Complex Fermatean pentapatitioned neutrosophic sets and its Applications

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**Abstract:**

Complex fuzzy sets and complex intuitionistic fuzzy sets are incapable of dealing with imprecise, indeterminate, inconsistent, and incomplete periodic information. To address this issue, we propose a complex fermatean pentapartitioned neutrosophic set whose complex valued truth membership function, complex valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, and complex valued false membership function are the combination of real valued truth amplitude term in association with phase term, real-valued contradiction amplitude term in association with phase term, real-valued unknown membership function, and complex valued false membership function. Complex fuzzy sets and complex intuitionistic fuzzy sets are incapable of dealing with imprecise, indeterminate, inconsistent, and incomplete periodic information. To address this issue, we propose a complex fermatean pentapartitioned neutrosophic set whose complex valued truth membership function, complex valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, and complex valued false membership function are the combination of real valued truth amplitude term in association with phase term, real-valued contradiction amplitude term in association with phase term, real-valued unknown membership function, and complex valued false membership function.

**Keywords:**

Complex fermatean pentapartitioned neutrosophic set, complex fermatean pentapartitioned neutrosophic relation, complex fermatean pentapartitioned neutrosophic applications.

1. **Introduction:**

Zedeh's significant paper was the first to propose fuzzy sets. This unique notion is successfully employed in modeling uncertainty in many real-world applications. A membership function with the range [0, 1] characterizes a fuzzy set. Fuzzy sets and their applications have been extensively researched in several areas. In 1986, Atanassov created intuitionistic fuzzy sets, which include the hesitation degree known as the hesitation margin. The hesitating margin is equal to one less the sum of membership and non-membership. As a result, the intuitionistic fuzzy set is defined by a membership function and a non-membership function with values between [0, 1].

Ramot et al. [6] developed a new approach for fuzzy set extension by introducing complex fuzzy sets in which the degree of membership is swapped by a complex value of the type

 where and are both in the range [0,1].The range in complex unit disk is . In 1995, Florentin Smarandache introduced the concept of Neutrosphic set, which enables knowledge of neutral thought by introducing a new factor in the set termed indeterminacy. As a result, the neutrosophic set was created, which includes the components of the truth membership function (T), indeterminacy membership function (I), and falsity membership function (F). Neutrosophic sets are concerned with the nonstandard interval [0, 1].

 Wang [4] (2010) developed the concept of single-valued nuetrosophic sets (SVNS), commonly known as an extension of intuitionistic fuzzy sets, which has since been a hot study area. Fermatean Pentapartitioned Single Valued Neutronosophic Sets, proposed by Rajashi Chatterjee et al., are based on Belnap's four logics and Smarandache's four numerically valued logics. In FPSVNS, indeterminacy is divided into two functions known as 'contradiction' (both true and false) and 'unknown' (neither true nor false), resulting in five components: TA , CA , KA , UA and FA, all of which lie in the non-standard unit interval [0, 1].

 This research applies Ramot et al. [6], Alkouri and Saleh [1], and Cai and Zhang et al. [8]'s work to neutrosophic sets. To introduce a sophisticated neutrosophic set, we basically follow the theory of Ramot et al. [6]. A complex valued truth membership function, a complex valued indeterminate membership function, and a complex valued falsity membership function define the complex neutrosophic. Furthermore, the complex neutrosophic set is widely used because it not only generalizes all present frameworks but also defines information in a thorough and comprehensive manner.

 A relation, like a set, is vital in all engineering, scientific, and mathematically oriented fields. Ralationise has a deep understanding of logic, approximation reasoning, rule-based systems, nonlinear simulation, synthetic evaluation, classification, pattern recognition, and control. The relationships between fuzzy sets and intuitionistic fuzzy sets have been widely researched in the neutrosophic environment. Yang et al. suggested and investigated single valued neutrosophic relations (SVRS).

 The rest of this paper is structured as follows: Section 2 recaps some of the core principles associated with complicated neutrosophic sets and CNSs. In Section 3, we define Complex Fermatean Pentapartitioned Neutron Sets (CFPNs) as a precursor to the concept of CNR. Following that, the notion of CNR will be defined. This component also generates a decision-making algorithm. Using the CFPNR features, this algorithm investigates the efficacy of a variety of training strategies. In Section 4, we define various fundamental operations on CFPNRs, such as complement, inverse, and composition. This section also includes projection definitions for CFPNRs. Section 5 compares CFPNR and other current approaches to the supremacy of our suggested strategy in detail. Section 6 summarizes the paper's conclusion and proposes future study topics.

1. **Preliminaries:**

A complicated neutrosophic set is defined as follows:

**Definition 2.1:**

A complex neutrosophic set A, of course, is defined on a universe X, for any x X, TA(x ) a truth membership function, an indeterminacy membership function IA(x ), and a falsity membership function FA(x ) assign a complex-valued grade of TA(x ), IA(x ), and FA(x ) in S. The values TA(x ), IA(x ), and FA(x ), as well as their sum, may all be within the complex plane's unit circle and so have the following form:

TA(x) =

IA(x) =

FA(x) = where ,, and ,, are respectively, real valued and ,, [0,1] such that 3.

 The complex neutrosophic set S can be expressed in set form as

,

 where , and

 , and

**Complex Neutrosophic Relations:**

In this part, we propose the Cartesian product of two CNSs, followed by the formal definition of the CNR.

**Definition 2.2**

Let X and Y represent two complex neutrosophic sets over U and V, respectively. The Cartesian product of X and Y, represented by X Y, is a CNS defined as

, where is a complex valued truth membership function is a complex valued indeterminacy membership function and is a complex- valued falsity membership function and ,

,

,

.

**Definition 2.3: [7]**

Consider X a universe. An object of the form A Fermatean pentapartitioned neutrosophic set (FPN) A on X is written as A = {< x, TA , CA , KA , UA , FA ,) >: x X }

 (TA)3 + (CA)3 + (KA)3+ (UA)3 + (FA)3 ≤ 3

Here, TA(x) is the truth membership.

CA(x) is contradiction membership,

KA(x) is ignorance membership

UA (x) is unknown membership,

FA(x) is the false membership.

1. **Complex Fermatean Pentapartitioned neutrosophic sets:**

The following is the definition of a complex fermatean pentapartitioned neutrosophic set:

**Definition 3.1:**

A complex fermatean pentapartitioned neutrosophic set A defined on a universe of course X, It is distinguished by a truth membership function TA(x ), CA(x) is contradiction membership function, KA(x) is ignorance membership function, UA (x) is unknown membership function and FA(x) that assigns a complex-valued grade of TA(x ), CA(x), KA(x) , UA (x) ,and FA(x) in S for any x X. The values TA(x ), CA(x), KA(x) , UA (x) and FA(x)is the false membership function. Their entire can take the following form and be contained entirely within the unit circle in the complex plane.

TA(x) =

CA(x) =

KA(x) =

UA (x) =

FA(x)=where,,,,and ,,,,

are respectively, real valued and ,,,, [0,1] such that

 ,,,, 3.

 The complex fermatean pentapartitioned neutrosophic set S can be expressed in the set form as

 where , ,

 ,, and

 , and

**Definition 3.2.**

Given X to be a universal set, a complex fermatean pentapartitioned neutrosophic set A is defined as:

A = {< x, TA(x ) , CA(x) , KA(x) , UA (x) , FA(x)) >: x X } where TA(x ) is the degree of truth membership,CA(x) is called the degree of contradiction membership, KA(x) is called the degree of ignorance membership ,

UA (x) is called the degree of unknown membership and FA(x) is the degree of false membership that has a mapping TA : X → { z1 : z1 ∈ C : | z1| ≤ 1}

CA : X → { z2 : z2 ∈ C : | z2| ≤ 1}, KA : X → { z3 : z3 ∈ C : | z3| ≤ 1}, UA : X → { z4 : z4 ∈ C : | z4| ≤ 1}

And FA : X → { z5 : z5 ∈ C : | z5| ≤ 1}. For every x ∈ X, the degree of truth membership is

 TA(x) = , the degree of contradiction membership is CA(x ) = , the degree of ignorance membership KA(x ) = , the degree of unknown membership

 UA(x) = and the degree of false membership is FA(x) = respectively.

Where TA , CA , KA , UA , FA ∈ [0,1] , ,

 , .

++ + +.

* 1. **Interpretation of complex fermatean pentapartitioned neutrosophic sets**
* The concept of complex valued truth membership function, complex-valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, complex valued false membership function and complex valued is not a simple task in understanding. Real valued truth membership function, real-valued contradiction membership function, real-valued ignorance membership function, real valued unknown membership function, real-valued false membership function in the interval [0, 1] can be easily perceptive.

Understanding the concept of a complex fermatean pentapartitioned neutrosophic set is straightforward when looking at its membership functions, which are represented by the following: ignorance, truth, contradiction, unknown, and false.

**Definition 3.3.1.**

TA(x) =

CA(x) =

KA(x) =

UA (x) =

FA(x) =

It is clear that a truth amplitude term and a truth phase term define the complex grade of truth membership function. Similarly, the complex grade of contradiction membership function is defined as a contradiction amplitude term and a contradiction phase term ; the complex grade of ignorance membership function is defined as an ignorance amplitude term and a contradiction phase term ; the complex grade of unknown membership function is defined as an unknown amplitude term and a contradiction phase term ; and the complex grade of false membership function is defined as a false amplitude term and a false phase term , respectively. The truth amplitude term equal to, the amplitude term . The contradiction amplitude is equal to, ignorance amplitude is equal to , unknown amplitude is equal , and false amplitude terms is equal to .

Neutronosophic sets are generalized into complex fermatean pentapartitioned sets. Representing a neutrosophic set as a complex fermatean pentapartitioned neutrosophic set is a simple task. As an illustration, the real-valued truth membership function, contradiction membership function, ignorance membership function, unknown membership function, and false membership function all describe the neutrosophic set S. Complex fermatean pentapartitioned neutrosophic sets are generalizations of neutrosophic sets. A neutrosophic set in the form of a complex fermatean pentapartitioned neutrosophic set is an easy task. For example, the neutrosophic set S is characterized by a real-valued truth membership function, a contradiction membership function , an ignorance membership function , an unknown membership function , and a false membership function . By setting the truth amplitude term is equal to , and the truth phase term equal to zero for all x, and similarly, we can set the contradiction amplitude term equal to and a contradiction phase term equal to zero, the ignorance membership amplitude term equal to and a ignorance phase term equal to zero, the unknown amplitude term equal to and an unknown phase term equal to zero, and the false amplitude term equal to and a false phase term equal to zero for all x. Consequently, it is observed that a fermatean pentapartitioned neutrosophic set can be readily generated from a complex fermatean pentapartitioned set. The explanation that follows identifies the following: the truth amplitude term corresponds to the real-valued grade of the truth membership function; the contradiction amplitude term, to the real-valued grade of the contradiction membership function; the ignorance amplitude term, to the real-valued grade of the ignorance membership function; the unknown amplitude term, to the real-valued grade of the unknown membership function; and the false amplitude term, to the real-valued grade of the false membership function.

The truth phase term, contradiction phase term, ignorance phase term, unknown phase term, and false phase term are the sole variables that set them apart. This sets apart the regular neutrosophic set from the complex fermatean neutrosophic set. Put another way, the complicated fermatean pentapartitioned neutrosophic set will effectively transform into the fermatean pentapartitioned neutrosophic set if all five phase terms are eliminated. The fact that , , and and have range [0, 1], which is the real-valued grade of membership in the truth, the real-valued grade of membership in contradiction, the real-valued grade of membership in ignorance, the real-valued grade of membership in the unknown, and the real-valued grade of false membership, supports every discussion in this article.

Accordingly, complex fermatean pentapartitioned neutrosophic sets are the most sophisticated generalization of all existence techniques; as a result, complex fermatean neutrosophic sets represent a unique and noteworthy idea.

* 1. **Numerical example of a complex fermatean pentapartitioned neutrosophic set**

**Example 3.4.1.**

Let us consider that there is a universe of discourse. Then S is a complex fermatean pentapartitioned neutrosophic set in X, as given below:

 +

Set theoretic operations on a complex fermatean pentapartitioned neutrosophic set

Ramot et al. [6] calculated the complement of the membership phase term by several possible methods, such as Zhang [8] defined the complement of the membership phase term by taking the rotation of by radian as.

To define the complement of a complex fermatean pentapartitioned neutrosophic set, we simply take the neutrosophic complement for the truth amplitude term , the contradiction amplitude term , the ignorance amplitude term , the unknown amplitude term , and the false amplitude term . For phase terms, we take the complement defined in [6]. We now proceed to define the complement of the complex fermatean pentapartitioned neutrosophic set.

**Definition 3.5. Complement of complex fermatean pentapartitioned neutrosophic set.**

Let us consider A = {< x, TA(x),CA(x), KA(x), UA (x), FA(x)) >: x X} being a complex fermatean pentapartitioned neutrosophic set in X. Then the complement of a complex fermatean pentapartitioned neutrosophic set A is defined as c (A) and is defined by

 ,

Where = c ( , ,,

, in which c(= c( is such that c( and or .

Similarly, and or .

 and or .

 and or .

 Finally, where and

 Or.

**Proposition 3.6**

Let us consider A be a complex fermatean pentapartitioned neutrososphic set on so, c(c(A)) = A.

Proof By definition 3.1. , it is clearly demonstrated.

**Proposition 3.7.**

Let us consider A and B to be two complex fermatean pentapartitioned neutrososphic sets on X.

Then.

**Definition 3.8. Union of complex fermatean pentapartitioned neutrososphic sets.**

The union of two complex fermatean pentapartitioned neutrososphic sets A and B was defined as follows by Ramot et al. [6]: Let and be the complex-valued membership functions of A and B, respectively. Then the membership union of is given by . Since and are real valued and belonging to [0,1], the operators max and min may be used on them to determine the membership union of. Several methods are provided for computing the phase term. We now define the following as the union of two complex fermatean pentapartitioned neutrososphic sets:

Given two complex fermatean pentapartitioned neutrososphic sets X, A and B are characterized as follows:

A = {< x, TA(x ) , CA(x) , KA(x) , UA (x) , FA(x)) >: x X } and

B = {< x, TB(x) , CB(x) , KB(x) , UB (x) , FB(x)) >: x X }

Then the union of A and B is represented as A∪B and is provided as

Here the truth membership function, the contradiction membership function, the ignorance membership function, the unknown membership function and false membership function are described by

Where and indicate the max and min operators respectively. To calculate phase terms and We clarify the following:

**Definitions 3.9**

Given two complex fermatean pentapartitioned neutrososphic sets in X, A and B have the following membership functions: complex-valued truth TA(x ) and TB(x ); complex-valued contradiction CA(x ) and CB(x ); complex-valued ignorance KA(x ) and KB(x ); complex-valued unknown UA(x ) and UB(x ); and complex-valued false FA(x ) and FB(x ), respectively. The function associated with the union of the complex fermatean pentapartitioned neutrososphic sets A and B is represented by:

 X

.

* A complex value is imposed by, that is, for all x X,

 , ,,

and.

The following axiomatic conditions must be met, at the very least, by this function: Regarding any

:

Axiom 1: , , ,

 and ( boundary conditions).

Axiom 2 : , ,

 and (commutative condition).

Axiom 3 : If , then ,

, and

(Monotonic condition).

Axiom 4 : , ,

, ,

and ) (associative condition).

 It may be possible in some cases that the following are also held:

Axiom 5: (continuity).

Axiom 6 : , ,

 , and (super idempotency).

Axiom 7: and , then ,

 and, then ,

 and, then

 and, then

and and , then

(Strict monotonicity).

The phase term of complex truth membership function, complex contradiction membership function ignorance membership function, unknown membership function, and false membership function pertains to (0,). In order to specify the phase terms, we assume

,,

 ,,.

We now use those forms to define the phase terms that Ramot et al [6] gave.

 respectively.

Sum :

.

Maximum :

.

Minimum :

.

“The game of winner, contradictor, ignorance person, unknown person, and loser”:

The game of winner, contradictor, ignorant person, unknown person, and loser is a novel concept, and it is the generalization of the concept “winner take all” introduced by Ramot et al. [6] for the union of phase terms.

**Example 3.9.1**

Consider to be a universe of discourse. Regarding complex fermatean pentapartitioned neutrosophic sets, let A and B be as follows:

 +,

and

 +

Then

**Definition 3.10: Intersection of complex fermatean pentapartitioned neutrosophic set.**

Considering two complex fermatean pentapartitioned neutrosophic sets in X, A and B are as follows:

A = {< x, TA(x) , CA(x) , KA(x) , UA (x) , FA(x)) >: x X } and

B = {< x, TB(x),CB(x) , KB(x) , UB (x) , FB(x)) >: x X }

Next, A and B's intersection is represented as A∩B and can be written as

Where the truth membership function, the contradiction membership function, the ignorance membership function, the unknown membership function and false membership function are expressed by

Where and indicate the max and min operators respectively. To compute the phase terms and we describe the following:

**Definitions 3.11**

Let us consider A and B to be two complex fermatean pentapartitioned neutrososphic sets in X with complex-valued truth membership functions  TA(x ) and TB(x ), complex-valued contradiction membership functions CA(x ) and CB(x ), complex-valued ignorance membership functions  KA(x ) and KB(x ), complex-valued unknown membership functions UA(x ) and UB(x ) and complex-valued false membership functions FA(x ) and FB(x ) respectively. The intersection of the complex fermatean pentapartitioned neutrososphic sets A and B is described by, which is related to the function:

 X

.

 A complex value is allocated by, that is, for all x X,

 , ,,

and .

At the very least, this function needs to meet the following axiomatic requirements: Any of a, b, and c

:

Axiom 1: , , ,

 and ( boundary conditions).

Axiom 2: , ,

 and (commutative condition).

Axiom 3: If , then ,

, and

(Monotonic condition).

Axiom 4: ,,

, ,

 and) (associative condition).

 In certain circumstances, it's feasible that the following also holds true:

Axiom 5: (continuity).

Axiom 6 : , ,

 , and (super idempotency).

Axiom 7 : and , then ,

 and, then ,

 and, then

 and, then

and and , then

(Strict monotonicity).

On the same lines by winner, contradiction person, ignorance person, unknown person, and loser game, we can easily calculate the phase terms,,, and .

**Preposistion 3.12**

Let A, B, C be three complex fermatean pentapartitioned neutrososphic sets on X. Then,

Proof: We merely demonstrate that portion 1 here.

Assuming that A, B, and C are three complex fermatean pentapartitioned neutrososphic sets in X, we have A = {< x, TA(x) , CA(x) , KA(x) , UA (x) , FA(x)) >: x X } ,

B = {< x, TB(x),CB(x), KB(x), UB (x), FB(x)) >: x X} and

C = {< x, TC(x) , CC(x) , KC(x) , UC (x) , FC(x)) >: x X }, according to be their complex valued truth membership function, complex-valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, complex valued false membership function.

Next, we have

=

 = min (max (

 = max (min (

 =max ((

 = (=

Likewise, we may demonstrate it for

and in the same way.

**Preposition 3.13**

Consider two complex fermatean pentapartitioned neutrososphic sets on X, denoted by A and B. Thus,

Proof: We establish it in part 1. Given A and B two complex fermatean pentapartitioned neutrosophic sets in X, let A = {< x, TA(x) , CA(x) , KA(x) , UA (x) , FA(x)) >: x X } and

B = {< x, TB(x),CB(x), KB(x), UB (x), FB(x)) >: x X} be the corresponding complex valued truth, complex-valued contradiction, complex valued ignorance, complex valued unknown membership function, and complex valued false membership functions.

Next,

=

 = min (max (

 = .

Likewise, we may demonstrate it for

,

and respectively.

**Definition 3.14.**

Consider two complex fermatean pentapartitioned neutrososphic sets on X, denoted by A and B. Thus, ,,,

 , and

,,

 be the corresponding complex-valued truth membership function, complex-valued contradiction membership function, complex-valued ignorance membership function, complex-valued unknown membership function, complex-valued false membership function. The complex fermatean pentapartitioned neutrososphic. The product of A and B is shown as A o B and is specified by the functions, ,

,

,

,

,

**Example 3.14.1.** Let be a universe of discourse. Assume that A and B are two complex fermatean pentapartitioned neutrosophic sets with the following characteristics:

 +

And

 +

Then

 ,

**Definition 3.15.**

Let be N complex fermatean pentapartitioned neutrosophic sets on X (n=1, 2, 3….N), and

,,,

 , be their complex-valued truth membership function, complex-valued contradiction membership function, complex-valued ignorance membership function, complex-valued unknown membership function, complex-valued false membership function.

As represented by the function, the Cartesian product of is.

 = min ((

 = min (()

 = max ((

 = max ((

And

= max ((

Where. (N times X)

**4. Distance measure and -equalities of complex fermatean pentapartitioned neutrosophic sets**

We reviewed the distance measure and additional operational characteristics of complex feramtean pentapartitioned neutrosophic sets in this section.

**Definition 4.1**

Assume that all complex feramtean pentapartitioned neutrosophic sets on X are collected in CN(X). Then, A and B are members of CN(x). If is such that the amplitude terms and a truth phase term and such that the amplitude terms and a contradiction phase term , amplitude terms and the phase terms , amplitude terms and a contradiction phase terms, amplitude terms and a falsity phase terms respectively, then A is included in or equal to B ie., B .

**Definition 4.2**

Two complex fermatean pentapartitioned neutrosophic sets A and B are said to be equal if and only if ,,, and are for amplitude terms and ,,, and are for phase terms (arguments).

**Definitions: 4.3**

A distance of complex fermatean pentapartitioned neutrosophic sets is a function

, such that for any A, B, C

,

 ,

.

Let be a function which is defined as

**Theorem 4.4**

The above-described function is a function of complex fermatean pentapartitined neutrosophic sets on X.

Proof: The proof is simple to understand.

**Definition 4.5**

Assume that A and B are two complex fermatean pentapartitioned neutrosophic sets on X.

Then,

,,,

 , and

,,

 are their complex-valued truth membership function, complex-valued contradiction membership function, complex-valued ignorance membership function, complex-valued unknown membership function, complex-valued false membership function.

Then A and B are said to be -equal, if and only if, where it is indicated by .

**Preposition 4.6**

The following is true for complex fermatean pentapartitioned neutrosophic sets A, B, and C.

1. A = (0) B.
2. A = (1) B if and only if A = B.
3. If if and only if
4. andthen
5. If, then for all, where J is an index set.
6. If andthere exist a unique such that, then for all A,B
7. If and, then where.

Proof: Properties 1-4 and 6 are readily demonstrable. Only 5 and 7 are proven by us.

Therefore, for all, we have

Accordingly,

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,

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, and

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Thus,

Hence, .

7. Since, we have

Which implies

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Also we have , so

Which implies

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, and

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,

,

Now

Now

 = = +-1),

From definition 4.3,

 . Therefore, -\* = 1- , where =\*. Thus,.

**Theorem 4.7**

If , then, where c(A) and c(B) are the complement of the complexfermatean pentapartitioned neutrosophic sets A and B.

Proof: Since

 = .

**5. Relations between the CFPNS with their applications in Decision Making**

**Definition 5.1**

Consider AN to be N CFPNSs on X, (n = 1,2,3,….,N), and be a complex-valued truth membership function, CA(x) = be a complex-valued contradiction membership function, KA(x) = be a complex-valued ignorance membership function,

 UA (x) = be a complex-valued unknown membership function and

FA(x) = be a complex-valued false membership function. The Cartesian products of indicated as, are described by the functions:

,

,

,

,

**5.2. Complex fermatean pentapartitioned neutrosophic relation**

 This section presents the formal definition of the CFPNR followed by the definition of the Cartesian product between two CFPNSs. We also solve an MCDM problem using the CFPNR approach. The Cartesian product of two CFPNSs is defined as follows:

**Definition 5.3.**

For each of the two universal sets, U and V, let X and Y be two CFPNSs. X × Y represents the Cartesian product of X and Y, which is a CFPNS that is defined as

,

where is a complex valued truth membership function is a complex valued contradiction membership function, is a complex valued ignorance membership function, is a complex valued unknown membership function, and is a complex- valued falsity membership function and ,

,

,

,

,

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Now, let's define the term CFPNR as follows:

**Definition 5.4.**

Consider X and Y to be two CFPNSs in the universes U and V, respectively. A complex neutrosophic subset of X×Y is a complex fermatean pentapartitioned neutrosophic connection from X to Y. Consequently, a CFPNS from X to Y is represented by R(X, Y), where R(X,Y)⊆ X×Y. R(X,Y) is always represented as the set of ordered sequences

, where,, TR(u,v) =

CR(u,v) =

KR(u,v) =

UR (u,v) =

FR(u,v) = .

The values TR(u,v) ,CR(u,v) ,KR(u,v) ,UR (u,v) ,FR(u,v) are within the unit circle in the complex plane and both the amplitude terms and the phase terms are real valued such that

TR(u,v) ,CR(u,v) ,KR(u,v) ,UR (u,v) ,FR(u,v) and

.

We now present a practical use of CFPNR to demonstrate how well it describes and analyzes actual occurrences.

**6 Complex fermatean pentapartitioned neutrosophic relation Education:**

When time is a significant component and indeterminacy is inevitable, CFPNR can be used to quantify the interaction between various education variables efficiently. We now present a relation between two CFPNSs as an example.

**Example 6.1.**

Assume assessment is done to find the most effective teaching method that raises student achievement. Let U denote the collection of educational strategies used with a particular set of pupils, where

U = {.

Let V represent a collection of metrics measuring the interaction and academic accomplishment of the student, where

V= {.

Consider two CFPNSs over U and V, respectively, X and Y, which can be described as follows:

We now calculate the relationship between the two CFPNSs, X and Y, to look into how students' performance is impacted by contemporary teaching techniques. The CFPNS represented by R(X, Y) is such that R(X, Y) ⊆ X×Y.

Assume that a 12-month period is used to measure the relationship between X and Y. In our example, the terms of truth amplitude, contradiction amplitude, ignorance amplitude, unknown amplitude and false amplitude, of R(X, Y) measure the truth membership degree of the impact of the modern methods in education on the student’s performance, the contradiction membership degree of the effect of the recent techniques in education on the student’s effectiveness, the ignorance membership degree of the effect of the recent techniques in education on the student’s effectiveness, the unknown membership degree of the effect of the recent techniques in education on the student’s effectiveness, and false membership degree of the effect of the recent techniques in education on the student’s effectiveness respectively, furthermore the truth phase term, the contradiction phase term, the ignorance phase term, the unknown phase term and false phase term of R(X, Y) represent the period of time in which the recent techniques influence the student’s effectiveness, the time frame within which we cannot say whether or if the use of contemporary methods affects students' performance and the time frame within which such methods have no effect on the achievement of learners, respectively. Given that the phase terms in R(X, Y) denote time intervals and that R(X, Y) depicts the relationship between contemporary educational practices and students' performance over a 12-month period, the range values of each complex fermatean pentapartitioned neutrosophic value, the range value of each of the truth, contradiction, ignorance, unknown, and false phase terms should fall between 0 and 1.

**6.2. Operations on complex fermatean pentapartitoned neutrosophic relation**

Now, let's introduce some fundamental CFPNS operations, like complement, inverse, and composition of CFPNRs. We will begin by presenting a definition for the complement of CFPNR.**Definition 6.3**

Consider R to be a CFPNR on, where

.Then, the complex fermatean pentapartitioned neutrosophic complement relation R represented by, and it can be written as

, where

 = = .

 = =

 = =

 = =

 = .=

The definition of a CFPNR's inverse as well as a preposition about it will be discussed next.

**Definition 6.4.** Let R be a CFPNR from X to Y. The inverse of R is represented by and is a CFPNR from Y to X, then it can be written as

 .

Where and.

.

**Preposition 6.5**

Let X and Y be two CFPNSs over U and V, correspondingly. Assume that two CFPNS from X to Y

are R and S. Then, the following outcomes are valid:

If then.

Proving and, we have

,

Where,

,,

,. This implies that.

If, then

,

Likewise, we can demonstrate that

,

,

.

This section presents an example that demonstrates the practical use of the axiomatic definition of the composition of CFPNRs. Next, we present two theorems on the composition idea.

**Definition 6.6.**

Over the related universes U, V, and W, let X, Y, and Z denote three CFPNSs. Assume that S is a CFPNR connecting Y and Z, and let be a CFPNR connecting X and Y. A CFPNR spanning from X to Z can be defined as the combination of CFPNRs R and S in the following manner:

 and, = ,

Where and ,

= ,

Where and,

= ,

Where and ,

= ,

Where and ,

= ,

Where and ,

This association can be written as.

The application of the CFPNRs' composition in practical situations is shown in the example that follows.

**Example 6.7.** Assume that X, Y, and Z are three CFPNSs that, in turn, reflect the sets of financial and public opinion indicators from Malaysia, China, and Malaysia. Let us assume that the CFPNRs R and S are used to measure the interactions between these sets over a 12-month period. R(X,Y) denotes the impact of Chinese financial indicators on Malaysian financial indicators, while S(Y,Z) represents the impact of Japanese financial indicators on Malaysian public perception indicators.

 A new CFPNR T(X, Z) is created by combining CFPNRs R(X, Y) and S(Y, Z), and it shows how Malaysian public opinion indicators are impacted by Chinese financial indicators. For the sake of illustration, it will suffice in this example to consider the composition of the following two approximations in the CFPNRs R(X, Y) and S(Y, Z).

(,

Where u ϵ X and v ϵ Y stand for, respectively, the Malaysian inflation rate and the Chinese Yuvan trade. This approximation assesses the degree and phase (period) of the Chinese Yuvan's exchange rate's influence on Malaysia's inflation rate, accounting for truth, contradiction, ignorance, unknown, and misleading information.

 (,

where w ϵ Z and v ϵ Y stand for the trust in the Malaysian economy and the country's inflation rate, respectively. This approximation assesses the degree and phase (period) of the influence of the Chinese Yuvan exchange rate on the confidence in the Malaysian economy, accounting for truth, contradiction, ignorance, unknown, and misleading information. The result of this composition is:

The components measure respectively the truth, the contradiction, ignorance, unknown and falsity for both degree and phase (period) of the influence of the inflation rate in Malaysia on the Confidence in the Malaysian financial system.

 Considering Definition 10, we demonstrate the subsequent outcomes.

**Theorem 6.8.**

Let X, Y, and Z represent three types of complex fermatean pentapartitioned neutrosophic sets over the respective universes U, V, and W. A CFPNR from X to Y is denoted by R, and a CFPNR from Y to Z by S. Next.

Proof: For all and, Let

,

and

To demonstrate the equality, we have to prove that,,

, and.

Therefore,

 = max [ = max [

 = max [min (, min (

 = max [min (, min (

 = max [

 =,

Which implies =, similarly we can show that =, proving that

=.

In a similar way, we can demonstrate that it also holds for the terms contradiction, ignorance, unknown, and falsity, completing the proof.

**Theorem 6.9** Let X, Y, Z and W be CFPNSs over the universes U, V, L and M respectively. Let R be a CFPNR from X to Y, S a CFPNR from Y to Z and T a CFPNR from Z to W. Then.

Proof. For all and , . Let

.

To prove the equality, we have to show that,

, and.

Therefore , = max [ = max [

 = max [min (, max [min (,

 = max [min (, min (

 = max [max [min (

 = max [max [,

 = max [,

 = ,

Which implies that. Similarly, we can show that , proving that .

This completes the evidence because the proofs for the terms contradiction, ignorance, unknown, and falsity can all be demonstrated in a similar manner..

 To gain a better understanding of this topic, we define projection and CFPNSs.

**Definition 6.10.**

Let U and V be two universe and R be a CFPNR on. Then for all and

The projection of R on U is a CFPNS. Defined respectively, by the complex valued truth, contradiction, ignorance, unknown and falsity membership functions:

(u) =,

(u) =

(u) =

(u) =

(u) =

The projection of R on V is a CFPNS , named respectively by the complex valued truth, contradiction, ignorance, unknown and falsity functions:

(v) =,

(v) =

(v) =

(v) =

(v) =

**Example 6.11.** Let and be two universes. Let R be a CFPNR on defined as follows:

The projection of the CFPNR R on U is given by:

(u)=

(u)=

The projection of the CFPNR R on V is given by:

(v)=

(v)=

**7. Conclusion:**complex valued membership functions for truth, contradiction, ignorance, unknown, and falsehood characterize a complicated fermatean pentapartitioned neutrosophic set. Consequently, a complex valued truth membership function is just a regular truth membership function plus one more term.

The additional term is referred to as the phase term, while the conventional truth membership function is known as the truth amplitude term. Thus, uncertainty is represented by the truth amplitude term in this sense, and periodity in the uncertain state is represented by the phase term. Therefore, uncertainty with periodicity as a whole is represented by a complex-valued truth membership function. Comparably, a complex-valued membership function for contradiction denotes a periodicity, while a complex-valued membership function for ignorance denotes a periodicity, a complex-valued membership function for unknown denotes a periodicity, and a complex-valued membership function for falsehood denotes a periodicity for falsity. In this study, we also explored various fundamental features of sets, including complement, union, intersection, complex fermatean pentapartitioned neutrosophic product, and cartesean product. Additionally, we provided an interpretation of the complex fermatean pentapartitioned neutrosophic set. Additionally, we have explored here the δ– equalities of complex fermatean pentapartitioned neutrosophic sets. The CFPNSs are derived, examined, and utilized in this study to explain and address a real-world decision-making challenge. Before defining the CFPNR, the Cartesian product between two CFPNSs must be defined. Next, we introduced several basic operators on the CFPNR, including the inverse and complement of the CFPNR. The axiomatic definition of CFPNR composition is also provided, along with an example that shows how to apply this idea to combine two CFPNRs to obtain practical information. We deduced certain properties using an instructive example and offered some theorems on the previous operation. Additionally, the notion of projection for CFPNRs is described and given examples.

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