**Darcy-Forchheimer flow of Casson nanofluid past a stretching cylinder in the presence of magnetic field**

Gouri Sankar Barik1,\*, Kharabela Swain2

1,\*Research Scholar, Department of Mathematics, Gandhi Institute For Technology, Bhubaneswar, Odisha-752054, India

\*E-mail: kharabela1983@gmail.com

2Department of Mathematics, Gandhi Institute For Technology, Bhubaneswar,

Odisha-752054, India

\*E-mail: kharabela1983@gmail.com

**Abstract**

The magnetic field may be used to efficiently regulate fluid flow and heat transfer. Additionally, it can be employed to improve thermodynamic effectiveness. There are several methods for improving heat transmission, including the use of porous media and nanofluids. Along with boosting heat transmission, porous material may also enhance pressure drop. The flow and heat transfer of Casson nanofluid across a stretched cylinder embedded in a porous media in the presence of a magnetic field are studied numerically in this work. By employing appropriate similarity transformations, the governing partial differential equations (PDEs) are transformed into non-linear ordinary differential equations (ODEs), which are then numerically solved using a successful shooting approach with the use of MATLAB software. Through graphs and tables, the effects of effective parameters on velocity, temperature profiles, skin friction coefficient, and Nusselt number are explained.

**Keywords:** MHD,Darcy-Forchheimer flow, Stretching cylinder, Casson fluid.

**1. Introduction**

Casson fluid, a non-Newtonian fluid with yield stress. Blood exhibits Newtonian behaviour when it is flowing at high shear rates from bigger diameter arteries. But when it passes via narrow arteries with modest shear rates, it behaves in a non-Newtonian manner. Casson [1] thought about the Casson fluid model in relation to blood flow and concluded that the yield stress for blood is nonzero at low shear rates. The peristaltic movement of blood in tiny vessels was researched by Misra and Pandey [2]. The Casson fluid flow with higher order chemical reaction was explored by Animasaun [3]. The Casson nanofluid flow across a stretched sheet has been quantitatively examined by Senapati et al. [4] in a three dimensional study.

Numerous industrial and engineering uses for the flow and heat transfer across a stretched sheet include the cooling of metallic plates, the polymer industry, the creation of paper, etc. The mixed convection flow across a stretched sheet was investigated by Bhukta et al. [5]. In the presence of a chemical reaction, Swain et al. [6–8] examined the flow, heat, and mass transfer of nanofluids over a stretched sheet. Thermal radiation was used by Hayat et al. [9] to study the flow and heat transmission through a stretched cylinder. The effects of Joule heating and viscous dissipation on Sisko nanofluid along a stretched cylinder were investigated by Hussain et al. [10]. The flow of Casson fluid through a stretched cylinder was investigated by Tamoor et al. [11]. Mahdy and Chamkha [12] and Hayat et al. [13] both looked into the Maxwell fluid flow in non-Darcy porous media with chemical reaction. Darcy-Forchheimer nanofluid flow across a nonlinear stretching sheet was seen by Rasool et al. [14].

The primary goal of the current work is to use the Darcy-Forchheimer model to examine the flow and heat transfer of Casson fluid via a stretched cylinder. Using appropriate similarity solutions, the controlling PDEs are transformed into coupled non-linear ODEs. These ODEs are numerically solved using a successful shooting approach with the aid of the MATLAB programme. Operating parameters are numerically calculated to the necessary level of precision. Tables and graphs are used to discuss the consequences of various factors.

**2. Mathematical Formulation**

Consider a steady two dimensional electrically conducting Darcy-Forchheimer flow of Casson fluid past an elongating cylinder in a saturated porous medium. The flow is caused by the linear stretching of cylinder with velocity . The flow is along axial (*x*) direction whereas radial direction is perpendicular to *x*. A uniform transverse magnetic field of strength is applied normally to the surface of cylinder (along *r* direction). The flow is subjected to an additional body force due to the presence of porous matrix. It is assumed that

* Magnetic Reynolds number of fluid is very low, negating the impact of an induced magnetic field.
* No slippage occurs between the fluid and the bounded surface.
* Viscous dissipation caused by internal resistance is handled by heat transfer. Joule heating is handled by a resistance to the flow of electric current.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid (Senapati et al. [4]) is expressed as



where is the rate of strain tensor,  is the component of stress tensor,  is the Casson coefficient of viscosity,  is the product of the rate of strain tensor with itself,  is the critical value of the product of the rate of strain tensor with itself,  is the yield stress of the fluid and is the Casson fluid parameter.

Under the boundary layer approximation and above assumptions, the governing equations of continuity, momentum and energy are given by

 (1)

 (2)

 (3)

where are velocity components in *x* and *y* directions respectively, is the magnetic field strength, is the kinematic viscosity, is the electrical conductivity, is the density, is the thermal conductivity, is the temperature of the fluid, is the specific heat, and is the quadratic drag coefficient.

The boundary conditions are

 (4)

where is the reference velocity,  is the characteristic length,  is the heat transfer coefficient, and is the ambient temperature.

Consider the stream function  such that dimensionless similarity transformations 

Therefore, the equation (1) is identically satisfied and equations (2) and (3) convert

 (5)

 (6)

The corresponding boundary conditions (4) become

 (7) where is the Hartman number , is the porosity parameter, is the curvature parameter, is the local inertia parameter, is the Prandtl number, and is the conjugate parameter.

The surface conditions of engineering interest such as the skin friction and Nusselt number are given by andrespectively.

Here wall shear stress and wall heat flux and is the local Reynolds number.

**3. Results and Discussion**

The dimensionless coupled non-linear ODEs (5) - (6) with appropriate boundary conditions (7) are solved numerically by shooting technique using MATLAB software with step length and the error tolerance. During calculation we fix the parameters as and unless otherwise the values are mentioned. The values of is calculated to validate our results with previously published results of Tamoor et al. [11] for various values of as shown in Table 1. It is found that our numerical results are in good agreement.

**Table 1** Comparison of  for various values of when and 

|  |  |
| --- | --- |
|  |  |
|  | Tamoor et al. [11] | Present study |
| 0.0 | 1.00000 | 1.0000083 |
| 0.2 | 1.01980 | 1.0198092 |
| 0.5 | 1.11803 | 1.1180343 |
| 0.8 | 1.28063 | 1.2806249 |
| 1.0 | 1.41421 | 1.4142136 |

Fig. 1 displays the influence of Hartman number  on velocity profile in the presence and absence of porous matrix. It is seen that in absence of porous matrix the velocity of the fluid is higher than that of presence of porous matrix. Further,  declines with an increase in. Physically, in presence of magnetic field generates a resistive force called Lorentz force which deceleratesto contribute a thinner momentum boundary layer. The impact of inertia coefficient  onis presented in Fig. 2 for both Casson fluidand Newtonian fluid. It is seen that Casson parameter enhances the velocity profile, whereas the Forchheimer parameter (local inertia parameter), which is responsible for the inertia drag has a reverse effect on the fluid velocity. Fig. 3 portrays the outcome of radius of curvatureon. The velocity profile slightly declines with higher values of near the cylinderbut increases with increase values of. The same observation was made by Hayat et al. [9].

From Fig. 4 it is observed that the temperature profileenhances with increase values ofand . Physically, Lorentz force resists the fluid flow and consequently, causes extra heat producing a thicker thermal boundary layer. Fig. 5 shows the behaviour of on  for both Casson fluidand Newtonian fluid. It is perceived that decreases with an increase in. Physically, higher Prandtl number fluid having lower thermal diffusivity which reduces the conduction and consequently thermal boundary layer decreases. Moreover, Casson fluid has lower velocity than that of Newtonian fluid. Fig. 6 describes the influence of curvature parameter and conjugate parameteron. It is seen that as  increases, more amount of heat is transferred from heated surface of the cylinder to cooled surface of the fluid. Consequently, thickness of thermal boundary layer enhances but the adverse effect is observed on the fluid temperature in case of.



Fig. 1 Effect of andon velocity profile



Fig. 2 Effect of on velocity profile



Fig. 3 Effect of on velocity profile



Fig. 4 Effect of andon temperature profile



Fig. 5 Effect of  on temperature profile



Fig. 6 Impact of and on temperature profile

**Table 2** Computation of and when 

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 0.1 | 0.1 | 0.1 | 2.325000 | 1.497189 |
| 0.3 |  |  | 2.377730 | 1.493493 |
| 0.5 |  |  | 2.480170 | 1.486296 |
|  | 0.5 |  | 2.638612 | 1.477482 |
|  | 1 |  | 2.824700 | 1.467178 |
|  |  | 0.3 | 3.091958 | 1.904015 |
|  |  | 0.5 | 3.354159 | 2.237804 |

 Table 2 shows the computed values of skin friction coefficientand local Nusselt numberfor various values of operating parameters such as and. It is perceived that and enhance the skin friction coefficient but the rate of heat transfer at the surface decreases with increasing values ofand  but increases with. Thus there exists Reynolds analogy between skin friction and surface flux [15]. The same observation was made by Hussain et al. [10] and Tamoor et al. [11].

**4. Conclusions**

The key findings from the current study are:

* Lower values of the velocity profile are produced by higher values of the Hartman number, whereas higher values of the thermal resistance result in higher temperatures profiles.
* The Forchheimer parameter, which causes the inertia drag, has a reverse impact on the fluid velocity, whereas the Casson parameter improves the velocity profile.
* and enhance the skin friction coefficient but the rate of heat transfer at the surface decreases with increasing values ofand  but increases with.
* As the Prandtl number is increased, the temperature profile drops.

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