multipath clusters, whereas  $I_\nu(\cdot)$  is the modified Bessel function of the first kind and  $\nu^{\text{th}}$  order [28]. The  $k - \mu$  shadowed physical model comprises clusters of multipath waves, that travel in a non-homogeneous environment. Within a cluster, the waves consist of one dominant conversely a line of sight (LOS) component in addition to

multiple scattered components. The scattered multipath components have random phases, same power, and similar time delays. The power of the dominant components is treated as arbitrary. The LOS components available in the clusters fluctuate randomly due to shadowing. Intracluster scattered multipath components have relatively shorter delay times as compared to intercluster scattered components [4].

### 3. OP ANALYSIS

The OP is a prime performance metric that is utilized to specify a wireless communication system. It is explained as the probability that the received SNR drops under a certain threshold constant  $\gamma_{th}$  [29]. An outage happens in the DF relaying system, whenever the instantaneous SNR of one or two of the hops falls under  $\gamma_{th}$ . Therefore, it ( $P_{out}$ ) can be described based on total SNR [8][15] as

$$P_{out} = \Pr\{\min(\gamma_1, \gamma_2) \leq \gamma_{th}\} \\ = 1 - \Pr(\gamma_1 > \gamma_{th})\Pr(\gamma_2 > \gamma_{th}). \quad (3)$$

Whereby  $\Pr(\gamma_1 > \gamma_{th})$  can be evaluated according to  $K_G$  fading for the  $S-R$  link as

$$\Pr(\gamma_1 > \gamma_{th}) = \int_{\gamma_{th}}^{\infty} f_{\gamma_1}(\gamma_1) d\gamma_1. \quad (4)$$

Expressing the modified Bessel function as given in [30, (03.04.03.0004.01)], the PDF expression in (1) can be found as

$$f_{\gamma_1}(\gamma_1) = \frac{e^{-2(\Xi\gamma_1)^{\frac{1}{2}}}}{\Gamma(m)\Gamma(k_1)} \sqrt{\pi} \sum_{j=0}^{|\alpha|-\frac{1}{2}} \frac{\left(j+|\alpha|-\frac{1}{2}\right)!}{4^j j! \left(-j+|\alpha|-\frac{1}{2}\right)!} \\ \times \Xi^{\frac{2\beta+1-2j}{4}} \gamma_1^{\frac{2\beta-3-2j}{4}}. \quad (5)$$

Putting the value of  $f_{\gamma_1}(\gamma_1)$  from (5) into (4) and after simplification, the expression of (4) is written as

$$\Pr(\gamma_1 > \gamma_{th}) = \frac{\sqrt{\pi}}{\Gamma(m)\Gamma(k_1)} \\ \times \sum_{j=0}^{|\alpha|-\frac{1}{2}} \frac{\left(j+|\alpha|-\frac{1}{2}\right)! \Xi^{\frac{2\beta+1-2j}{4}}}{4^j j! \left(-j+|\alpha|-\frac{1}{2}\right)!} \\ \times \int_{\gamma_{th}}^{\infty} e^{-2(\Xi\gamma_1)^{\frac{1}{2}}} \gamma_1^{\frac{2\beta-3-2j}{4}} d\gamma_1. \quad (6)$$

Using [31, (3.381.9)] in (6), the expression of  $\Pr(\gamma_1 > \gamma_{th})$  is simplified as

$$\Pr(\gamma_1 > \gamma_{th}) = \frac{\sqrt{\pi}}{\Gamma(m)\Gamma(k_1)} \\ \times \sum_{j=0}^{|\alpha|-\frac{1}{2}} \frac{2^{\frac{1-2\beta+2j}{2}} \left(j+|\alpha|-\frac{1}{2}\right)!}{4^j j! \left(-j+|\alpha|-\frac{1}{2}\right)!} \\ \times \Gamma\left(\frac{2\beta+1-2j}{2}, 2(\Xi\gamma_{th})^{\frac{1}{2}}\right). \quad (7)$$

If the  $R-D$  link experiences  $k-\mu$  fading distribution,  $\Pr(\gamma_2 > \gamma_{th})$  is determined as

$$\Pr(\gamma_2 > \gamma_{th}) = \int_{\gamma_{th}}^{\infty} f_{\gamma_2}(\gamma_2) d\gamma_2. \quad (8)$$

Putting the value of  $f_{\gamma_2}(\gamma_2)$  from (2) into (8) and converting the involved Bessel function as infinite series [31, (8.445)], the expression of (8) is can be given as

$$\Pr(\gamma_2 > \gamma_{th}) = \sum_{t=0}^{\infty} \frac{\mu^{\mu+2t} (1+k_2)^{\mu+t} k_2^t}{t! \Gamma(\mu+t) (\bar{\gamma}_2)^{\mu+t}} e^{\mu k_2} \\ \times \int_{\gamma_{th}}^{\infty} \gamma_2^{\mu+t-1} e^{-\frac{\mu(1+k_2)\gamma_2}{\bar{\gamma}_2}} d\gamma_2. \quad (9)$$

Simplifying (9), by making use of [31, (3.381.3)],

$$\Pr(\gamma_2 > \gamma_{th}) = \sum_{t=0}^{\infty} \frac{\mu^t k_2^t}{t! \Gamma(\mu+t) e^{\mu k_2}} \\ \times \Gamma\left(\mu+t, \frac{\mu(1+k_2)\gamma_{th}}{\bar{\gamma}_2}\right). \quad (10)$$

Inserting (7) and (10) into (3),  $P_{out}$  can be derived as

$$P_{out} = 1 - \left[ \frac{\sqrt{\pi}}{\Gamma(m)\Gamma(k_1) e^{\mu k_2}} \right. \\ \times \sum_{j=0}^{|\alpha|-\frac{1}{2}} \sum_{t=0}^{\infty} \frac{2^{\frac{1-2\beta-2j}{2}} \mu^t k_2^t \left(j+|\alpha|-\frac{1}{2}\right)!}{j! t! \left(-j+|\alpha|-\frac{1}{2}\right)! \Gamma(\mu+t)} \\ \times \Gamma\left(\frac{2\beta+1-2j}{2}, 2(\Xi\gamma_{th})^{\frac{1}{2}}\right) \\ \left. \times \Gamma\left(\mu+t, \frac{\mu(1+k_2)\gamma_{th}}{\bar{\gamma}_2}\right) \right]. \quad (11)$$

### 4. ABEP ANALYSIS

The ABEP of distinct digital modulation systems under fading channels can be determined using [16, (12)] as

$$\bar{P}_e = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp\left(-\frac{\omega^2}{2}\right) F_{\gamma_{equ}}\left(\frac{\omega^2}{\xi}\right) d\omega. \quad (12)$$

Where  $\xi = 1$  and  $\xi = 2$  for QPSK and BPSK modulation schemes, respectively. Based on (11), the expression of (12) can be written as

$$\begin{aligned} \bar{P}_e &= \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp\left(-\frac{\omega^2}{2}\right) d\omega \left[ \frac{1}{\Gamma(m)\Gamma(k_1)e^{\mu k_2}} \right. \\ &\times \sum_{j=0}^{\lfloor \frac{|\alpha| - \frac{1}{2}}{2} \rfloor} \sum_{t=0}^{\infty} \frac{2^{-\beta-j} \left(j + |\alpha| - \frac{1}{2}\right)! \mu^t k_2^t}{j! t! \left(-j + |\alpha| - \frac{1}{2}\right)! \Gamma(\mu + t)} \\ &\times \int_0^\infty e^{-\frac{\omega^2}{2}} \Gamma\left(\frac{2\beta+1-2j}{2}, 2\left(\frac{\Xi \omega^2}{\xi}\right)^{\frac{1}{2}}\right) \\ &\times \Gamma\left(\mu + t, \frac{\mu(1+k_2)\omega^2}{\xi \bar{\gamma}_2}\right) d\omega \Big]. \quad (13) \end{aligned}$$

The expression of (13) is rewritten as

$$\bar{P}_e = W_1 - W_2. \quad (14)$$

Applying [31, (3.321.3)], the first term in (14) becomes

$$W_1 = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp\left(-\frac{\omega^2}{2}\right) d\omega = 0.5. \quad (15)$$

Using [29, (8.352.7)], the second term in (14) is written as

$$\begin{aligned} W_2 &= \frac{1}{\Gamma(m)\Gamma(k_1)e^{\mu k_2}} \\ &\times \sum_{j=0}^{\lfloor \frac{|\alpha| - \frac{1}{2}}{2} \rfloor} \sum_{t=0}^{\infty} \sum_{p=0}^{\frac{2\beta+1-2j-1}{2}} \sum_{n=0}^{\mu+t-1} \frac{2^{p-\beta-j} \left(j + |\alpha| - \frac{1}{2}\right)!}{j! p! n! t! \left(-j + |\alpha| - \frac{1}{2}\right)!} \\ &\times \frac{\left(\frac{2\beta+1-2j}{2} - 1\right)! \mu^{t+n} k_2^t (\Xi)^{\frac{p}{2}} (1+k_2)^n}{\xi^{\frac{p}{2}+n} (\bar{\gamma}_2)^n} \\ &\times \int_0^\infty e^{-2\omega\left(\frac{\Xi}{\xi}\right)^{\frac{1}{2}} - \frac{\xi \bar{\gamma}_2 \omega^2 + 2\mu(1+k_2)\omega^2}{2\xi \bar{\gamma}_2}} \omega^{p+2n} d\omega. \quad (16) \end{aligned}$$

Solving the integration term, with the help of [31, (3.462.1)] and after simplification, the expression of (16) can be written as

$$W_2 = \frac{1}{\xi^{\frac{p}{2}} e^{\frac{\Xi \bar{\gamma}_2}{\xi \bar{\gamma}_2 + 2\mu(1+k_2)} - \mu k_2}} \Gamma(m)\Gamma(k_1)$$

$$\begin{aligned} &\times \sum_{j=0}^{\lfloor \frac{|\alpha| - \frac{1}{2}}{2} \rfloor} \sum_{t=0}^{\infty} \sum_{p=0}^{\frac{2\beta+1-2j-1}{2}} \sum_{n=0}^{\mu+t-1} \frac{2^{p-\beta-j} \left(j + |\alpha| - \frac{1}{2}\right)!}{j! p! n! t! \left(-j + |\alpha| - \frac{1}{2}\right)!} \\ &\times \frac{\left(\frac{2\beta-1-2j}{2}\right)! \mu^{t+n} k_2^t (\Xi)^{\frac{p}{2}} (1+k_2)^n}{\left(\xi \bar{\gamma}_2 + 2\mu(1+k_2)\right)^{\frac{p+2n+1}{2}}} \\ &\times \left(\bar{\gamma}_2\right)^{\frac{p+1}{2}} \Gamma(p+2n+1) D_\Lambda(z). \quad (17) \end{aligned}$$

Where  $D_\Lambda(z)$  is the parabolic cylinder function.

$$\Lambda = -(p+2n+1) \text{ and } z = \sqrt{\frac{4\Xi \bar{\gamma}_2}{\xi \bar{\gamma}_2 + 2\mu(1+k_2)}}.$$

## 5. ERGODIC CAPACITY ANALYSIS

The ergodic capacity is achieved as [32]

$$C_{erg} = \frac{CB}{2\ln(2)} \int_0^\infty \ln(1+\gamma) f_{\gamma_1}(\gamma) d\gamma, \quad (18)$$

where,  $CB$  is the bandwidth of the channel as well as  $f_{\gamma_1}(\gamma)$  is the SNR PDF at the system output as written below [33]

$$f_{\gamma_1}(\gamma) = P_{SR} \delta(\gamma) + (1 - P_{SR}) f_{\gamma_2}(\gamma). \quad (19)$$

Here,  $P_{SR}$  is the probability for arising outage with  $S-R$  link and  $f_{\gamma_2}(\gamma)$  stands for SNR PDF of  $R-D$  link. By definition  $P_{SR} = F_{\gamma_1}(\gamma_{th})$ , considering CDF of SNR for  $S-R$  link as  $F_{\gamma_1}(\cdot)$ . With the help of (5) and applying [31, (3.381.8)], the following expression can be obtained.

$$\begin{aligned} F_{\gamma_1}(\gamma_{th}) &= \frac{\sqrt{\pi}}{\Gamma(m)\Gamma(k)} \sum_{j=0}^{\lfloor \frac{|\alpha| - \frac{1}{2}}{2} \rfloor} \frac{\left(j + |\alpha| - \frac{1}{2}\right)!}{j! \left(-j + |\alpha| - \frac{1}{2}\right)!} \\ &\times \frac{g\left(\frac{2\beta+1-2j}{2}, 2\left(\Xi \gamma_{th}\right)^{\frac{1}{2}}\right)}{\left(2\right)^{\frac{2\beta-1+2j}{2}}}. \quad (20) \end{aligned}$$

Where,  $g(\cdot, \cdot)$  is the lower incomplete Gamma function.

Placing (19) into (18), and writing

$$\ln(1+\varpi) = G_{2,2}^{1,2}\left(\varpi \begin{matrix} 1, 1 \\ 1, 0 \end{matrix}\right) \text{ from [34], } C_{erg} \text{ can be}$$

simplified as

$$C_{erg} = \frac{CB(1-P_{SR})}{2\ln(2)} \int_0^\infty G_{2,2}^{1,2} \left( \gamma \left| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right. \right) f_{\gamma_2}(\gamma) d\gamma. \quad (21)$$

Where  $G_{y,z}^{w,x}(\cdot)$  is the Meijer G function. Putting (2) into (21) and expressing  $I_\nu(\cdot)$  as infinite series [31, (8.445)],

$$C_{erg} = \frac{CB(1-P_{SR})}{2\ln(2)} \sum_{t=0}^\infty \frac{\mu^{\mu+2t} (1+k_2)^{\mu+t} k_2^t}{t! \Gamma(\mu+t) (\bar{\gamma})^{\mu+t}} e^{\mu k_2} \times \int_0^\infty \gamma^{\mu+t-1} e^{-\frac{\mu(1+k_2)\gamma}{\bar{\gamma}}} G_{2,2}^{1,2} \left( \gamma \left| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right. \right) d\gamma. \quad (22)$$

Simplifying with the aid of [31, (7.813.1)],

$$C_{erg} = \frac{CB(1-P_{SR})}{2\ln(2)} \sum_{t=0}^\infty \frac{\mu^t k_2^t}{t! \Gamma(\mu+t) e^{\mu k_2}} \times G_{3,2}^{1,3} \left( \frac{\bar{\gamma}}{\mu(1+k_2)} \left| \begin{matrix} 1-\mu-t, 1, 1 \\ 1, 0 \end{matrix} \right. \right). \quad (23)$$

## 5. RESULTS AND DISCUSSIONS

The derived expressions of OP, ergodic capacity as well as ABEP in the aforesaid sections for dual-hop DF relay system under  $K_G$  as well as  $k-\mu$  fading links are evaluated. In Fig. 1 and Fig. 2,  $P_{out}$  versus average SNR per hop are depicted for the outage threshold values  $\gamma_{th} = 2dB$  and  $\gamma_{th} = 6dB$ . It is detected that outage performance develops when the outage threshold SNR value is low. In Fig. 1,  $k_2 = 2$  and  $\mu = 1$  are kept constant. The outage performance boosts with the advance in the values of  $k_1$  as well as  $m$ . An increase in  $k_1$  means reduced in the shadowing effect, hence as expected, the system's outage improves with an enhance in  $k_1$  and for a fixed value of  $m$ . Furthermore, for the constant weight of  $k_1 = 1.5$ , the outage performance strengthens as the amount of  $m$  hikes, analogous to the small-scale fading becoming less severe.

In Fig. 2,  $P_{out}$  versus average SNR per hop is depicted for  $k_1 = 5.5$  and  $m = 2$ . The outage performance increases with an advance in the values of  $k_2$  as well as  $\mu$ . The  $k-\mu$  fading model is activated for LOS wireless communications and the parameter  $k_2$  points out the power of the dominant component. Consequently, the outage improves with the increment in  $k_2$  and for a fixed value of  $\mu$ . The parameter  $\mu$  is the actual expansion of the quantity of clusters [27]. As

predicted, the increment in parameter  $\mu$  boosts the performance of the system.

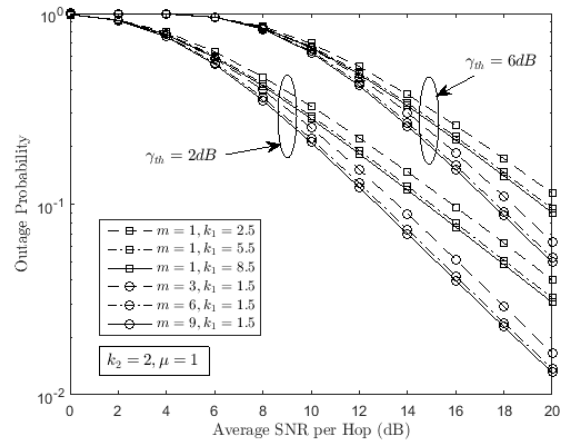
In Fig. 3, ABEP versus average SNR per hop is presented for  $k_2 = 2$  and  $\mu = 1$ , considering BPSK and QPSK modulation techniques. It can be noticed that ABEP performance is improving for BPSK modulation as compared to a QPSK modulation scheme. The ABEP performance of the analyzed system is better for larger the values of  $k_1$  and  $m$ .

In Fig. 4, ABEP versus average SNR per hop is plotted for  $k_1 = 5.5$  and  $m = 2$  employing BPSK and QPSK modulation techniques. It is shown in Fig. 4 that the ABEP of BPSK improves as compared to QPSK for two-hop DF relaying over considered fading environments. As expected, ABEP performance enhances with an addition in the values of  $k_2$  and  $\mu$ .

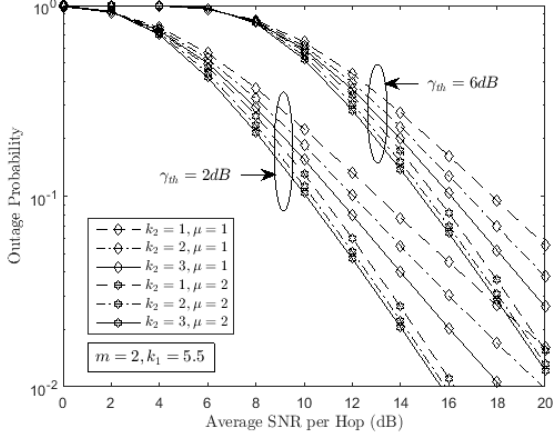
In Fig. 5,  $C_{erg}$  versus average SNR per hop is framed for  $k_2 = 2$  and  $\mu = 1$  with  $\gamma_{th} = 2dB$  and  $\gamma_{th} = 6dB$ . The ergodic capacity increases with an increment in the values of  $k_1$  as well as  $m$ . It is noticed that  $C_{erg}$  degrades when the outage threshold SNR value is high.

In Fig. 6,  $C_{erg}$  versus average SNR per hop is plotted for  $k_1 = 5.5$  and  $m = 2$ . The  $C_{erg}$  increases with an advancement in the values of  $k_2$  as well as  $\mu$ . The ergodic capacity improves with the decrease in threshold SNR from  $\gamma_{th} = 6dB$  to  $\gamma_{th} = 2dB$ .

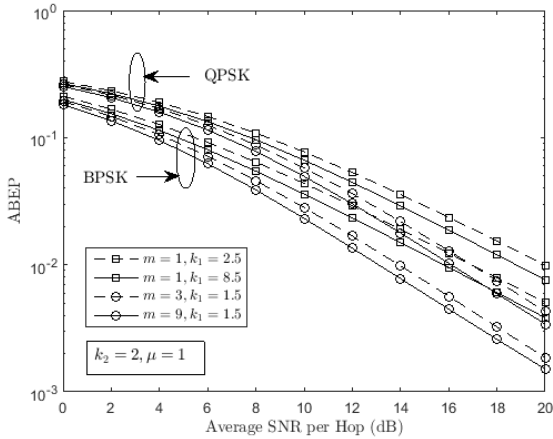
In the analytical computation of derived expressions, the infinite series terms are truncated to obtain correctness up to the 7th place of decimal digit. The explanatory curves are plotted from (11) and (13), considering 21 summations and in case of (23), the number of 51 summations are considered to obtain the correctness of the results.



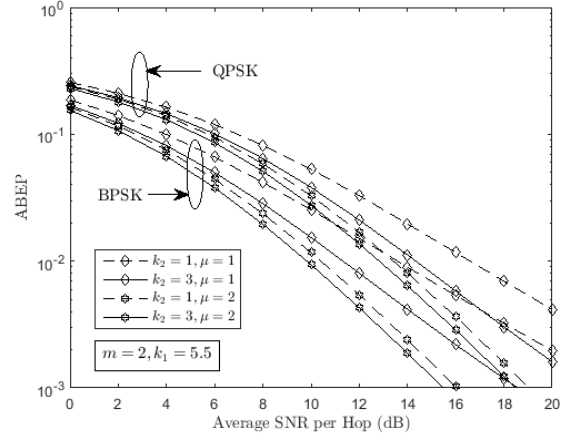
**Fig. 1:** Outage probability  $P_{out}$  of dual-hop DF relay system over  $K_G$  along with  $k - \mu$  fading channels for  $k_2 = 2$ ,  $\mu = 1$  and  $\bar{\gamma}_1 = \bar{\gamma}_2$ .



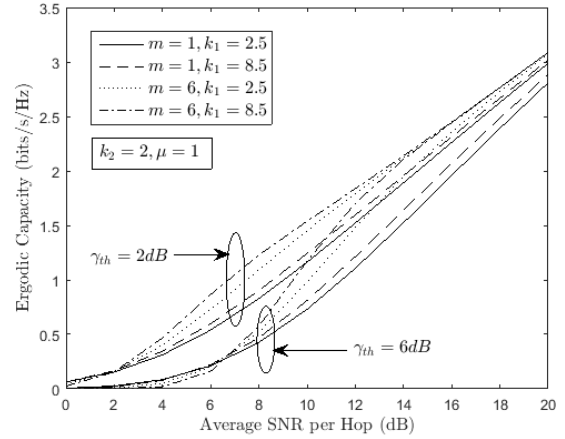
**Fig. 2:** Outage probability  $P_{out}$  of dual-hop DF relay system under  $K_G$  along with  $k - \mu$  fading channels for  $k_1 = 5.5$ ,  $m = 2$  and  $\bar{\gamma}_1 = \bar{\gamma}_2$ .



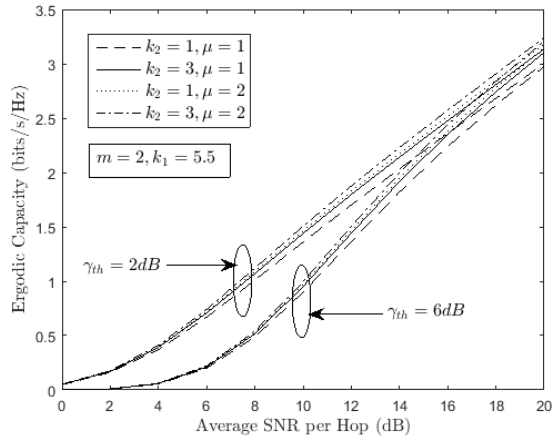
**Fig. 3:** ABEP of dual-hop DF relay system over  $K_G$  along with  $k - \mu$  fading channels for  $k_2 = 2$ ,  $\mu = 1$  and  $\bar{\gamma}_1 = \bar{\gamma}_2$ .



**Fig. 4:** ABEP of dual-hop DF relay system over  $K_G$  along with  $k - \mu$  fading channels for  $k_1 = 5.5$ ,  $m = 2$  and  $\bar{\gamma}_1 = \bar{\gamma}_2$ .



**Fig. 5:**  $C_{erg}$  of dual-hop DF relay system over  $K_G$  along with  $k - \mu$  fading channels for  $k_2 = 2$ ,  $\mu = 1$  and  $\bar{\gamma}_1 = \bar{\gamma}_2$ .



**Fig. 6:**  $C_{erg}$  of dual-hop DF relay system over  $K_G$  along with  $k - \mu$  fading channels for  $k_1 = 5.5$ ,  $m = 2$  and  $\bar{\gamma}_1 = \bar{\gamma}_2$ .

## 6. CONCLUSIONS

Closed-form  $P_{out}$ , ABEP and  $C_{erg}$  expressions of dual-hop DF relay system over  $K_G$  as well as  $k - \mu$  fading channels have been derived using a PDF-based approach. This combined fading channels model is suitable for precisely modelling numerous practical scenarios of two-hop wireless communication systems. The ABEP of the system is determined by employing BPSK and QPSK signalling schemes. The expression of ABEP is presented in terms of parabolic cylinder function. All the expressions are validated by computer simulations.

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