Lower Bounds on the Multiplicative S-index of Various Operations on Graph

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ABSTRACT

The branch of mathematical chemistry known as theory of chemical graphs applies graph theory to the mathematical modelling of chemical phenomena. A molecular graph is a simple graph in chemical graph theory in which the vertices and edges represent the atoms and chemical bonds between them. Topological index is a numerical descriptor of a molecule; it is found that there is strong correlation between the properties of chemical compounds and their molecular structure based on a specific topological feature of the corresponding molecular graph. We introduce a new graph invariant known as the Multiplicative S-index of a graph. In this paper, we establish the lower bounds for the multiplicative S-index of the graph operations such as Join, Cartesian product, Composition, Strong product, Corona product, Corona join product.

Keywords— Zagreb, S-index, Graph operations

# INTRODUCTION

Graph theory has given chemists many useful tools, such as topological indices. Molecules and molecular compounds are frequently represented by molecular graphs. A topological index can be thought of as the conversion of a chemical structure into a real number. Topological and graph invariants based on distances between graph vertices are widely used for characterizing molecular graphs, establishing relationships between structural and property, properties of molecules, predicting biological activities of chemical compounds, and developing chemical applications. Topological indices have the significance of being able to be used directly as simple numerical descriptors in comparison with physical, chemical, or biological parameters of molecules in Quantitative Structure Property Relationships (QSPR) and Quantitative Structure Activity Relationships (QSAR) (QSAR). There are several types of topological indices, including distance-based topological indices, degree-based topological indices, and counting-related polynomials 2 and graph indices. In medicinal chemistry and bioinformatics, the current trend of numerical coding of chemical structures with topological indices or topological coindices has been quite successful [6,12,19].

Let's consider two simple connected graphs, and , each with disjoint vertex and edge sets. For, and represent the number of vertices and edges. The degree of a vertex is the number of edges incident on the vertex and is expressed as for every vertex .

In 1972, I. Gutman and N. Trinajstic [2] defined the first and second Zagreb index of a graph as:

B. Furtula and I. Gutman defined the F-index as [5] in 2015:

In 2020, Abdu Alameri and Noman AI-Naggar [10] introduced the Y-index, which is defined as:

In 2021, S. Nagarajan , G. Kayalvizhi and G. Priyadharsini defined the S-index as [20]:

In 2010, R. Todeschini and D. Ballabio [8] introduced the first and second Multiplicative Zagreb indices of a graph, which is defined as:

In 2019, Asghar Yousefi and Ali Iranmanesh [1] introduced the Multiplicative forgotten topological index, which is defined as:

In (2013) C.D. Kinkar and Y. Aysum [9] derived graph operations in Multiplicative Zagreb indices . K. Xu and K.C. Das [3] computed the Multiplicative Zagreb coindices in (2013). In [1] Y. Asghar and Ali Iranmanesh (2019) derived the Multiplicative F-index of graph operations. M. Radhakrishnan and M. Suresh [15] derived the F-sum operations on graphs in multiplicative zagreb indices in [2020]. In [2012] Liu J and Q. Zhang [16] explained upper bounds on multiplicative zagreb of connected graphs. M. Azari and A. Iranmanesh [17] presented lower bounds of multiplicative sum zagreb index of graphs in [2015]. In [2013] M. Eliasi and D. Vukicevic [11] compared multiplicative indices. M. Azari [14] expressed the lower bounds on narumi-katayama index of graphs in 2014. In this paper, we evaluated the lower bounds for the Multiplicative S-index of various graph operations. Investigators interested in learning more about graph operations can consult to [4,,7,10,13].

**Definition 1.1:** The Multiplicative S-index of a graph G is defined as the product of graph’s four degree vertices and is denoted by:

# MAIN RESULTS

In this section we derived some graph operations in lower bound on the multiplicative S-index.

**Lemma 2.1:** [18] (AM-GM inequality)

Let be a nonnegative numbers. Then

holds with equality if and only if .

**Join:**

The join of graphs and with vertex sets and and edge sets and is the graph union together with all the edges between and . Obviously, and.

**Theorem 2.2:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

**Cartesian Product:**

The Cartesian product of graphs and has the vertex set and is an edge of if and , or and . Obviously, and .

**Theorem 2.3:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

**Composition:**

The Composition of graphs and with disjoint vertex sets and and edge sets and is the graph with vertex set and is adjacent to whenever is adjacent to or and is adjacent to .

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**Theorem 2.4:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

**Strong product:**

The Strong product of a graphs and is a graph with vertex set and any two vertices and are adjacent if and only if or or ..

**Theorem 2.5:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

**Corona product:**

The Corona product  of graphs and with disjoint vertex sets and and edge sets and is the graph obtained by one copy of and copies of and joining the vertex of to every vertex in copy of . Obviously,.

**Theorem 2.6:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

**Corona join product**:

Let and be simple connected graphs, and the Corona join graph of and is obtained by taking one copy of , copies of , and joining each vertex of the copy of with all vertices of . The Corona join product of and is denoted by

**Theorem 2.7:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

# Conclusion

Topological indices are defined and used in many fields to investigate the properties of various objects such as atoms and molecules. Mathematicians and chemists have defined and studied a number of topological indices. We investigated lower bound for the Multiplicative S-index of various graph operations such as Join, Cartesian product, Composition, Strong product, Corona product, Corona join product in this work.

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