Some Fixed Point Results on OWC Mappings for IFMS (Intuitionistic Fuzzy Metric Space).

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Abstract

OWC(occasionally weakly compatible mapping), Implicit relations, Complete intuitionistic fuzzy metric spaces, and a common fixed point.

Introduction

In 1965, Zadeh invented fuzzy set theory [17].Numerous writers have presented and analyzed diverse fuzzy metric space concepts in various ways [9], [4], and [5], as well as proven fixed point theorems with intriguing follow-up findings in fuzzy metric spaces [6]. The idea of an intuitionistic fuzzy metric space was recently introduced by Park [12], and Alaca and et al. [2] and Mohamad [10] have since examined the fixed point outcomes in these spaces. The idea of sporadically weakly compatible maps was first suggested by Al-Thagafi and N. Shahzad[3].

In this paper, we show standard fixed point theorems for intuitionistic fuzzy metric spaces as an application of occasionally weakly compatible mappings.

Preliminaries

Definition 1.1[14] A binary operation $\circ : [0,1]\times[0,1] \to [0,1]$ is *continuous t-norm* if $\circ$ is satisfying the following conditions :

(i) $\circ$ is commutative and associative,
(ii) $\circ$ is continuous,
(iii) $a \circ 1 = a$ for all $a \in [0,1]$,
(iv) $a \circ b \le c \circ d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$.

Definition 1.2 A binary operation $\triangle: [0,1] \times [0,1] \to [0,1]$ is *continuous t-conorm* if $\triangle$ is satisfying the following conditions :

(i) $\triangle$ is commutative and associative,
(ii) $\triangle$ is continuous,
(iii) $a \triangle 0 = a$ for all $a \in [0,1]$,
(iv) $a \triangle b \le c \triangle d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$.

Definition 1.3[2] A 5-tuple $(X, M, N, \circ, \triangle)$ is called an intuitionistic fuzzy metric space if X is an arbitrary set, $\circ$ is a continuous t-norm, and M, N are fuzzy sets on $X^2 \times (0, \infty)$ that satisfy the following conditions: For all $\nu, \omega, \lambda \in X$ and $s, \tau > 0$

(I-1) $M(\nu, \omega, \tau) + N(\nu, \omega, \tau) \le 1$,
(I-2) $M(\nu, \omega, 0) = 0$,
(I-3) $M(\nu, \omega, \tau) = 1$ if and only if $\nu = \omega$,

(I-4) $M(\nu, \omega, \tau) = M(\omega, \nu, \tau)$,
(I-5) $M(\nu, \omega, \tau) \circ M(\omega, \lambda, s) \leq M(\nu, \lambda, \tau + s)$,
(I-6) $M(\nu, \omega, .) : (0, \infty) \rightarrow (0,1]$ is left continuous,
(I-7) $\lim_{\tau\rightarrow\infty} M(\nu, \omega, \tau) = 1$,
(I-8) $N(\nu, \omega, 0) = 1$,
(I-9) $N(\nu, \omega, \tau) = 0$ if and only if $\nu = \omega$,
(I-10) $N(\nu, \omega, \tau) = N(\omega, \nu, \tau)$,
(I-11) $N(\nu, \omega, \tau) \triangle N(\omega, \lambda, s) \leq N(\nu, \lambda, \tau + s)$,
(I-12) $N(\nu, \omega, .) : (0, \infty) \rightarrow (0,1]$ is right continuous,
(I-13) $\lim_{\tau\rightarrow\infty} N(\nu, \omega, \tau) = 0$,

(M, N) is then referred to as an intuitionistic fuzzy metric on X. The functions $M(\nu, \omega, \tau)$ and $N(\nu, \omega, \tau)$ represent the degree of nearness and non-nearness of ν and ω with respect to τ, respectively.

Remark 1.4 Every fuzzy metric space $(X, M, \circ)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1\text{-}M, \circ, \triangle)$ such that t-norm $\circ$ and t-conorm $\triangle$ are associated i.e. $\nu \triangle \omega = 1\text{-} ((1\text{-}\nu) \circ (1\text{-}\omega))$ for all $\nu, \omega \in X$.

Example 1.5(Induced intuitionistic fuzzy metric space) Let (X, d) be a metric space. Define $a \circ b = ab$ and $a \triangle b = \min\{1, a + b\}$ for all $a, b \in [0, 1]$ and let M_d and N_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(\nu, \omega, \tau) = \frac{\tau}{\tau+d(\nu,\omega)}, \quad N_d(\nu, \omega, \tau) = \frac{d(\nu,\omega)}{\tau+d(\nu,\omega)}.$$

Then $(X, M_d, N_d, \circ, \triangle)$ is an intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric space.

Definition 1.6[2] Let $(X, M, N, \circ, \triangle)$ be an intuitionistic fuzzy metric space. Then

(a)A sequence $\{\nu_n\}$ in X is said to be *convergent to a* point ν in X if and only if $\lim_{n\rightarrow\infty} M(\nu_n, \nu, \tau) = 1$ and $\lim_{n\rightarrow\infty} N(\nu_n, \nu, \tau) = 0$ for each $\tau > 0$.

(b)A sequence $\{\nu_n\}$ in X is called *Cauchy sequence* if $\lim_{n\rightarrow\infty} M(\nu_{n+p}, \nu_n, \tau) = 1$ and $\lim_{n\rightarrow\infty} N(\nu_{n+p}, \nu_n, \tau) = 0$ for each $p > 0$ and $\tau > 0$.

(c)An intuitionistic fuzzy metric space $(X, M, N, \circ, \triangle)$ is said to be *complete* if and only if every Cauchy sequence in X is convergent in X.

Lemma 1.7[13] Let $\{\nu_n\}$ be a sequence in an intuitionistic fuzzy metric space $(X, M, N, \circ, \triangle)$ with $\tau \circ \tau \geq \tau$ and $(1\text{-}\tau) \triangle (1\text{-}\tau) \leq (1\text{-}\tau)$ for all $\tau \in [0, 1]$. If $\exists$ a number $\kappa \in (0, 1)$ such that $M(\nu_{n+2}, \nu_{n+1}, \kappa\tau) \geq M(\nu_{n+1}, \nu_n, \tau)$ and $N(\nu_{n+2}, \nu_{n+1}, \kappa\tau) \leq N(\nu_{n+1}, \nu_n, \tau)$, for all $\tau > 0$ and $n \in \mathbf{N}$, then $\{\nu_n\}$ is a Cauchy sequence in X.

Proof :– For $\tau > 0$ and $\kappa \in (0, 1)$ we have,

$$M(\nu_2, \nu_3, \kappa\tau) \geq M(\nu_1, \nu_2, \tau) \geq M(\nu_0, \nu_1, \tau/\kappa)$$

or $$M(\nu_2, \nu_3, \tau) \geq M(\nu_0, \nu_1, \tau/\kappa^2)$$

By simple induction, we have for all $\tau > 0$ and $n \in \mathbf{N}$

$$M(\nu_{n+1}, \nu_{n+2}, \tau) \geq M(\nu_1, \nu_2, \tau/\kappa^n)$$

Thus for any positive number p and real number $\tau > 0$, we have

$$M(\nu_n, \nu_{n+p}, \tau) \geq M(\nu_n, \nu_{n+1}, \tau/p) \circ \ldots\ldots\ldots\ldots \circ M(\nu_{n+p-1}, \nu_{n+p}, \tau/p) \quad \text{[By I – 5]}$$
$$\geq M(\nu_1, \nu_2, \tau/p\kappa^{n-1}) \circ \ldots\ldots\ldots\ldots \circ M(\nu_1, \nu_2, \tau/p\kappa^{n+p-2})$$

Therefore by I – 7, we have

$$M(\nu_n, \nu_{n+p}, \tau) \geq 1 \circ \ldots\ldots\ldots\ldots \circ 1 \geq 1,$$

Similarly, for $\tau > 0$ and $\kappa \in (0, 1)$ we have,

$$N(\nu_2, \nu_3, \kappa\tau) \leq N(\nu_1, \nu_2, \tau) \leq N(\nu_0, \nu_1, \tau/\kappa)$$

or $$N(\nu_2, \nu_3, \tau) \leq N(\nu_0, \nu_1, \tau/\kappa^2)$$

By simple induction, we have for all $\tau > 0$ and $n \in \mathbf{N}$

$$N(\nu_{n+1}, \nu_{n+2}, \tau) \leq N(\nu_1, \nu_2, \tau/\kappa^n)$$

Thus for any positive number p and real number $\tau > 0$, we have

$$N(\nu_n, \nu_{n+p}, \tau) \leq N(\nu_n, \nu_{n+1}, \tau/p) \triangle \ldots\ldots\ldots\ldots \triangle N(\nu_{n+p-1}, \nu_{n+p}, \tau/p) \quad \text{[By I – 11]}$$
$$\leq N(\nu_1, \nu_2, \tau/p\kappa^{n-1}) \triangle \ldots\ldots\ldots\ldots \triangle N(\nu_1, \nu_2, \tau/p\kappa^{n+p-2})$$

Therefore by IFM – 13, we have

$$N(\nu_n, \nu_{n+p}, \tau) \leq 0 \triangle \ldots\ldots\ldots\ldots \triangle 0 \leq 0,$$

$\Rightarrow \{\nu_n\}$is a Cauchy sequence in X. This completes the proof ♦

Lemma 1.8[13] Let $(X, M, N, \circ, \triangle)$ be an intuitionistic fuzzy metric space. If $\forall\ \nu, \omega \in X$ and $\tau > 0$ with positive number $\kappa \in (0, 1)$ and $M(\nu, \omega, \kappa\tau) \geq M(\nu, \omega, \tau)$ and $N(\nu, \omega, \kappa\tau) \leq N(\nu, \omega, \tau)$, then $\nu = \omega$.

Proof :– If for all $\tau > 0$ and some constant $\kappa \in (0, 1)$, then we have

$$M(\nu, \omega, \tau) \geq M(\nu, \omega, \tau/\kappa) \geq M(\nu, \omega, \tau/\kappa^2) \geq \ldots\ldots \geq M(\nu, \omega, \tau/\kappa^n) \geq \ldots,$$

and $$N(\nu, \omega, \tau) \leq N(\nu, \omega, \tau/\kappa) \leq N(\nu, \omega, \tau/\kappa^2) \leq \ldots\ldots \leq N(\nu, \omega, \tau/\kappa^n) \leq \ldots,$$

$n \in \mathbf{N}$ and for all $\tau > 0$ and $\nu, \omega \in X$. Let $n\to\infty$, we have $M(\nu, \omega, \tau) = 1$ and $N(\nu, \omega, \tau) = 0$ and thus $\nu = \omega$ ♦

Definition 1.9[7] Two self mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, \circ, \triangle)$ are called *compatible* if $\lim_{n\to\infty} M(AS\nu_n, SA\nu_n, \tau) = 1$, $\lim_{n\to\infty} N(AS\nu_n, SA\nu_n, \tau) = 0$ whenever $\{\nu_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} A\nu_n = \lim_{n\to\infty} S\nu_n = \nu, \text{ for some } \nu \in X.$$

Definition 1.10[8] Two self mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, \circ, \triangle)$ are called *weakly compatible* if they commute at their coincidence points. i.e. if $Au = Su$ for some $u \in X$, then $ASu = SAu$.

Definition 1.11[1]Two self mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, \circ, \triangle)$ are called *occasionally weakly compatible (owc)* if and only if a point x in X which is coincidence point of A and S at which A and S commute.

Lemma 1.12[1] Let A and S are two owc self mappings of an intuitionistic fuzzy metric space $(X, M, N, \circ, \triangle)$. If A and S have unique point of coincidence, $\mu = A\nu = S\nu$, then w is unique common fixed point of A and S.

Proof: Since A and S are owc, there exists a point ν in X such that $\mu = A\nu = S\nu$ and $AS\nu = SA\nu$. Thus, $AA\nu = AS\nu = SA\nu$, which says that $AA\nu$ is also a point of coincidence of A and S. Since the point of coincidence $\mu = A\nu$ is unique by hypothesis, $SA\nu = AA\nu = A\nu$, and $\mu = A\nu$ is a common fixed point of A and S.

Moreover, if λ is any common fixed point of A and S then $\lambda = A\lambda = S\lambda = \mu$ by the uniqueness of the point of coincidence♦

M.A.Al-Thagafi and Naseer Shahzad [3] shown that occasionally weakly is weakly compatible but converse is not true.

Example 1.13[3] Let R be the usual metric space. Define S, T: $R\to R$ by $Sx = 2x$ and $Tx = x^2$ for all $x \in R$. Then $Sx = Tx$ for $x = 0, 2$ but $ST0 = TS0$, and $ST2 \neq TS2$. S and T are occasionally weakly compatible self maps but not weakly compatible.

Main Results

Following theorem is given by [12]

Theorem Let $(X, M, \circ)$ be a complete fuzzy metric space and Let A, B, S and Tbe self mappings of X. Let the pairs {A, S} and {B, T} be OWC. If there exist $q \in (0, 1)$ such that

$$M(Ax, By, qt) \geq \alpha_1 M(Sx, Ty, t) + \alpha_2 M(Ax, Ty, t) + \alpha_3 M(By, Sx, t)$$

For all $x, y \in X$, where $\alpha_1, \alpha_2, \alpha_3 > 0$, $\alpha_1 + \alpha_2 + \alpha_3 > 1$ then there exist a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of A, B, S and T.

Here we generalized this theorem in intuitionistic fuzzy metric spaces as follows:

Theorem 2.1 Let the pairs (A, S) and (B, T) are occasionally weakly compatible self mappings on complete intuitionistic fuzzy metric space $(X, M, N, \circ, \triangle)$ for any $\nu, \omega \in X$ and $\tau > 0$ with positive number $\kappa \in (0, 1)$ such that

$$M(A\nu, B\omega, \kappa\tau) \geq a_1 M(S\nu, T\omega, \tau) + a_2 M(A\nu, T\omega, \tau) + a_3 M(B\omega, S\nu, \tau) \quad \ldots(i)$$

and

$$N(A\nu, B\omega, \kappa\tau) \leq b_1 N(S\nu, T\omega, \tau) + b_2 N(A\nu, T\omega, \tau) + b_3 N(B\omega, S\nu, \tau), \quad \ldots(ii)$$

Inequalities (i) and (ii) are true for all $\nu, \omega \in X$, where $a_1, a_2, a_3, b_1, b_2, b_3 > 0$, $a_1 + a_2 + a_3 > 1$ and $b_1 + b_2 + b_3 < 1$ then there exist a unique point $\mu \in X$ such that $A\mu = S\mu = \mu$ and a unique point $\lambda \in X$ such that $B\lambda = T\lambda = \lambda$. Moreover, $\lambda = \mu$, so that there is a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} be owc, so there are points $\nu, \omega \in X$ such that

$A\nu = S\nu$ and $B\omega = T\omega$. We claim that, $A\nu = B\omega$. If not, by inequality (i)

$$M(A\nu, B\omega, \kappa\tau) \geq a_1 M(S\nu, T\omega, \tau) + a_2 M(A\nu, T\omega, \tau) + a_3 M(B\omega, S\nu, \tau)$$

$$= a_1 M(A\nu, B\omega, \tau) + a_2 M(A\nu, B\omega, \tau) + aw_3 M(B\omega, A\nu, \tau)$$

$= (a_1 + a_2 + a_3) M(A\nu, B\omega, \tau)$

this gives contradiction since $a_1 + a_2 + a_3 > 1$.

Similarly, by inequality (ii)

$$N(A\nu, B\omega, \kappa\tau) \le b_1 N(S\nu, T\omega, \tau) + b_2 N(A\nu, T\omega, \tau) + b_3 N(B\omega, S\nu, \tau)$$
$$= b_1 N(A\nu, B\omega, \tau) + b_2 N(A\nu, B\omega, \tau) + b_3 N(B\omega, A\nu, \tau)$$
$$= (b_1 + b_2 + b_3) N(A\nu, B\omega, \tau)$$

We get contradiction, because $(b_1 + b_2 + b_3) < 1$. And by Lemma 1.8 $A\nu = B\omega$, i.e. $A\nu = S\nu = B\omega = T\omega$. Let us assume that there is another point λ such that $A\lambda = S\lambda$, then by (i) and (ii), we have $A\lambda = S\lambda = B\omega = T\omega$. This gives, $A\nu = A\lambda$ and $\mu = A\nu = S\nu$,we have conclude that μ is the unique point of coincidence of A and S. By Lemma 1.12 μ is the only common fixed point of A and S, i. e. $\mu = A\mu = S\mu$. Similarly there is a unique point $\lambda \in X$ such that $\lambda = B\lambda = T\lambda$.

Let us assume that $\mu \ne \lambda$. We have,

$$M(\mu, \lambda, \kappa\tau) = M(A\mu, B\lambda, \kappa\tau)$$
$$\ge a_1 M(S\mu, T\lambda, \tau) + a_2 M(A\mu, T\lambda, \tau) + a_3 M(B\lambda, S\mu, \tau)$$
$$= a_1 M(\mu, \lambda, \tau) + a_2 M(\mu, \lambda, \tau) + a_3 M(\lambda, \mu, \tau)$$
$$= (a_1 + a_2 + a_3) M(\mu, \lambda, \tau)$$

this gives contradiction since $(a_1 + a_2 + a_3) > 1$.

Similarly,

$$N(\mu, \lambda, \kappa\tau) = N(A\mu, B\lambda, \kappa\tau)$$
$$\le b_1 N(S\mu, T\lambda, \tau) + b_2 N(A\mu, T\lambda, \tau) + b_3 N(B\lambda, S\mu, \tau)$$
$$= b_1 N(\mu, \lambda, \tau) + b_2 N(\mu, \lambda, \tau) + b_3 N(\lambda, \mu, \tau)$$
$$= (b_1 + b_2 + b_3) N(\mu, \lambda, \tau)$$

again contradiction, because $(b_1 + b_2 + b_3) < 1$. And by Lemma 1.8, $\lambda = \mu$. Also by Lemma1.12, λ is the common fixed point of A, B, S and T. The uniqueness of the fixedpoint holds from (i) and (ii) ♦

Theorem 2.2 Let the pairs (A, S) and (B, T) are occasionally weakly compatible self mappings on complete intuitionistic fuzzy metric space $(X, M, N, \circ, \triangle)$ for any $\nu, \omega \in X$ and $\tau > 0$ with positive number $\kappa \in (0, 1)$ such that

$$M(A\nu, B\omega, \kappa\tau) \ge a_1 \min\{M(S\nu, T\omega, \tau), M(S\nu, A\nu, \tau)\} + b_1 \min\{M(B\omega, T\omega, \tau), M(A\nu, T\omega, \tau)\} + c_1 M(B\omega, S\nu, \tau) \quad \ldots\text{(iii)}$$

and

$$N(A\nu, B\omega, \kappa\tau) \le a_2 \min\{N(S\nu, T\omega, \tau), N(S\nu, A\nu, \tau)\} + b_2 \min\{N(B\omega, T\omega, \tau), N(A\nu, T\omega, \tau)\} + c_2 N(B\omega, S\nu, \tau) \quad \ldots\text{(iv)}$$

Inequalities (iii) and (iv) are true for all $\nu, \omega \in X$, where $a_1, a_2, b_1, b_2, c_1, c_2 > 0$, $a_1 + b_1 + c_1 > 1$ & $a_2 + b_2 + c_2 < 1$ then there exist a unique point $\mu \in X$ such that $A\mu = S\mu = \mu$ and a unique point $\lambda \in X$ such that $B\lambda = T\lambda = \lambda$. Moreover, $\lambda = \mu$, so that there is a unique common fixed point of A, B, S and T.

Proof : - It is given that the pairs {A, S} and {B, T} are owc, so there are points ν, ω in IFM Such that $A\nu = S\nu$ and $B\omega = T\omega$. We claim that, $A\nu = B\omega$. If not, by inequality (iii)

$$M(A\nu, B\omega, \kappa\tau) \ge a_1 \min\{M(S\nu, T\omega, \tau), M(S\nu, A\nu, \tau)\} + b_1 \min\{M(B\omega, T\omega, \tau), M(A\nu, T\omega, \tau)\} + c_1 M(B\omega, S\nu, \tau)$$
$$= a_1 \min\{M(A\nu, B\omega, \tau), M(A\nu, A\nu, \tau)\} + b_1 \min\{M(B\omega, B\omega, \tau), M(A\nu, B\omega, \tau)\} + c_1 M(B\omega, A\nu, \tau)$$
$$= a_1 \min\{M(A\nu, B\omega, \tau), 1\} + b_1 \min\{1, M(A\nu, B\omega, \tau)\} + c_1 M(B\omega, A\nu, \tau)$$
$$= a_1 M(A\nu, B\omega, \tau) + b_1 M(A\nu, B\omega, \tau) + c_1 M(A\nu, B\omega, \tau)$$
$$= (a_1 + b_1 + c_1) M(A\nu, B\omega, \tau)$$

Above inequality gives us contradiction because $a_1 + b_1 + c_1 > 1$

Similarly, by inequality (iv)

$N(A\nu, B\omega, \kappa\tau) \leq a_2 \min\{N(S\nu, T\omega, \tau), N(S\nu, A\nu, \tau)\} + b_2 \min\{N(B\omega, T\omega, \tau), N(A\nu, T\omega, \tau)\} + c_2 N(B\omega, S\nu, \tau)$

$= a_2 \min\{N(A\nu, B\omega, \tau), N(A\nu, A\nu, \tau)\} + b_2 \min\{N(B\omega, B\omega, \tau), N(A\nu, B\omega, \tau)\} + c_2 N(B\omega, A\nu, \tau)$

$= a_2 \min\{N(A\nu, B\omega, \tau), 1\} + b_2 \min\{1, N(A\nu, B\omega, \tau)\} + c_2 N(B\omega, A\nu, \tau)$

$= a_2 N(A\nu, B\omega, \tau) + b_2 N(A\nu, B\omega, \tau) + c_2 N(A\nu, B\omega, \tau)$

$= (a_2 + b_2 + c_2) N(A\nu, B\omega, \tau)$

We get again contradiction, since $a_2 + b_2 + c_2 < 1$. And by Lemma 1.8, $A\nu = B\omega$, i.e. $A\nu = S\nu = B\omega = T\omega$. Suppose that there is another point λ such that $A\lambda = S\lambda$, then by (iii) and (iv), we have $A\lambda = S\lambda = B\omega = T\omega$. Hence we have, $A\nu = A\lambda$ and $\mu = A\nu = S\nu$ is the unique point of coincidence of A and S. By Lemma 1.12, μ is the only common fixed point of A and S, i. e. $\mu = A\mu = S\mu$. Similarly there is a unique point $\lambda \in X$ such that $\lambda = B\lambda = T\lambda$.

Let(Hyp.) $\mu \neq \lambda$. We have,

$M(\mu, \lambda, \kappa\tau) = M(A\mu, B\lambda, \kappa\tau)$

$\geq a_1 \min\{M(S\mu, T\lambda, \tau), M(S\mu, A\mu, \tau)\} + b_1 \min\{M(B\lambda, T\lambda, \tau), M(A\mu, T\lambda, \tau)\} + c_1 M(B\lambda, S\mu, \tau)$

$= a_1 \min\{M(\mu, \lambda, \tau), M(\mu, \mu, \tau)\} + b_1 \min\{M(\lambda, \lambda, \tau), M(\mu, \lambda, \tau)\} + c_1 M(\lambda, \mu, \tau)$

$= a_1 \min\{M(\mu, \lambda, \tau), 1\} + b_1 \min\{1, M(\mu, \lambda, \tau)\} + c_1 M(\mu, \lambda, \tau)$

$= a_1 M(\mu, \lambda, \tau) + b_1 M(\mu, \lambda, \tau) + c_1 M(\mu, \lambda, \tau)$

$= (a_1 + b_1 + c_1) M(\mu, \lambda, \tau)$

We again get contradiction since $a_1 + b_1 + c_1 > 1$

Similarly,

$N(\mu, \lambda, \kappa\tau) = N(A\mu, B\lambda, \kappa\tau)$

$\leq a_2 \min\{N(S\mu, T\lambda, \tau), N(S\mu, A\mu, \tau)\} + b_2 \min\{N(B\lambda, T\lambda, \tau), N(A\mu, T\lambda, \tau)\} + c_2 N(B\lambda, S\mu, \tau)$

$= a_2 \min\{N(\mu, \lambda, \tau), N(\mu, \mu, \tau)\} + b_2 \min\{N(\lambda, \lambda, \tau), N(\mu, \lambda, \tau)\} + c_2 N(\lambda, \mu, \tau)$

$= a_2 \min\{N(\mu, \lambda, \tau), 1\} + b_2 \min\{1, N(\mu, \lambda, \tau)\} + c_2 N(\mu, \lambda, \tau)$

$= a_2 N(\mu, \lambda, \tau) + b_2 N(\mu, \lambda, \tau) + c_2 N(\mu, \lambda, \tau)$

$= (a_2 + b_2 + c_2) N(\mu, \lambda, \tau)$

Again gives contradiction, because $a_2 + b_2 + c_2 < 1$. And by Lemma 1.8 $\lambda = \mu$. Also by Lemma1.12, λ is the common fixed point of A, B, S and T. The uniqueness of the fixed point holds from (iii) and (iv) ♦

Conclusion

We establish common fixed point solutions for occassionally weakly compatible in intuitive fuzzy metric spaces, which enhances and generalizes the work of several writers who have previously presented their findings in the fixed point theory of fuzzy metric spaces.

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